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PHASE SHIFT ANALYSIS OF $\pi^-p \rightarrow K^0\Lambda$

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A B S T R A C T

We have performed a phase shift analysis of $\pi^-p \rightarrow K^0\Lambda$ at twenty energies below 2 GeV/c. After extensive random searches, the single-energy solutions found were linked by a shortest path technique with smoothing. S_{11} and P_{11} resonances near 1700 MeV mass are definitely required, with an optional P_{13} or D_{13} resonance. The well-known D_{15} and F_{15} resonances decouple from $K\Lambda$. We also give details on the method, important for appreciating recent K^+p work.

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1. INTRODUCTION

This is a revised version of our 1968 phase shift analysis ¹⁾ of $\pi^- p \rightarrow K^0 \Lambda$. The random searches have been completely redone with much new data, and an improved shortest path program used which gives smoother solutions. However our conclusions are the same.

In the last few years πp phase shift analyses have led to the discovery of many new N^* resonances ²⁾. Since they are all very inelastic, it is natural to look at coupled channels for confirmation. The $K\Lambda$ channel is especially favourable, since its two particles make true phase shift analysis possible, while the Λ decay automatically provides polarization data. Even before 1968, there had been rumours for many years of inexplicable resonances in $\pi p \rightarrow K\Lambda$, but these were based on model fits ³⁾ rather than proper phase shift analyses. Typically they considered K^* and u -channel exchange plus contributions from older N^* resonances, especially the F_{15} .

In 1968 we concluded ¹⁾ that the well-known $F_{15}(1690)$ and $D_{15}(1680)$ resonances hardly contributed at all to the $K\Lambda$ channel, which was instead dominated by their $S_{11}(1700)$ and $P_{11}(1750)$ grand-daughters. Independently, Deans et al. ⁴⁾ performed a resonance fit using the newer N^* 's and concluded that $S_{11}(1700)$ and $P_{11}(1470)$ were most important. We think we can trace the precise reason for the small $P_{11}(1750)$ in their analysis: their parametrization did not allow for a rotation of the resonance circle by interference with background. A relative rotation between the S_{11} and P_{11} resonances is definitely needed, since these waves must stay 90° out of phase near 1700 MeV to keep the polarization ~ 1 . The nearest their computer could get, was to put all the P_{11} into the tail of the Roper resonance, thus giving this an anomalously large $K\Lambda$ coupling, very difficult to reconcile with $SU(3)$ (as they remark). It also makes the polarization too big near threshold. Subsequent energy-dependent phase shift analyses have been performed by Orito and Sakai ¹⁷⁾ and Orito ¹⁸⁾, concluding that $S_{11}(1700)$ and $P_{11}(1750)$ indeed dominate. Two analyses of $\gamma p \rightarrow K^+ \Lambda$ have led to the same result ¹⁹⁾. Thus our heretical S_{11} - P_{11} dominant theory of the $K\Lambda$ channel has become orthodox in the

two years between the preliminary ¹⁾ and final version of this paper. However, the energy-dependent analyses mentioned do not seem to have used the most recent $\pi^- p \rightarrow K^0 \Lambda$ data.

Since this is the first long paper on any CERN II phase shift analysis, we have given a general account of the method. This is relevant also for $K^+ p$ work. Section 2 describes the single energy searches, Section 3 the shortest path linkage.

2. SINGLE ENERGY SEARCHES

First we collected all available $K\Lambda$ data ⁵⁻¹⁶⁾ below 2 GeV/c, and arranged it into 20 energies (Table 1). These go from 1617 to 2137 MeV total mass, with rather uneven spacing. At each energy we made an exhaustive phase shift analysis. These were purely experimental, since dispersion relation predictions would be difficult in the $K\Lambda$ channel because of the large unphysical region, while there are too many anomalies for Regge fits to be reliable here yet in our opinion. Since we wanted independent evidence about N^* resonances, we avoided using πN information, except in two respects :

a) We neglected G_{19} and H waves, since they are observed to be elastic in πN phase shift analysis ²⁾. For the same reason, we neglected F_{17} and G_{17} below 1810 MeV mass. At 1617 MeV, we only included S and P waves because of the centrifugal barrier, and at 1627, 1641 MeV only S, P, D waves.

b) In an inelastic channel, there is no optical theorem. Therefore all partial waves can be multiplied by the same over-all phase, without changing any observable. This ambiguity can never be resolved experimentally. However, for a pure Breit-Wigner resonance, the ratio of the amplitudes in any two channels is real. Now in πN phase shift analyses, $D_{15}(1680)$ and $F_{15}(1690)$ have very regular Breit-Wigner shapes ²⁾. We therefore required the $K\Lambda$ amplitudes for these waves to be real multiples of the πN up to 1810 MeV ^{*)}. Above this mass,

*) This does not mean the $K\Lambda$ phase is the same as the πN δ , since zero is at the bottom of the unitary circle for an elastic amplitude, but in the middle for an inelastic.

we used G_{17} in the same way. The values of this real multiplier were varied independently at the different energies. This leaves a sign ambiguity, which we did not attempt to settle except by continuity. Each of our solutions can be entirely multiplied by -1 . Columns 4-5 of Table 1 summarize which waves were varied at each energy.

Each angular experiment was assigned a normalization variable x , and these were varied in all the fits. This means that all differential cross-section (d.c.s.) data from those authors were multiplied by e^x , while $(x/E_n)^2$ was added to χ^2 . Here E_n is the normalization error, which ranged from 0.04 for good new experiments up to 1 for unnormalized old data. This is important for $K\Lambda$, since there is some conflict and uncertainty about the integrated cross-section. Where experimentalists quoted polarization times d.c.s., this was renormalized simultaneously, but not where the d.c.s. had been divided out. However, the normalization of all polarization data depends on α_Λ , which we fixed at $\alpha_\Lambda = 0.645$, renormalizing when an experimentalist had assumed a different value. We took considerable trouble to disentangle the sign conventions of the earlier experiments. The publication habits of some bubble chamber groups also make double-counting a non-trivial problem.

To preserve the unitarity limit, each partial wave was parametrized by

$$T_{l\pm} = re^{i\theta} / (1 + r^2), \quad (1)$$

where r and θ are variables for that wave and energy. The d.c.s. and polarization are then given by standard meson-baryon formulae ²⁰⁾, taking χ^2 from the initial channel.

The single energy searches were performed by minimizing χ^2 by a variable metric technique ²¹⁾, from a large number of different starting points. The program was SEARCH II (written by C. Lovelace and F. Wagner, with earlier help from J. Iliopoulos). It has also been used in recent CERN K^+p phase shift analyses ^{22),23)}, and in coupled channel fits to $\bar{K}N$ at Munich ²⁴⁾. For $K\Lambda$ each complete minimization took about one

second on a CDC 6600, except where there was a lot of data. In the 1968 analysis, anticipating criticism of our conclusion that F_{15} and D_{15} were small, we had random searched with unprecedented thoroughness (over 1000 trials at 1691 MeV alone). In the present analysis, we used ~ 300 -500 starting points at each energy (8578 in all), consisting of all solutions from 1968 plus many more random starting points (Table 1, column 6).

It is quite exceptional for a phase shift analysis to give well-defined distinct minima of χ^2 . The usual situation, which certainly occurs at all energies for $K\Lambda$, is to have long winding valleys with local wobbles in their floors. Neither the minimum nor its statistical error is then of much physical significance - what really matters is the χ^2 acceptance contour on the valley wall. Phase shift analysis depends on the dimensionality of the valley floor being small compared to the total number of parameters. The valleys at different energies will be slanted relative to each other, so continuity then selects particular points in them, as giving simple solutions as a function of energy. Since different partial waves are always highly correlated, this valley structure will not be evident to the eye on Argand diagrams of the single energy solutions. Looking at the errors on separate waves, greatly underestimates the data. However, the relation of the valleys at different energies can be explored mechanically by the shortest path program. Our single-energy searches were therefore designed to feed it with a complete representation of the acceptable χ^2 region at each energy. It should be evident from what we have just said that the minima and their associated statistical errors do not give a complete representation. We would feel very suspicious of any shortest path analysis which relied on minima (i.e., any not done with the CERN programs), unless the single energy solutions were already almost unique.

We therefore stopped minimizing χ^2 as soon as it reached an acceptable level [$P(\chi^2) \geq 0.02$ usually], and counted the result as a distinct solution if it was sufficiently far from all previous solutions. However, if a minimum was reached with $10^{-4} \leq P(\chi^2) < 0.02$, we counted this as a solution also, to guard against systematic errors in the data. The shortest path program has provision for a subsequent χ^2 cut if required. To decide whether two solutions were distinct, we calculated their distance

$$d(a,b) = 100 \left[\sum_{\ell,j} \frac{1}{2} (\ell+j+\frac{1}{2}) |T_{\ell,j}^{(a)} - T_{\ell,j}^{(b)}|^2 \right]^{\frac{1}{2}}, \quad (2)$$

and considered them identical if $d(a,b) < d_0$, where d_0 is a certain minimum distance. Obviously the smaller d_0 , the more distinct solutions will be found. The total number of solutions at each energy is restricted to 200, by space considerations in the shortest path program. Therefore the best policy is to choose d_0 at each energy such that the number of distinct solutions there is exactly 200. We arranged this by doing the random searches in batches, and adjusting d_0 between, so that about half the solutions found were continually rejected as duplicate. [The proportion rejected because $P(\chi^2) < 10^{-4}$ should be small if the data are consistent, and the minimization program is working efficiently.] As the solution store fills up, it gets more difficult to find really different ones, and d_0 has to be made smaller. The measure of the uniqueness of each energy is then not the total number of solutions found, but rather its final value of d_0 (last column of Table 1).

At the end of this process, we therefore have a "quantized" representation of the acceptable χ^2 region at each energy by 200 points. These fill the region $P(\chi^2) \geq 0.02$. By rejecting ~ 200 more solutions as duplicates, we have ensured that our selected ones are evenly distributed over the whole of this acceptable region. The χ^2 probability of each selected solution is remembered, along with its partial waves. Also outlying minima with $10^{-4} \leq P(\chi^2) < 0.02$ are kept. Requiring these bad solutions to be minima, means that there will be relatively few of them among the 200.

At 1683 MeV mass the data are inconsistent, because of obvious conflicts in the integrated cross-section. Some ^{9),16)} but not all ^{8),11)} experiments claim to see a narrow bump at this energy. In this case, we included both sets of data and accepted all solutions with χ^2 less than 3.5 times its expected value. They can then choose one experiment or the other, or compromise. In practice all shortest paths ran through solutions with no spike in the integrated cross-section. This does not

exclude a spike - shortest path is obviously biased against them - but we do not think existing experiments sufficient to establish it.

In the 1700 MeV mass region, the important partial waves are S_1 , P_1 , P_3 and D_3 . There does not even exist any solution at energies with good data in which the F_{15} or D_{15} resonances dominate, let alone a shortest path.

3. ENERGY CONTINUATION

The shortest path method ²⁵⁾ is the best for imposing continuity in the energy. A path is a series of solutions, one at each energy, starting from zero at threshold (1613 MeV mass). Equation (2) also defines a distance between two solutions at neighbouring energies. Each solution at the highest energy (2137 MeV) will have a shortest path leading up to it, according to this distance formula. It can be constructed by iteration (see below). In practice it turns out that the shortest paths to all 200 solutions at the top energy have a common tree structure, passing through the same solutions at lower energies. If the paths differ only at the last five energies or less, we count them as the same and choose the shorter. By this definition, there are usually only about 5-20 distinct shortest paths among the 200 possible. Even these usually branch from each other at lower energies. The tree of shortest paths also has "dead branches". These are shortest paths up to some solution at an energy below the top, which are distinct from all shortest paths to the top energy. When constructing the shortest path tree, we also record and examine all branches which died out after being distinct for at least five energies. These dead branches can be continued by "planting". This means that we remove all other solutions at the lower energies, thus forcing all shortest paths to the top energy to pass through this branch (or a selected part of it).

The shortest path method produces those solutions which have the least possible energy dependence. Our distance formula (2) will penalize jumps in high waves more than in low ones. This is in accord both with the centrifugal barrier, and with the relative size of their statistical

errors. However, it is desirable to penalize wobbles as well as jumps. We do this by a smoothing method, which is a little complicated to explain. Take a solution p at the N^{th} energy. Let q be a shortest path up to the $(N-1)^{\text{th}}$ energy, with length $s(q)$. Now take the last K energies on q , and do linear fits (in the mass) to the real and imaginary parts of each partial wave. Thus extrapolate the path q to the N^{th} energy. Call this extrapolated solution $E(q)$, and let $r(p,q)$ be its distance from p . Next calculate $\text{Min}_q [s(q)+r(p,q)]$, over all paths q to solutions at the $(N-1)^{\text{th}}$ energy. This gives the smoothed shortest path to p at the N^{th} energy, and its length $s(p)$. Taking $K=1$ (the number of previous energies used in the extrapolation) gives shortest paths with no smoothing. Increasing K , selects for smoothness as well as continuity. The resulting reduction of wobbles is very noticeable in the Argand diagrams of the shortest paths. If the energies are closely spaced, we found $K=3$ or 4 the optimal value. For K too big, the paths develop sudden jumps between long straight sections. If energies are far apart, $K=1$ or 2 is better. Note that the extrapolation is always done by a linear fit. By choosing $K > 2$, we make this overdetermined. This stops the shortest path from responding to statistical fluctuations at a single energy. Of course, it is easy to think of more elaborate extrapolation formulae, but we wanted to avoid anything which would impose resonance circles.

This shortest path construction was done by the CERN program SLINK IV (written by F. Wagner and C. Lovelace). The 1968 $K\Lambda$ analysis ¹⁾ and the published K^+p work ²²⁾ used an earlier version with only $K=1$ or 2 smoothing. The technical problem in shortest path programs is space rather than time. Ours was designed to handle 200 solutions at each of 60 energies, with each solution consisting of 15-70 numbers.

If the experiments are consistent, the smoothness of the final shortest paths depends also on the close packing of the single energy solutions. By this criterion, we have always found 200 points adequate at energies where polarization and d.c.s. are both measured, as always in $K\Lambda$. In other processes, if there is no polarization at one energy, a good trick is to use solutions at adjacent energies as starting points for the χ^2 minimization. The 200 solutions resulting will then be the ones most disposed to continuity.

If the single-energy searches were inadequate, it may be impossible to find any reasonable looking path. It seems to us that this happened in the $K\Lambda$ phase shift analysis of Doyle ⁸⁾, who used only 2-6 solutions at each energy in his shortest path analysis (as against our 200). Even with very thorough searches, and with $K > 2$ smoothing, the shortest path method does not protect against clockwise circles, large zigzags or slow hairpin bends. Almost always, a certain proportion of shortest paths can therefore be discarded immediately on physical grounds. The less accurate and complete the experiments, the greater the number of unreasonable shortest paths, and the bigger the probability that the true one will be killed by competition before it gets to the top energy. This can be partly overcome by "planting dead branches", as described above, which looked reasonable at the lower energies. This increases the total number of shortest paths, and thus the survival chances of physical ones. In this respect, $K\Lambda$ is worse than K^+p where, in the Reggeized analysis, the shortest path of all is the most plausible solution physically ²³⁾, but very much better than K^-p where in 1968 we found the experiments so indeterminate that no amount of gardening could produce anything sensible. Of course, this selection of physically reasonable solutions introduces subjective bias. The big advantage of the shortest path technique is that the bias comes only at the end. People who do energy-dependent fits, never find anything that they did not anticipate finding when they wrote the program. In practice, it seems to us that shortest path has proved the most effective way of constructing alternatives.

In $K\Lambda$, the shortest path tree for $K = 3$ smoothing had five distinct paths complete up to 2137 MeV, plus a further eight branches which died out between 1688 and 2137 MeV. For $K = 4$ smoothing, we got nine live and eleven dead branches. Unfortunately all these live branches contained zigzags, hairpin bends or big clockwise circles, though distorted S_1 and P_1 resonances were apparent in all the readable ones. At first we suspected the inconsistent data at 1683 MeV. However, omitting this energy completely had no noticeable effect, so we put it back. The trouble is definitely due to the large and sudden gap in the experiments between 1694 and 1741 MeV mass, which "derails" the shortest path program. To overcome this, we found that $K = 4$ smoothing

below the gap, $K = 1$ (no smoothing) while crossing it, and $K = 3$ above, gave best results. To get more alternatives, we first examined the twenty distinct paths up to 1694 MeV, and selected the six which were tidy that far. Then we "planted" each of them separately (see above). This gave us fifty-seven paths to 2137 MeV, of which ten looked entirely reasonable physically.

All ten contain resonances in S_1 and P_1 . Furthermore, all the others rejected as unphysical also have strong structure in these waves, but compressed sideways or otherwise deformed. The choice for S_1 and P_1 is between circles and cigars, definitely not between circles and straight lines. These ten solutions can be divided into three classes, according to the behaviour of P_3 and D_3 . Type A (three solutions) have a P_3 resonance, type B (five solutions) a D_3 resonance, while type C (two solutions) have neither. D_5 and F_5 are small always. Because of this, our Breit-Wigner constraints were not very effective in resolving the over-all phase ambiguity. In fact, the various solutions within each type differ mainly by an over-all phase rotation (not always 180°). The neatest solution from each class is shown in Fig. 1. The numbers in Table 2 are their real and imaginary parts, in units of 10^{-2} times the diameter of the unitarity circle for $\pi^-p \rightarrow K^0\Lambda$. To get the $I = \frac{1}{2}$ isostate, multiply by $\sqrt{\frac{3}{2}}$. The second column of Table 2 is the χ^2 probability in the form $10 + \log_{10} P(\chi^2)$. Thus eight corresponds to $P(\chi^2) = 0.01$.

Typical data fits are shown in Figs. 2-3. Both differential cross-sections and polarizations are very similar for all three solutions, except sometimes the forward d.c.s. It may not be easy to resolve this ambiguity by improved experiments. Comparison with Regge theories seems more hopeful, since the predicted A and B amplitudes are drastically different at the higher energies for the three types *).

*) We warn against comparing Regge fits with shortest path solutions at their top energy, since they are most arbitrary there. Rather they should be compared as a series of energies, and only those features of the shortest path solution which are stable with energy considered. Also beware of the over-all phase uncertainty in inelastic channels.

Figure 4 shows the integrated $K\Lambda$ cross-section. In all three solutions, the shortest path method takes advantage of the experimental normalization errors to minimize the cross-section. Thus, as usual in phase shift analysis, we get a lower bound on the structure. Since we are far from the unitarity limit in any wave, the fit to Fig. 4 could be improved by scaling the whole solution.

Shortest paths are meant for proving the existence, spin and parity of resonances, and not for deducing accurate masses and widths. Energy-dependent fits are certainly better for the latter. Therefore we have simply calculated Table 3 by dividing the resonance circles into quadrants, as shown in Fig. 5. We give mass E_r , width Γ_{tot} , diameter d , and phase rotation ϕ by background. There is no evidence for any other background. The width is averaged over the upper and lower sides, which often show big differences. For comparison, we have quoted Orito's energy-dependent fit ¹⁸⁾, which is clearly our solution A. This gives some idea of distortion due to the high threshold in the final channel. The diameter d of the resonance circle is related to the branching fraction by

$$d = \sqrt{\frac{2}{3}} \left[\Gamma_{K\Lambda} \Gamma_{\pi N} \right]^{\frac{1}{2}} / \Gamma_{tot}, \quad (3)$$

where $\sqrt{2/3}$ comes from initial state isospin.

Using known πN parameters ²⁾, these three shortest paths give upper limits for the $K\Lambda$ partial widths of the $D_{15}(1680)$ and $F_{15}(1690)$ resonances

$$\begin{aligned} D_{15} &: \Gamma_{K\Lambda} = (.2 \pm .2) \text{ MeV}, \\ F_{15} &: \Gamma_{K\Lambda} = (.1 \pm .1) \text{ MeV}. \end{aligned} \quad (4)$$

Allowing for the strong centrifugal barrier, these are not ²⁶⁾ in disagreement with $SU(3)$.

4. CONCLUSIONS

At low energies, the process $\pi^- p \rightarrow K^0 \Lambda$ is dominated by twin S_{11} and P_{11} resonances near 1700 MeV mass. Solutions exist (A, B, C) in which these are supplemented by either a smaller P_{13} or D_{13} resonance, or neither. In contrast the well-known F_{15} and D_{15} resonances are very weakly coupled. As quoted in the last column of Table 3, πN phase shift analyses ²⁾ have produced S_{11} , P_{11} and P_{13} resonances in this region, and there are indications of something in D_{13} . However, except for the D_{13} , the total widths deduced from $K\Lambda$ always seem narrower than the πN ones, even allowing for the higher threshold. Solution C shows signs of a D_{13} resonance at higher mass, which could be $D_{13}(2050)$.

The $K\Lambda$ channel can be recommended to people who want to study daughter resonances. The most urgent new experiments required are :

- (I) - The disagreement in the integrated cross-section around 1690 MeV mass (Fig. 4) should be resolved. A narrow spike has been claimed here ¹⁶⁾, which does not correspond to anything in the present analysis. In fact, bad normalization is an important cause of $K\Lambda$ uncertainty over the whole energy range.
- (II) - The gap between 1694 and 1741 MeV mass should be filled. This is particularly unfortunate for the shortest path method, since it is near the tops of the resonances.
- (III) - Experiments at the higher energies need to be more closely spaced and accurate, especially forward and backward to determine high waves and join to Regge fits. Only our solution C has appreciable coupling of the $G_{17}(2200)$ resonance, but it is impossible to tell the difference with the present data (Fig. 3).
- (IV) - More accurate polarizations near threshold will be needed to deduce good parameters for the P_{11} resonance, since it is obviously distorted by the final state centrifugal barrier.

We are grateful to Drs. T.O. Binford and C.J. Doyle for sending us their data ^{7), 8)} before publication.

Mass MeV	P _{lab} MeV/c	Data References	Phase and modulus varied	Modulus only varied	Trials	d ₀
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1617.5	904	5, 6	S ₁ , P ₁ , P ₃	-	380	2
1626.6	920	6, 16	S ₁ P ₁ P ₃ D ₃	D ₅	450	2.5
1640.8	945	5, 8, 9, 16		250	2.5	
1652.7	966	5, 6, 8, 16		310	4	
1663.9	986	5, 8		414	6	
1666.7	991	8, 16		D ₅	648	4
1675.0	1006	5, 6, 8		404	5	
1682.8	1020	8, 9, 11, 16	D ₃ F ₅	F ₅	449	3.5
1688.3	1030	8, 10		454	3	
1693.8	1040	5, 8, 9, 16		475	2.5	
1741.5	1128	7, 9		677	1.5	
1763.8	1170	8, 12, 16		355	2.5	
1797.3	1234	7, 9, 16		321	5	
1818.9	1248	7	S ₁	G ₇	375	8
1856.5	1350	7, 9, 12	P ₁		363	5
1930.5	1500	12, 13, 14	P ₃		469	8
1978.3	1600	14, 15	D ₃		253	6
2025.0	1700	14	D ₅		486	10
2097.6	1860	14	F ₅		444	10
2137.4	1950	14	F ₇		601	10

- Table 1 -

Single-energy searches

Mass	$10+\log P(\chi^2)$	S_1	P_1	P_3	D_3	D_5	F_5	F_7	G_7
1613	0.060	0	0	0	0	0	0	0	0
1617	9.698	5	-1	0	0	0	0	0	0
1627	8.331	7	0	0	0	0	0	0	0
1641	9.696	9	1	1	0	0	0	0	0
1653	7.692	14	2	2	1	0	0	0	0
1664	6.444	17	0	4	1	0	0	0	0
1667	7.223	16	8	3	2	0	0	0	0
1675	6.762	14	9	5	2	0	0	0	0
1683	6.000	13	-3	6	1	0	0	0	0
1688	7.982	10	-4	1	-2	2	0	0	0
1694	7.641	14	-7	4	1	0	1	0	0
1741	6.545	4	-10	6	0	0	0	0	0
1764	8.397	-2	-7	6	0	0	0	0	0
1797	8.608	-5	-8	6	0	1	0	0	0
1819	8.173	-6	-8	4	-3	0	0	0	0
1857	7.009	-6	-8	1	-2	0	0	1	-1
1930	6.508	-10	-6	-1	-2	1	-2	0	0
1978	9.248	-5	-5	0	0	2	0	2	-1
2025	8.375	-2	-2	-6	1	0	-2	0	0
2098	6.334	-5	-2	-9	0	0	-1	-1	0
2137	5.659	-5	-1	-8	1	-1	-1	-2	-1
			0	-9	0	-1	0	0	-2

Solution A

- Table 2A -

Table 2 : 10^2 X real and imaginary parts of partial waves

Mass	$10+\log P(\chi^2)$	S_1	P_1	P_3	D_3	D_5	F_5	F_7	G_7
1815	6.000	0,	0,	0,	0,	0,	0,	0,	0
1817	9.596	5,	-1,	0,	0,	0,	0,	0,	0
1827	6.331	7,	0,	0,	0,	0,	0,	0,	0
1841	9.698	9,	2,	1,	0,	0,	0,	0,	0
1853	7.692	14,	0,	2,	1,	0,	0,	0,	0
1864	6.444	17,	0,	3,	2,	0,	0,	0,	0
1867	7.223	16,	-1,	5,	2,	0,	0,	0,	0
1875	8.762	14,	-3,	6,	1,	0,	0,	0,	0
1883	0.000	13,	-4,	1,	-2,	0,	0,	0,	0
1888	7.440	6,	-6,	-2,	-4,	0,	0,	0,	0
1894	7.684	6,	-11,	-2,	-6,	0,	0,	0,	0
1741	6.677	2,	-11,	-1,	-6,	1,	0,	0,	0
1764	8.515	2,	-14,	-1,	-5,	1,	0,	0,	0
1797	6.361	-3,	-14,	-1,	-5,	0,	0,	0,	0
1819	7.675	-8,	-10,	-2,	0,	1,	0,	0,	0
1857	7.146	-9,	-7,	0,	-1,	0,	-1,	1,	0
1930	8.432	-6,	-2,	-2,	0,	4,	0,	0,	0
1978	9.378	-4,	0,	-2,	-2,	4,	0,	0,	0
2025	6.395	0,	1,	-2,	0,	2,	0,	-1,	0
2098	6.795	-1,	2,	-2,	1,	-2,	0,	-2,	0
2137	7.317	-1,	2,	-2,	1,	1,	1,	-2,	-1

Solution B

- Table 2B -

Mass	$10+\log P(\chi^2)$	S_1	P_1	P_3	D_3	D_5	F_5	F_7	G_7
1613	0.000	0,	0,	0,	0,	0,	0,	0,	0,
1617	9.696	5,	-1,	0,	0,	0,	0,	0,	0,
1627	8.331	7,	0,	1,	0,	0,	0,	0,	0,
1641	7.698	9,	2,	2,	1,	0,	0,	0,	0,
1653	7.892	14,	6,	-2,	0,	0,	0,	0,	0,
1664	8.444	-2,	7,	-2,	0,	0,	0,	0,	0,
1667	7.223	-1,	0,	-2,	0,	0,	0,	0,	0,
1675	8.762	16,	9,	-2,	0,	0,	0,	0,	0,
1683	0.000	14,	11,	0,	0,	0,	0,	0,	0,
1686	7.869	6,	13,	0,	3,	2,	0,	0,	0,
1694	7.291	13,	-4,	1,	-2,	0,	0,	0,	0,
1741	6.436	10,	-3,	0,	-2,	0,	0,	0,	0,
1764	8.330	14,	10,	0,	1,	0,	1,	0,	0,
1797	8.780	12,	-9,	2,	-2,	0,	-1,	0,	0,
1819	8.202	7,	-10,	1,	-2,	0,	0,	0,	0,
1857	7.102	10,	-13,	2,	-5,	0,	0,	0,	0,
1920	8.326	3,	-13,	-1,	-2,	-1,	0,	0,	-2,
1978	9.084	2,	-10,	0,	-5,	1,	0,	1,	-2,
2025	8.454	6,	-2,	-3,	-3,	-2,	0,	0,	0,
2098	8.401	1,	-2,	-4,	-2,	0,	0,	1,	-2,
2137	6.116	0,	0,	-2,	-7,	0,	-1,	0,	-2,
		2	-1,	-3,	-6	-2,	-2,	-1	-5

Solution C

- Table 2C -

Solution		Orito	A	B	C	τ N
S ₁₁	E_R	1712	1685	1685	1685	1700
	Γ_{tot}	131	110	140	140	300
	d	0.20	0.19	0.17	0.17	0.8
	ϕ		50°	40°	40°	
P ₁₁	E_R	1653	1685	1740	1740	1750
	Γ_{tot}	245	160	180	220	350
	d	0.14	0.13	0.17	0.13	0.3
	ϕ		120°	140°	130°	
P ₁₃	E_R	1868	1800			1850
	Γ_{tot}	372	220			300
	d	0.09	0.10			0.2
	ϕ		60°			
D ₁₃	E_R			1780		1700 ?
	Γ_{tot}			160		50 ?
	d			0.08		0.2 ?
	ϕ			-130°		
D ₁₃	E_R				?	2050
	Γ_{tot}				?	300
	d				0.07	0.25
	ϕ				180°	
G ₁₇	E_R				?	2200
	Γ_{tot}				?	300
	d				0.06	0.35
	ϕ				180°	

- Table 3 -

K $\wedge$ Resonance Parameters

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FIGURE CAPTIONS

Figure_1 Argand diagrams of our solutions A, B, C. The boundary is half the $\pi^-p \rightarrow K^0\Lambda$ unitarity limit.

Figure_2 Fits at 1688 MeV (1030 MeV/c), against data of Refs. 8) and 10). (a) differential cross-section, (b) polarization times d.c.s.

Figure_3 Fits at 2097 MeV (1860 MeV/c), against data of Ref. 14). (a) differential cross-section, (b) polarization times d.c.s.

Figure_4 Fits to integrated $\pi^-p \rightarrow K^0\Lambda$ cross-section, against all data.

Figure_5 How our resonance parameters (Table 3) were deduced from the Argand circles.

Figure_6 Speed of the S_1 resonance :

$$S = \frac{1}{2} \left[\frac{|T_{i+1} - T_i|}{E_{i+1} - E_i} + \frac{|T_i - T_{i-1}|}{E_i - E_{i-1}} \right].$$

FIG.1a

SOLUTION C

SOLUTION B

SOLUTION A

SOLUTION A

SOLUTION B

SOLUTION C

FIG.1b

POLARIZATION TIMES DIFFERENTIAL CROSS SECTION
AT 2098 MeV [1860 MeV/c]

FIG.3b

FIG.4

FIG.5

FIG. 6