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LOW ENERGY DATA, ANALYTICITY
AND THE VIOLATION OF POMERANCHUK'S THEOREM

R. Wit ^{*)}
CERN - Geneva

A B S T R A C T

In the case of $\pi^{\pm}$ p scattering the low energy data do not seem to agree with the predictions following from the recently introduced parametrizations violating the Pomeranchuk theorem.

^{*)} On leave of absence from the Jagellonian University, Cracow (permanent address after 1 January 1971).

As realized by many authors ¹⁾ the usual forward dispersion relation for the crossing odd amplitude is not a very powerful tool for analyzing the high energy data. For different (but "reasonable") asymptotic behaviours of the scattering amplitudes the low energy predictions are not distinguishable from each other. Therefore, it is obvious one should look for further, more restrictive constraints on the possible asymptotic behaviour of the total cross-sections.

In this letter we derive an integral representation for the crossing odd amplitude valid only if the Pomeranchuk theorem is violated. Present high and low energy data suggest, however, that this relation is unlikely to be satisfied. For the sake of simplicity we restrict ourselves to the case of π^+p scattering. The analysis given below can be, however, extended to the cases of KN and NN scattering with some obvious changes.

Our interest will be concentrated on the high energy behaviour of the crossing odd amplitude ^{*)} $T_2(k) = (T_-(k) - T_+(k))/2$. Its asymptotic properties are very often discussed in connection with the possible violation of Pomeranchuk's theorem [cf., Ref. 3) and the references contained therein]. In this case the high energy parametrization of $T_2(k)$ takes usually the form ¹⁾

$$T_2(k) = \gamma_- \omega (\ln k - B - i\pi/2) + i\gamma_+ e^{-i\pi/2} \omega^{0.5} \quad (1)$$

where, from the beginning γ_- is assumed to be different from zero (the best fit solutions yield γ_- small and negative).

In what follows we would like to make use of two observations:

i) if (1) holds, then

$$\frac{k}{|T_2(k)|} \underset{k \rightarrow \infty}{\approx} \frac{1}{\ln k} \rightarrow 0; \quad (2)$$

^{*)} We follow here the notation adopted in Ref. 2).

ii) in the large momentum interval $1.65 \text{ GeV}/c \leq k \leq 65 \text{ GeV}/c$ the imaginary part of $T_2(k)$ does not change its sign ⁴⁾. The results of several fits to the existing high-energy data (in the region, say, $10 \text{ GeV}/c \leq k \leq 65 \text{ GeV}/c$) extrapolated to higher energies do not change this property of $\text{Im} T_2(k)$. Therefore, we take for granted the inequality

$$\sigma_2(k) \geq 0 \quad (3)$$

above $k = 1.65 \text{ GeV}/c \equiv k_N$.

Let us recall the most general form of dispersion relation for $T_2(k)$ ⁵⁾

$$\frac{\mu}{\omega} \text{Re} T_2(k) = \text{Re} T_2(0) - \frac{2f^2}{\mu} \frac{k^2}{k^2 + \bar{\mu}^2} + \frac{2}{\pi} k^2 \int_0^\infty \frac{dk' \mu \text{Im} T_2(k')}{k' \omega' (k'^2 - k^2)} \quad (4)$$

where $\bar{\mu}^2 = \mu^2 [1 - (\mu/M)^2]$. This equation follows in fact from the Cauchy formula (valid also for complex values of k) which can be written as

$$\begin{aligned} \frac{T_2(z)}{\omega} + \frac{2f^2}{\mu^2} \frac{z}{z + \bar{\mu}^2} - \frac{\text{Re} T_2(0)}{\mu} - \frac{z}{\pi} \int_0^{k_N^2} \frac{dk'^2 \text{Im} T_2(k')}{k'^2 \omega' (k'^2 - z)} \\ = \frac{z}{\pi} \int_{k_N^2}^\infty \frac{dk'^2 \text{Im} T_2(k')}{k'^2 \omega' (k'^2 - z)}, \quad z = k^2. \end{aligned} \quad (5)$$

There is no difficulty in calculating the left-hand side of this equation for $z = k_N^2$ since $\text{Im} T_2(k_N) = 0$ by definition and the coupling constant as well as the threshold value $\text{Re} T_2(0)$ are rather well known ⁶⁾.

Now, let us introduce an auxiliary function

$$F(z) = -\alpha^2 + \frac{z}{\pi} \int_{k_N^2}^{\infty} \frac{dk'^2 \gamma_m T_2(k')}{k'^2 \omega' (k'^2 - z)}$$

where α^2 is chosen in such a way that $F(k_N^2) < 0$. In this case $F(z)$ does not have any zero and the holomorphy domain for $F(z)$ and $1/F(z)$ is the same. Combining this information with the asymptotic behaviour of $F(z)$ [cf., assumption (2)] one can write the Cauchy formula for $1/F(z)$ in the form

$$-\frac{1}{F(z)} = \frac{1}{\pi} \int_{k_N^2}^{\infty} \frac{dk'^2 \gamma_m F(k')}{|F(k')|^2 (k'^2 - z)} \quad (6)$$

To simplify our discussion we put $z=0$; then

$$\frac{1}{\alpha^2} = \frac{1}{\pi} \int_{k_N^2}^{\infty} \frac{dk'^2 \gamma_m F(k')}{k'^2 |F(k')|^2} \quad (7)$$

The integral in this equation converges but probably rather slowly. Indeed, the integral behaves like $(\sqrt{x} \ln x)^{-2}$ for $x \rightarrow \infty$. Therefore, it may very well happen that the relation (7) will be more sensitive to a proper choice of the parameters describing the high-energy behaviour of $T_2(k)$ than, e.g., to the original dispersion formula (4). Actually, one can interpolate very nicely the existing high-energy experimental points with quite different functional forms of the scattering amplitudes [cf., Refs. 1), 7) and 8)] but the extrapolation to the asymptotic region is far from being unique. Thus, relations similar to (7) may appear to be helpful in rejecting some (otherwise acceptable) models proposed recently for the high-energy analysis of the existing experimental data.

The numerical analysis of Eq. (7) can be performed in the following way. First, we calculate the contribution to the right-hand side of (7) coming from the interval $k_N \leq k \leq 8 \text{ GeV/c}$. We have taken the values of $\text{Re } T_2(k)$ and $\text{Im } T_2(k)$ from the compilations ^{*)}, 9). Then, $\text{Re } F(k)$ can be calculated from the left-hand side of (5). Higher momenta were described by the asymptotic expansion (1) with the parameters taken from Ref. 1a) and from some other independent fits to the data. So, one knows again the behaviour of $\text{Re } F(k)$ from the left-hand side of (5). Calculations were performed for different values of α^2 ; the main result remained the same: the ratio of the right-hand side to the left-hand side of Eq. (7) was at most equal to 0.64. The absolute value of the difference between the left-hand side and the right-hand side depends, of course, on the choice of units [note that the amplitude T_2 appearing in the formula (4) differs by a factor 4π from that used in Ref. 8) and that the transition from the system of units where $\hbar = c = \text{GeV} = 1$ to the natural system of units introduces another factor $\simeq 7.194$].

A more difficult question is what error one assigns to the numerical result given above. Evaluating the integral in Eq. (7) we observe rather strong cancellations among the contributions coming from the left-hand side of Eq. (5). The integral over the interval $0 \leq k \leq 1.65 \text{ GeV/c}$ appears to be large so that the contributions coming from $k \geq 65 \text{ GeV/c}$ in Eq. (7) are almost negligible. Therefore, the way of analyzing the high-energy data proposed in this note seems to be strongly correlated with the low energy input.

Possible deviations from the estimated value of the considered ratio were investigated in the following way. As a guiding relation we have chosen Eq. (7) evaluated at $z = k_N^2$ (the difference between a simple g term contribution and a contribution violating the Pomeranchuk theorem was almost not seen). Varying the values of the coupling constant and $\text{Re } T_2(0)$ between the numbers given in Ref. 6b) we have observed the following general picture: either the guiding relation is fulfilled and Eq. (7) is violated or Eq. (7) is almost satisfied but then Eq. (5) is badly violated (in

^{*)} This procedure is not inconsistent with the remarks made in Refs. 1).

the extreme case of about 200% deviation). Changing the values of f^2 and $\text{Re } T_2(0)$ in a range "allowed" by Eq. (5) we have obtained no more than 15% change in the ratio of the right-hand side to the left-hand side of Eq. (7). Fortunately, the value of $\text{Re } T_2(k_N)$, being close to a local maximum of $\text{Re } T_2(k)$, has a good chance of being well estimated. It is, however, obvious that the analysis described above should be repeated for all energies where $\text{Re } T_2(k)$ was measured, in full analogy with the usual dispersion relations (actually, our conclusion is at the moment essentially a "one point result"). Any disagreement with the data would indicate a non-violation of Pomeranchuk's theorem since, e.g., (6) was derived under the assumption (2) [the present experimental data do not give any indication for a change of sign by $\sigma_2(k)$ at momenta greater than 65 GeV/c]. Therefore, the relation (6) represents in fact a stronger constraint than a similar dispersion formula²⁾ derived for the crossing even amplitude where only the non-vanishing asymptotic limit of the averaged total cross-section $\sigma_1 = (\sigma_- + \sigma_+)/2$ was assumed.

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