CLUSTERS OF GALAXIES
AND THE DIFFUSE GAMMA RAY
BACKGROUND

Sergio Colafrancesco\textsuperscript{a} and Pasquale Blasi\textsuperscript{b}

\textsuperscript{a}Osservatorio Astronomico di Roma, Via dell’Osservatorio 2, I-00040
Monteporzio, Italy

\textsuperscript{b}Dept. of Astronomy & Astrophysics, E. Fermi Inst., University of Chicago,
Chicago, IL 60637-1433, USA

Abstract

We discuss the diffuse emission of gamma rays and neutrinos from galaxy clusters in the viable models for structure formation in the universe. We use a self-consistent picture for cluster formation and evolution starting from a primordial density perturbation spectrum, and a realistic modelling for the distribution of the intergalactic medium which is abundantly present within galaxy clusters. We find that an evolving population of clusters can produce a fraction $\sim 0.5 \div 2\%$ of the diffuse gamma-ray background (DGRB) observed by EGRET. This result is robust and is weakly dependent on the cosmological scenario and on the degree of evolution of the intergalactic medium (IGM) in distant clusters, because the bulk of the sources contributing to the DGRB is located at redshifts $z \lesssim 0.2$. We also found a correlation between the non-thermal, gamma-ray and the thermal X-ray emissions from these structures. Using this result, we derived a list of gamma-ray clusters observable with the next generation $\gamma$-ray detectors. Finally, we briefly discuss the possible relevance of galaxy clusters for neutrino astronomy and for very high energy particle astronomy.

1 Introduction

The EGRET experiment on board CGRO revealed the existence of a diffuse gamma-ray background (hereafter DGRB) at the level of $I_{\text{DGRB}} = 9.6 \cdot 10^{-7} E_{\text{GeV}}^{-2.11 \pm 0.05} \text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{GeV}^{-1}$ [1] in the energy range $0.03 \div 10$ GeV. However, a recent reanalysis of the EGRET data [2] found that the level of the DGRB is systematically lower by a factor $\sim 20\%$ in the energy range $\sim 0.1 \div 4$ GeV. The DGRB is observed at high galactic latitudes $b > 10$ deg.
and such an evidence suggested an extragalactic origin for this diffuse background. Nonetheless, the specific origin of the DGRB is still under debate. In fact, the EGRET experiment [3] has a poor angular resolution ($\theta_{\text{min}} \sim 1$ deg) so that it is hard to discriminate among different origins of this extragalactic background. Specifically, it is still difficult to discriminate between a purely diffuse nature of the DGRB (see e.g. [4]) and the option of a DGRB made by a superposition of unresolved, discrete sources.

The large number of identified AGNs and flat spectrum radio quasars (hereafter FSRQ) in the EGRET sky ([5] [6] [7]) suggested that most of the DGRB can be produced by a non-resolved population of AGNs, the actual fraction of the DGRB produced by FSRQ and BLLacs being in the range $\sim 40 \div 95\%$ (see e.g. [8]). Separately, it has been evaluated that a fraction $\sim 42 \div 97\%$ of the DGRB could be ascribed to blazars [9]. However, the flatness of the spectrum of the DGRB seems to favour the possibility that BLLacs could be the major contributors to the DGRB of extragalactic origin [10]. Erlykin et al. [73] reviewed the various AGN contributions and quoted that the fraction of the DGRB produced by the observed AGNs is $\sim 65\%$. The DGRB fractions previously reported may be subject to a revision ($\sim 25\%$ increase) if the recent reanalysis [2] of the EGRET data is adopted.

On account of the large theoretical uncertainties and of the present observational precision of the EGRET detectors, it is still hard to discriminate among the different proposed possibilities, even though a fluctuation analysis of the EGRET data should give more precise indications on the nature and origin of the DGRB.

Beside the discrete, unresolved source case pictured for the origin of the DGRB, there have been some pioneering works [11] [12] [13] [14] suggesting that a relevant fraction of the DGRB could be produced by extended sources through hadronic collisions of cosmic ray (hereafter CR) protons interacting with the protons of the Inter Galactic Medium (hereafter IGM) which is abundantly present within galaxy clusters (see [15] for a review). In this alternative picture, the CR’s are assumed to be produced within clusters (we will discuss in Sect.4 some of the possible sources) where also a population of protons and electrons is residing in the form of a hot (with temperatures $T \sim 10^7 \div 10^8$ K), tenuous (with electron number densities $n_e \sim 10^{-3}$ cm$^{-3}$), chemically enriched and massive (with mass fractions $M_{\text{IGM}}/M \sim 0.05 \div 0.3$) plasma: the IGM.

The proposed mechanism has an essential ingredient in the confinement of the CR’s within clusters where they are produced; this point, already realized by some authors [13] [14], is responsible for the net increase in the probability of interaction per proton with respect to the case of a straight line propagation. The increase factor can be estimated to be $\sim ct_\text{cl}/R_\text{cl} \gtrsim 600$, where $t_\text{cl} \lesssim H_0^{-1}$ is the age of the cluster and $R_\text{cl}$ is its size. Cosmic rays produced within a cluster
during all its lifetime can thus produce gamma rays through the production and the subsequent decay of neutral pions:

\[ p + p \rightarrow \pi^0 + X, \quad \pi^0 \rightarrow \gamma + \gamma. \]  

(1)

Note that in the same interactions, charged pions are also produced, which determine a neutrino emission through the following channels:

\[ p + p \rightarrow \pi^\pm + X, \quad \pi^\pm \rightarrow \mu^\pm \nu_\mu(\bar{\nu}_\mu), \quad \mu^\pm \rightarrow e^\pm + \bar{\nu}_\mu(\nu_\mu) + \nu_e(\bar{\nu}_e). \]  

(2)

We will also discuss the relevance of these last processes in Section 7 below.

Using the gamma ray production from clusters of galaxies according to eq. (1), Houston et al. [11] suggested that the total extragalactic gamma ray intensity detected above 35 MeV [16], \( I_\gamma \approx 5.5 \times 10^{-5} \text{ cm}^{-2}\text{s}^{-1}\text{sr}^{-1} \), could be ascribed, for a large fraction, to galaxy clusters. They predicted a level \( I_\gamma \approx 5 \times 10^{-5} \text{ cm}^{-2}\text{s}^{-1}\text{sr}^{-1} \) at energies above 35 MeV, assuming an observed local cluster space density, \( n_d \approx 7.3 \times 10^{-5} \text{Mpc}^{-3} \), integrated out to the Hubble radius, \( R_H = 6 \times 10^3 \text{Mpc} \), and neglecting any cosmological effect. More recently, Dar & Shaviv (hereafter DS [12]) reanalyzed the problem in the light of the EGRET data [1] and calculated the contribution to the DGRB from CR interactions in the intracluster gas, under the assumption that the energy density of CR’s in clusters is the same as in our own galaxy (universality). With this assumption, Dar & Shaviv [12] predicted a level \( I_\gamma(>100 \text{ MeV}) \approx 1.2 \times 10^{-5} \text{ photons cm}^{-2}\text{s}^{-1}\text{sr}^{-1} \), which could explain the whole amount of the DGRB of extragalactic origin. In a following paper, Berezinsky, Blasi & Ptuskin (hereafter BBP [13]) relaxed the ad hoc assumption of universality, and estimated the CR energy density in clusters due to various possible sources of CR, using the condition of diffusive confinement of CRs. In their approach, BBP [13] showed that it is impossible to fulfill the universality condition with the usual CR sources in clusters, emphasizing that the DGRB due to the CR interactions in clusters should be a small fraction of the total diffuse flux observed by EGRET. This conclusion was reached by the previous authors under the hypothesis that a large fraction of the baryons in the universe is contained inside clusters of galaxies (BBP considered that clusters are a fair sample of the baryons in the universe [17], [18], [19]) assumed to have a homogeneous inner distribution of gas, \( n_e = \text{const} \) (here \( n_e \) is the IGM electron number density). Because of these assumptions, their results depend only on overall cosmological parameters like the baryon fraction in the universe \( \Omega_b \), and on the cluster size.

Dar & Shaviv [12] also predicted the gamma ray fluxes from a few nearby clusters (Coma, Perseus and Virgo): for these three clusters they found \( \gamma \)-ray fluxes in the range \( F_\gamma(>100 \text{ MeV}) \approx 5 \div 20 \times 10^{-8} \text{cm}^{-2}\text{s}^{-1} \). In particular,
the value which they predicted for A/1/6/5/6/n28 Coma/n29, Fod/n0d/n3e/1/0/0 Me V/n29, is close to or slightly higher than the upper limits given by EGRET for this source. Similar results were obtained for these clusters by Ensslin et al. [20] assuming a population of CRs from radio sources located within galaxy clusters in almost equipartition with the IGM thermal energy.

We stress here that in all the previous works a uniform IGM density profile was assumed. Moreover, the cluster population was not assumed to evolve with cosmic time, and the same working hypothesis of no-evolution was assumed for the IGM content of each cluster.

However, X-ray studies of galaxy clusters, have shown that these cosmic structures are indeed well structured, having a gas density profile \( n(r) \propto [1 + (r/r_c)^2]^{-\beta/2} \), with core radii \( r_c \approx 0.1 \div 0.3 \ h^{-1} \) Mpc and \( \beta \approx 0.6 \div 0.8 \) (see e.g. [21]; see also [15] and references therein). Beside this, the IGM is indeed evolving as indicated by its sensitive metal enrichment, \( F_e/H \sim 0.2 \div 0.5 \) (in solar units, see e.g. [22], [23]), shown even for the brighter clusters observable at redshifts \( z \sim 0.5 \) [24]. Nonetheless, there is also an increasing debate on the possible evolution of the X-ray luminosity function observed out to \( z \lesssim 0.5 \) with the EINSTEIN [25] [26] and ROSAT satellites [27] [28] and on the possible evolution of the cluster temperature function [29] [30] [31]. If an evolution is present in the cluster population this can be, in fact, understood as a result of two competing effects:

\( i) \) a luminosity evolution, where the cluster X-ray luminosity, \( L \propto n^2 T^{1/2} R^3 \) (mainly due to thermal bremsstrahlung), changes with redshift due to variations in the gas mass density, \( n \propto f_g \rho_d \) (where the cluster gas mass is taken to be a fraction \( f_g \equiv M_{\text{gas}}/M \) of the total cluster mass), and/or changes in the IGM temperature \( T \) at fixed mass, \( M \propto \rho_d R^3 \) (here \( \rho_d \) is the cluster total mass density);

\( ii) \) a change in redshift of the number density, \( N(M, z) \), (usually referred to as mass function, hereafter MF) of clusters that are found to be collapsed (or virialized) in the mass range \( M, M + dM \) at redshift \( z \).

Detailed studies of cluster evolution in X-rays (see e.g. [32], [33], [29]) considered in fact that a combination of the previous mechanisms is responsible for the actual cluster evolution when they fit the available data (see [32] for a detailed discussion).

In this paper we predict the amount of high energy, non-thermal, gamma-ray emission from galaxy clusters using detailed modelling of the realistic cluster structure, as well as viable modelling for the evolution of the IGM and of the cluster MF. Based on these phenomenological cluster models, we predict the amount of DGRB that can be produced in the viable cosmological models: here we consider flat and low-density (open or vacuum-dominated) CDM models.
as well as mixed Dark Matter models with a fraction $\Omega_\nu \approx 0.3$ of the total density of the universe in form of massive neutrinos. We use $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$ throughout the paper unless otherwise specified.

The plan of the paper is the following. In Sect. 2 we briefly summarize the cluster formation history in hierarchical scenarios for structure formation. In Sect. 3 we describe a model for the production of diffuse gamma-ray emission due to the interaction of CR’s with the target protons present in the extended, diffuse IGM. We consider in Sect. 4 different CR sources that can be found in connection with galaxy clusters. We discuss in Sect. 5 the correlation between the extended gamma-ray emission and the much better known thermal X-ray emission coming from the IGM. Based on these properties, we construct a list of predicted $\gamma$-ray fluxes for a compilation of X-ray clusters with detailed informations on their IGM structure, IGM temperatures and X-ray fluxes. In Sect. 6 we present predictions for the amount of DGRB produced by galaxy clusters in different cosmological scenarios. We briefly discuss in Sect. 7 the extended neutrino fluxes emerging from these objects and their contribution to a possibly detectable diffuse neutrino background (hereafter DNB). Finally, in Sect. 8 we discuss our results in the light of the current limits obtained from EGRET and in the light of the future experiments for gamma-ray and neutrino astronomy.

2 Cluster formation: theory

Clusters of galaxies originate from small density perturbations at early times [34] that grow under the action of their own gravitational instability. In the viable cosmological scenarios, cluster evolution can be followed adopting a simple spherical collapse picture according to which a homogeneous, spherical perturbation detaches from the Hubble flow at time $t_m$ given by the equation, $t_m = [3\pi/32G\rho_b(t_m)]^{1/2}$, collapses at time $t_c \approx 2t_m$, and virializes at time $t_v \approx 3t_m$ [here $\rho_b(t_m)$ is the cosmological background density at the time $t_m$ of the maximum expansion of the perturbation, see [69] for details]. The relative density contrast of such a perturbation at the initial redshift $z_i$, $\delta_{i,v}$, depends on the cosmological model. Under the assumption of linear growth, the density contrast at $t_v$ is $\delta_v = \delta_{i,v}\mathcal{D}(t_v)/\mathcal{D}(t_i)$, where $\mathcal{D}(t_v)$ is the linear growth factor in the chosen cosmology (see [35] for details). For $\Omega_0 \rightarrow 1$, $\delta_v$ tends to the standard value $\delta_v \approx 2.2$, independent of $t_v$. The actual, non-linear density contrast, $\Delta = \rho/\rho_b$ (where $\rho$ is the perturbation density and $\rho_b$ is the background density at the time of virialization) of a cluster that virializes at redshift $z$ in a $\Omega_0 \leq 1$ cosmological model writes: $\Delta(\Omega_0, z) = 18\pi^2/[\Omega_0(H_0t)^2(1+z)^3]$ [in flat, vacuum dominated low-density models $\Delta$ has not an analytical expression (see, e.g. [35] and references therein)]. It tends to the standard value $\approx 400$, found in a flat ($\Omega_0 = 1$) universe.
Detailed imaging of galaxy clusters in the X-rays (see [15]) revealed that the IGM is concentrated in the cluster central region (the core) and its density decreases rapidly at large distances from the core. Thus, following Colafrancesco et al. [35], we relax the assumption of uniformity, \( n = \text{const} \), by considering a 3-D gas density profile:

\[
n(r) = n_c \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{-3\beta/2}, \tag{3}
\]

where \( n_c \) is the central electron number density and \( r_c \) is a core radius.

There are various open issues pertaining to the formation of hot gaseous cores of galaxy clusters. For our purposes here we can adopt the following simplified approach (see also [35]): shortly after a cluster forms and virializes, a gaseous core forms (probably as a result of tidal galactic interactions and other gas stripping processes) with the hot gas in local hydrostatic equilibrium in the potential wells of the cluster. Analytical studies of the cluster self-similar collapse [36] and numerical simulations [37] [44] indicate that the gas density profile scales following the total matter density of the cluster. Thus, the central gas density, \( n_c \), is related to the total density at the cluster centre, \( \rho_c \), through

\[
n_c = f_g(\rho_c/m_p)^{2/3} (1 + X),
\]

where \( m_p \) is the proton mass and \( X = 0.69 \) is the cosmic Hydrogen mass fraction. The mass within the outer radius, taken as \( R = q r_c \), is

\[
M(q, \beta) = 3 M_c \omega(q, \beta),
\]

where \( \omega(q, \beta) = \int_0^q dx x^2 (1 + x^2)^{-3\beta/2} \). Here \( M_c = (4\pi/3) r_c^3 \rho_c \). Because of the assumed profile, the ratio between the central and mean mass density of the cluster is

\[
\rho_c/\rho(\rho_0 \Delta) = q^3/3 \omega(q, \beta).
\]

Assuming that the cluster collapse is self-similar, we can infer the mass and redshift dependence of the core radius \( r_c \) from the equivalence:

\[
r_c = R_c/q = \frac{1}{q} (3 M/4 \pi \rho_0 \Delta)^{1/3} (1 + z)^{-1}.
\]

This gives:

\[
r_c(\Omega_0, M, z) = \frac{1.29 \, h^{-1} \, \text{Mpc}}{q} \left[ \frac{M}{10^{15} h^{-1} \, M_{\odot}} \right]^{1/3} \frac{\Delta(1, 0)}{\Omega_0 \Delta(\Omega_0, z)} \frac{1}{1 + z}.
\]

Hereafter we fix \( q = 10 \) to recover \( r_c \) values consistent with the observations [47] [21].

Under the standard assumption of the IGM in hydrostatic equilibrium with the potential well of a spherically-symmetric virialized cluster, the relation between IGM temperature and cluster mass is easily obtained by applying the virial theorem:

\[
T = -\mu m_p U/(3 k M),
\]

where \( \mu = 0.62 \) is the mean molecular weight (corresponding to a hydrogen mass fraction of 0.69), \( k \) is the Boltzmann constant and \( U \) is the cluster potential energy. If the cluster is assumed to be uniform, \( U = -(3/5) GM^2/R_v \) and \( T = T^{(u)} = (1/5)(\mu m_p/k)GM/R_v \), where \( R_v = [3 M/(4 \pi \rho_0 \Delta)]^{1/3} / (1 + z) \) is the cluster virial radius. In the case of the density profile in eq. (3), \( U = -(GM^2/r_c)^{\psi}/\omega^2 \) and \( T = 5q \psi T^{(u)}/(3\omega^2) \), where
\[
\psi = \int_0^\infty y dy / (1 + y^2)^{3/2} \int_0^\infty t^2 dt / (1 + t^2)^{3/2}. 
\]
Thus the cluster IGM temperature is
\[
T = 5.8 \, \text{keV} \ (1 + z) \left( \frac{M}{10^{15} M_\odot h^{-1}} \right)^{2/3} \left[ \frac{\Omega_0 \Delta(\Omega_0, z)}{\Delta(1, 0)} \right]^{1/3},
\]
(5)
where the normalization constant, \( T_0 = 5.8 \, \text{keV} \), is obtained for \( q = 10 \) and \( \beta = 2/3 \). For \( \beta = 1 \) we get \( T_0 \approx 8.2 \, \text{keV} \), where \( T_0 \) is the temperature of a \( 10^{15} M_\odot h^{-1} \) cluster at \( z = 0 \) in a \( \Omega_0 = 1 \) universe.

Although it is expected that the IGM mass fraction, \( f_g \), depends on \( z \) and \( M \), little is currently known about the exact form of these dependences. We adopt the simple parametrization (described in detail in Colafrancesco & Vittorio [32]) which is suggested by models of the IGM evolution driven by entropy variation in the cluster cores [38] [39] [40] and/or shock compression and heating [41] [42] [43]:
\[
f_g = f_{g,0} \left( \frac{M}{10^{15} h^{-1} M_\odot} \right)^\eta (1 + z)^{-s}.
\]
(6)
The normalization to \( f_{g,0} \approx 0.1 \), is based on a local, rich cluster sample. Values of \( \eta \approx 0.2 \div 0.9 \) and \( s \approx 0.5 \div 2 \) are consistent with the available data.

The previous information about \( n(r) \) [see eq. (3)], \( T \) [see eq. (5)] and the cluster extension, \( R = qr_c \), allow to predict both the gamma-ray and the X-ray emissivities which will be discussed in the following sections.

3 Diffuse High Energy Emission from Clusters of Galaxies

As recently discussed by BBP [13] and Volk et al. [14], most of the cosmic rays produced in clusters of galaxies remain confined within the cluster potential wells and produce high energy gamma rays and neutrinos by interactions with the intrachannel baryonic gas (the IGM). The confinement of these CR’s in the intrachannel space is a crucial mechanism for maximizing the efficiency of the \( \gamma \) (or neutrino) emission and is strictly related to the value and to the configuration of the magnetic field in clusters. This determines the diffusion coefficient, given by:
\[
D(p) = \frac{1}{3} cr_L(p) \int_1^{\infty} \frac{B^2}{P(k) dk}.
\]
(7)
Here \( P(k) \) is the power spectrum of the fluctuations in the magnetic field and \( r_L(p) = pe/(eB) \) is the Larmor radius of a particle with electric charge \( e \) and
momentum $p$. In this paper we assume that the fluctuations of the magnetic field have a Kolmogorov spectrum:

$$P(k) = P(k_0) \left( \frac{k}{k_0} \right)^{-5/3},$$

(8)

where $k_0 = 1/d_0$, and $d_0$ is the smallest spatial scale at which the magnetic field achieves homogeneity. The spectrum in eq. (8) is normalized as:

$$\int_{k_0}^{+\infty} P(k) dk \approx B^2,$$

(9)

which implies,

$$P(k_0) = \frac{2}{3} \frac{1}{k_0} B^2 = \frac{2}{3} d_0 B^2.$$

(10)

Finally, the diffusion coefficient for relativistic particles is obtained from eqs. (7) and (10):

$$D_{CR}(E) = \frac{1}{3} e d_0^{2/3} (eB)^{-1/3} E^{1/3},$$

(11)

where $E = pc$ is the energy of the relativistic particles.

Measurements of the overall magnetic field in single galaxy clusters yield values in the range $\sim 1 \div 10 \mu G$ ([49] [50] [51] [52] [53]), while a statistical analysis of a sample of clusters (see e.g. [54]) yielded typical, average values $B \sim 1 \mu G$ on homogeneity scales $d_0 \gtrsim 20 \text{ kpc}$. This size is compatible with the model of Jaffe [55] for the origin of the magnetic field in clusters: according to this model, the turbulence of the IGM produced by the large scale motions of the galaxies is responsible for the value and the homogeneity scale of the magnetic field.

Thus, from eq. (11) we obtain:

$$D_{CR}(E) \approx 2.3 \times 10^{26} E(eV)^{1/3} B^{-1/3} \mu G \text{cm}^2/\text{s}.$$

(12)

The diffusion time of particles with energy $E$ at distance $r$ from the CR source reads:

$$\tau = \frac{r^2}{6 D_{CR}(E)} \approx 6.9 \times 10^{21} r_{Mpc}^2 E(eV)^{-1/3} \text{s},$$

(13)
where the factor 6 in the previous expression for $\tau$ comes from the solution of the diffusion equation in spherical geometry (here we assumed $B_{\mu G} = 1$). This implies a confinement of cosmic rays, on typical scale of $R_d \approx 2\ Mpc$, if

$$E \lesssim 4.2 \times 10^{14} h^{-3}\ eV.$$  \hspace{1cm} (14)

These cosmic rays have diffusion times larger than the age of universe, for which we adopted the value $t_0 = 2.06 \times 10^{17} h^{-1}\ s$.

More difficult is to extrapolate, from the average confinement ability of the clusters, the confinement efficiency of the cluster core: the difficulty comes from the small size of this region ($r_c \sim 0.2 \div 0.5\ Mpc$) where a detailed knowledge of the structure of the magnetic field is needed (the typical separation between galaxies in clusters cores can be even smaller that the size of the largest galaxies often observed in the cores).

The confinement of cosmic rays inside clusters and the relevance of this process for the production of high energy radiation was already pointed out by BBP. In that work, however, the true density profile of clusters was not considered, and only an average IGM density entered the calculation. Indeed, for realistic IGM density profiles, like that in eq. (3), a large fraction of cosmic rays could be confined inside the dense cluster cores, thus determining a sensitive increase in the interaction rates.

Following the approach of BBP, and assuming a power-law spectrum of CR produced by a source located at the center of the cluster, the number of i-secondaries ($i=\gamma, \nu$) produced per unit time and unit volume, at energy $E$ and at distance $r$ from the CR source in a cluster, is:

$$q_i(E, r) = Y_i \sigma_{pp} n^2(r) \frac{[n_p(E, r)]}{n(r)},$$  \hspace{1cm} (15)

where $\sigma_{pp} \approx 3.2 \times 10^{-28}\ cm^{-2}$, $Y_i$ are the yields for the i-secondary production [56], $n(r)$ is the IGM density profile given by eq.(3) and the produced CR proton density, $n_p(E, r)$, is determined from the diffusion equation as:

$$n_p(E, r) = \frac{1}{4\pi r} \left[ \frac{Q_p(E)}{D_{CR}(E)} \right].$$  \hspace{1cm} (16)

In this last equation, $Q_p(E)$ is the emitted spectrum of CR from a source assumed to be located in the cluster core. Thus, the total number of i-secondaries
produced per unit time at energy $E$ reads:

$$Q_l(E) = 4\pi \int_0^R dr r^2 q_l(E, r) = Y_\sigma \sigma_{pp} n_0 c \left[ \frac{Q_p(E)}{D_{CR}(E)} \right] r_c^2 \zeta(q, \beta, E, z; M), \quad (17)$$

where

$$\zeta(q, \beta, E, z; M) = \frac{R_{diff}}{r_c} \int_0^{R_{diff}/r_c} dx x(1 + x^2)^{-3\beta/2}. \quad (18)$$

We note that the maximum length over which CR diffuse is $R_{diff} \equiv \ell_D = \sqrt{6D_{CR}(E) t(z, \Omega_0)}$, at each epoch $t(z, \Omega_0)$. This yields

$$\ell_D \approx 0.5 \, h^{-1} \, \text{Mpc} \left( \frac{t_0}{2 \cdot 10^{10} \text{yr}} \right)^{1/2} \cdot \left( \frac{D_{CR}}{10^{29} \text{cm}^2} \right)^{1/2}. \quad (19)$$

For a uniform cluster we reproduce the results of BBP. In the present approach, instead, the inclusion of a realistic density profile for the IGM produces a change from a pure power–law CR spectra, $Q_i(E) \propto E^{-\gamma_0}$ [if $Q_p(E) \propto E^{-\gamma_0}$ is assumed], due to the presence of an energy dependent term in the function $\zeta \propto [1 + (\ell_D/r_c)^2]^{-3\beta/2+1}$ in eq.(18). However, at the energies for which the CR confinement is mostly effective, $\ell_D^2 \ll r_c^2$, expanding eq.(17) in power series, yields:

$$Q_l(E) \approx 3Y_\sigma \sigma_{pp} n_0 c \ell(z) Q_p(E). \quad (20)$$

In this limit, the total number of i-secondaries produced per unit time is independent on the cluster size and depends only on the IGM density, $n_0$ at the cluster centre.

Note that the term $Q_p(E)$ can take into account the possible evolution of the luminosity of the CR source as well as the possible evolution of the number of CR sources that are present in the central regions of the clusters. Another important source of evolution in the $\gamma$ (or neutrino) diffuse emission from galaxy clusters comes from the possible evolution of the IGM content (see Sect. 2). We will discuss these points in the following.
4 Sources of Cosmic Rays in Clusters of Galaxies

In the previous sections we discussed the confinement of cosmic rays in galaxy clusters, irrespective of the way they are produced. In this section we shall deserve more attention to the possible CR sources and we will estimate their contribution in terms of CR luminosity.

4.1 Normal galaxies

The most natural candidates as sources of CR in clusters are normal galaxies. If we assume that our own galaxy is a typical one, the CR luminosity can be estimated to be \( L_{CR} \approx 3 \times 10^{40} \text{ erg/s} [56] \). The central number of galaxies \(^1\) \( N_0 \) (which is a measure of the cluster richness) is found to be correlated with the cluster temperature and it scales as \( N_0 \propto T^{0.8} [22] \). Thus the gamma ray luminosity associated with normal galaxies in clusters scales as \( L_{CR} \propto N_0 L_p \propto T^{0.8} L_p \), and for a Coma-like cluster with \( T \approx 8.3 \text{ keV} \), we find \( L_{CR} \approx 2 \times 10^{42} \text{ erg/s} \). In any case, even for the richest and hottest clusters observed (A2163 with \( T = 13.9 \text{ keV} \)) the CR luminosity associated to normal galaxies is found to be \( L_{CR} \lesssim 10^{43} \text{ erg/s} \).

4.2 Active galaxies

As discussed by BBP, on statistical grounds we can expect \( \sim 1 \) active galaxy per cluster, where the term active galaxy is used here to indicate several classes of luminous objects, such as AGN, radiogalaxies and cD galaxies. The mechanisms of CR production in active galaxies have been studied by many authors (see e.g. [57] and references therein) and typical luminosities of \( L_{CR} \approx 10^{44} \text{ erg/s} \) can be achieved.

In a recent paper, Ensslin et al. [20] proposed that jets of powerful radiogalaxies in clusters can establish a sort of equipartition between the thermal energy of the IGM and the cosmic ray energy density in clusters, due to particle acceleration in the jets. The previous authors calculated the gamma ray flux from single clusters due to the CR interactions in the case mentioned above and predicted a gamma ray flux of \( F_\gamma (> 100 \text{ MeV}) \approx 6 \times 10^{-8} \text{ cm}^{-2} \text{s}^{-1} \) for A1656 (Coma), a little bit in excess of the EGRET experimental limit. Similar predictions are given for other clusters (A426, Ophiucus, M87-Virgo) for which they found \( F_\gamma (> 100 \text{ MeV}) \) in the range \( \approx 3 \div 12 \times 10^{-8} \text{ cm}^{-2} \text{s}^{-1} \).

\(^1\) The central number density of galaxies \( N_0 \) is related to the total number of galaxies in a cluster by the expression \( N_g = 4\pi N_0 \sigma_b^3 \omega(q, \beta). \)
Given all these estimates, a CR luminosity $L_{CR} \approx 10^{44} \text{erg/s}$ can be expected from active galaxy interaction with the IGM of local clusters. At higher redshifts, the AGN luminosity (or the CR density) is expected to increase roughly as $\sim (1 + z)^2$ while the IGM density (the target proton density) is expected to decrease roughly as $\sim (1 + z)^{-1.2}$. The two evolutionary effects tend to balance out and the net result is a moderate increase, if any, of the effect due to distant, AGN populated clusters.

### 4.3 Cluster accretion shocks

Another important acceleration mechanism for CR’s is provided by accretion shocks formed in and around clusters of galaxies as a result of their collapse and virialization [36] [58] [59] [60]. These shocks have been recognized as possible sources of CR acceleration through the first order Fermi mechanism (see e.g. [13] [59]).

We shall estimate here the CR luminosity by adopting two models for the accretion of matter around galaxy clusters. In the first model we refer to the analytical calculations of Bertshinger [36], in which a self-similar analytical solution of the hydrodynamic equations is found. The solution, concerning the velocity and density of the accreting gas, is written in terms of the dimensionless variable, $\lambda = r/R_{ta}$, where $r$ is the distance from the center of the cluster (assumed to have spherical symmetry) and $R_{ta}$ is the turn-around radius, given by

$$R_{ta} = \left( \frac{8GMt_0}{\pi^2} \right)^{1/3},$$  

(here $M$ is the cluster gravitational mass). In these calculations [36] the stationary shock is located at $\lambda_{sh} \approx 0.347$. At the shock location, the gas velocity is found to be $v(\lambda_{sh}) \approx 1.5R_{ta}/t_0$, while the density of the gas is $\rho_{gas}(\lambda_{sh}) \approx 2.2 \times 10^{-29} \text{g/cm}^3$. This last number has been obtained assuming $\rho_{gas}(r) = 0.1\rho(r)$, where $\rho$ is the total mass density of the cluster (dark matter and baryonic gas have been assumed to have the same radial distribution).

Thus, the total energy per unit time available at the shock can be written as:

$$L = \frac{1}{2} \rho_{gas}(R_{sh})v(R_{sh})^2 4\pi R_{sh}^2 v(R_{sh}) \approx 2.4 \times 10^{45} \text{erg/s} \left( \frac{R_{ta}}{5 \text{Mpc}} \right)^5$$  

(we use here $h = 0.75$). The typical efficiency of CR acceleration at the shock is of order $\sim 0.1$ [56], so that $L_{CR} \approx (2 \div 3) \times 10^{44} \text{erg/s}$ is obtained, which is comparable with the CR luminosities derived in the previous Section 4.2 for the case of active galaxies within galaxy clusters.
An alternative way to estimate $L_{CR}$ for the accretion shock case is to write it through the matter accretion rate, $\dot{M}$, at the shock. Numerical simulations of the large scale structure formation ([61] [62]) give larger values for the shock size, of order $\sim 5 \, h^{-1} \text{Mpc}$. If we again assume that a fraction $\sim 0.1$ of the gravitational energy at the shock is converted in cosmic rays accelerated at the shock, we can write:

$$L_{CR} \approx 0.1 \frac{GM\dot{M}}{R_{sh}} \approx (1.3 - 2.5) \times 10^{44} \text{ erg/s}. \quad (23)$$

In this numerical estimate, we assumed that the cluster accretion rate at the shock is given by the time averaged value

$$\dot{M} \approx \frac{\dot{M}_{gas}}{t_0} \simeq 5 \times 10^{29} \text{ g/s}, \quad (24)$$

where $M_{gas}$ is the mass of the cluster in the form of IGM, assumed here to be $\sim 10\%$ of the total mass. Note that this last estimate is probably a lower value for the actual IGM mass in rich clusters (see e.g. [18] [63]).

To summarize, the CR luminosity obtained in the two accretion shock models yield comparable results for the local cluster CR luminosity, $L_{CR} \approx$ a few $10^{44} \text{ erg/s}$. At higher redshifts, the CR luminosity from cluster shocks should tend to decrease, if $f_g$ does.

On the basis of the previous considerations we are justified in assuming an average CR luminosity of $L_{CR} = 10^{44} \text{ erg/s}$ for a typical, rich cluster with $M = M_{15}$ (here $M_{15} = 10^{15} \text{ h}^{-1} \text{ M}_\odot$) at $z = 0$, either due to active galaxies and/or to accretion shocks around clusters.

5 The correlation between X-ray and Gamma-ray emission

From eq.(17) we can now calculate gamma-ray fluxes and luminosities for clusters with mass $M$ at redshift $z$. In Fig.1 we plot the cluster $\gamma$-ray luminosity against the cluster temperature $T$ expected in different, viable cosmological scenarios (see Fig.1 caption for details). In our specific predictions we find that the cluster $\gamma$-ray luminosity $L_\gamma$ [or equivalently $Q_i$ as written in eq.(17)] scales with the cluster properties (at fixed redshift $z$) as:

$$L_\gamma \propto \left[ \frac{Q_g(E)}{D_{CR}(E)} \right] \cdot n_0 \xi^2, \quad (25)$$
that yields the scaling:

\[ L_\gamma \propto \left[ \frac{Q_p(E)}{D_{CR}(E)} \right] T^{1+n\zeta}. \]  

(25)

We assumed here a virial equilibrium for the IGM which yields, \( T \propto GM/R \). In our specific model for the IGM we also consider a distribution of values for \( f_g = M_{\text{gas}}/M \propto M^n \) (with \( n \sim 0.2 \div 0.9 \)) from groups to galaxy clusters in order to recover phenomenologically the general trend indicated by the most recent data [63] [64] [48]. The function \( \zeta \) decreases with increasing cluster mass (temperature) and hence the actual scaling of \( L_\gamma \) with the cluster temperature reads \( L_\gamma \sim T^{0.5} \) that is weaker with respect to the scaling of the previous equation (see also Fig.1). The shape of the \( L_\gamma(T) \) curves also depends slightly on the considered cosmological model, with low \( \Omega_0 \) values providing higher \( L_\gamma \) at fixed \( T \) (see Fig.1). This mild cosmological dependence is caused by the sensitivity of the cluster parameters (mainly the core radius \( r_c \)) to the underlying cosmology: in fact, \( r_c \) increases with decreasing \( \Omega_0 \) [see eq.(4)].

In Fig.2 we plot the expected \( \gamma \)-ray fluxes at \( E > 100 \text{ MeV} \) for clusters with X-ray flux \( F_X(2-10 \text{ keV}) \) in the energy band \( 2-10 \text{ keV} \). We show this plot for clusters at \( z = 0.023 \), the redshift of Coma. For this cluster we have an upper limit on \( F_\gamma(> 100 \text{ MeV}) \) measured by EGRET (as indicated in Fig.2). We plot the fluxes expected in the cosmological models considered in Fig.1. The \( \gamma \)-ray luminosities shown in Fig.1 correspond to fluxes \( F_\gamma(> 100 \text{ MeV}) \lesssim 10^{-9} \) photons \( \text{s}^{-1} \text{ cm}^{-2} \) for clusters at \( z \sim 0.1 \), and \( F_\gamma(> 100 \text{ MeV}) \) in the range \( 7 \cdot 10^{-9} \div 1.5 \cdot 10^{-8} \) photons \( \text{s}^{-1} \text{ cm}^{-2} \) for Coma-like clusters (at \( z \sim 0.023 \)) in different cosmological models, as shown in Fig.2. These fluxes are a factor \( \sim 2 \div 5 \) smaller than the EGRET upper limit for Coma; such a limit was saturated by the predictions of Dar and Shaviv [12] and Ensslin et al. [20]. Smaller fluxes are obtained for flat (\( \Omega_0 = 1 \)) cosmologies. The differences in \( L_\gamma \) expected in different cosmological models (see Fig.1) are amplified when one compares the relative fluxes, \( F_\gamma = L_\gamma/4\pi d_L^2 \), by the \( \Omega_0 \) dependence of the luminosity distance \( d_L \). At redshifts \( z \simeq 0.3 \) the predicted \( F_\gamma(> 100 \text{ MeV}) \) for clusters fall below values \( \sim 10^{-10} \) photons \( \text{s}^{-1} \text{ cm}^{-2} \), and become hardly observable from any present and/or planned gamma-ray mission for the next coming years (INTEGRAL, GLAST, AMS).

From the previous results, we found a correlation between the X-ray luminosity of galaxy clusters and their gamma (or neutrino) luminosity due to CR interaction in the intracluster gas, as shown in Fig.2. Such a correlation is indeed expected because the protons in the IGM are both the target for the CR interactions and the responsible for the thermal bremsstrahlung radiation which provides the bulk of the cluster X-ray luminosity. In fact, the total X-ray
Luminosity from a cluster with an IGM temperature $T$ can be written as:

$$L_X = A(T)(kT)^{1/2} \int_0^R dr 4\pi r^2 n^2(r) .$$  (26)

Using eq. (3) for the IGM density profile, eq.(26) writes as:

$$L_X = 4\pi n_c^2 r_c^3 A(T)(kT)^{1/2} I(q, \beta) ,$$  (27)

where

$$I(q, \beta) = \int_0^q dx x^2 (1 + x^2)^{-3\beta} .$$  (28)

The function $A(T)$ in eqs. (26-27) contains the contribution of the Gaunt factors to the frequency integrated X-ray spectrum, and writes as:

$$A(T) = \frac{2^5\pi e^6}{3hmc^3} \left( \frac{2\pi}{3mc} \right)^{1/2} \int dy g_{\text{ff}}(T, y) e^{-y} ,$$  (29)

where $y = h\nu/kT$, $e$ and $m_e$ are the electron charge and mass, respectively, and $g_{\text{ff}}$ is the Gaunt factor.

Using eqs. (8) and (16) we can derive the expression for the ratio of the the gamma (or neutrino) flux coming from a single galaxy cluster and its corresponding X-ray flux, $F_X$:

$$\frac{F_{\gamma,\nu}}{F_X} = \frac{Y_{\gamma,\nu} \sigma_{pp} c}{4\pi A(T) I(q, \beta)} \left( \frac{1}{n_c r_c} \right) \frac{1}{(kT)^{1/2}} \left\{ \frac{Q_p(E)}{D_{CR}(E)} \right\} \zeta .$$  (30)

Thus, from the previous eq.(30) it is possible to estimate the cluster $\gamma$ (or neutrino) flux given a measurement of the cluster X-ray flux $F_X$, the central IGM density $n_c$, the cluster core radius $r_c$ and the cluster IGM temperature $T$:

$$F_{\gamma,\nu} = F_X \cdot C(E, \beta, z) \frac{1}{n_c r_c \sqrt{kT}} .$$  (31)

Here the quantity $C(E, \beta, z)$ reads:

$$C(E, \beta, z) = \frac{Y_{\gamma,\nu} \sigma_{pp} c}{4\pi A(T) I(q, \beta)} \left\{ \frac{Q_p(E)}{D_{CR}(E)} \right\} \zeta .$$  (32)
We will use here the previous results and the available data for $F_X, n_c, r_c, \beta$ and $T$ [21] [65] [67] to derive a list of estimated cluster fluxes in the high energy $\gamma$ and neutrino energy regions (see Table 1). The clusters in the sample listed in Tab.1 are those which have well measured values of $\beta, n_c, r_c, T$ and $F_X$ and are selected from the homogeneous sample of Jones and Forman [66] which is limited to redshifts $z \lesssim 0.1$. In Fig.3 we plot these data in the $F_\gamma(> 100\,\text{MeV}) - T$ plane, together with the predicted curves, $F_\gamma(>(100\,\text{MeV}))(T)$, expected in a CDM+$\Lambda$ model with $\Omega_0 = 0.4$ and $h = 0.6$ at redshifts $z = 0.023$ (solid line) and at $z = 0.072$ (dashed line). The theoretical predictions at the minimum and maximum redshifts of the clusters in the Jones & Forman sample encompass the data points. The $\gamma$-ray fluxes predicted from eq.(30-31) for our sample of clusters at $z \lesssim 0.1$ span over the range $F_\gamma(> 100\,\text{MeV}) \approx 5 \cdot 10^{-10} \div 8.5 \cdot 10^{-9}$ photons s$^{-1}$ cm$^{-2}$ and are distributed with some intrinsic dispersion along the theoretical $F_\gamma(> 100\,\text{MeV}) - T$ curves predicted at each redshift (see Fig.3) according to eq.(31). Note that the region at $T \lesssim 2$ keV is populated mainly by galaxy groups for which the determinations of the IGM temperatures are more difficult and uncertain due to their small amount of diffuse IGM (see [45] [46] for a discussion). In the following, we will consider mainly galaxy clusters with $T \gtrsim 2$ keV.

In Fig.4 we show how the clusters listed in Tab.1 distribute according to their predicted $\gamma$-ray flux. The distribution of the cluster $\gamma$-ray fluxes peaks at $F_\gamma(>100\,\text{MeV}) \approx 8 \cdot 10^{-10} \div 3 \cdot 10^{-9}$ photons s$^{-1}$ cm$^{-2}$. Our sample of clusters was extracted from an X-ray flux limited sample [66] and thus it can be considered as representative of the cluster population even in the $\gamma$-rays because of the existing correlation $F_\gamma - F_X$ of eq.(32). From Fig.4 we note that the optimal observational strategy to detect large samples of galaxy clusters in the $\gamma$-rays is to achieve sensitivities $\gtrsim 5 \cdot 10^{-10}$ photons s$^{-1}$ cm$^{-2}$; such a sensitivity level should be at hand with the next generation $\gamma$-ray telescopes (INTEGRAL, GLAST).

6 The contribution of galaxy clusters to the DGRB

The diffuse flux of $\gamma$ (or neutrino) emission at energy $E_i$ that is received from a cluster at redshift $z$ is given by

$$F_{\gamma,\nu}(E_i) = \frac{Q_i[E_i(1 + z), z]}{4\pi d_L^2} ,$$

(33)
where the luminosity distance
\[ d_L(z, \Omega_0) = \frac{c}{H_0}(1 + z) \int_0^z \left(1 + z'\right)^3 \Omega_a + 1 - \Omega_a \right)^{-1/2}, \] (34)
allows for the flux-luminosity conversion:
\[ L = 4\pi F d_L^2(z, \Omega_0). \] (35)

The total flux of γ-rays (or neutrino) due to the superposition of a population of evolving clusters is then:
\[ I_{\gamma, \nu}(E') = \int_0^{z_{max}} \frac{dV(z, \Omega_0)}{dz} \int_{M_{min}} \frac{Q_i(E(1 + z)z)}{4\pi d_L^2} dM N(M, z), \] (36)
where \( dV(z, \Omega_0)/dz \) is the cosmological volume element per unit steradian.

The space density of clusters at any mass \( M \) and redshift \( z \) is given by the mass function, \( N(M, z) \), (hereafter MF) usually derived by the Press & Schechter [68] theory as:
\[ N(M, z) = \sqrt{\frac{2}{\pi}} \frac{p_b}{M^2} \frac{\delta_v}{\sigma} \frac{d\ln \sigma}{\ln M} \exp[-\delta_v^2/2\sigma^2]. \] (37)

We remind the reader that \( p_b \) is the comoving background density of the universe, \( M \) is the total cluster mass and \( \delta_v \) is the linear density contrast of a perturbation that virializes at redshift \( z \) (see Sect.2). The variance \( \sigma \) of the (linear) density fluctuation field at the scale \( R = (3M/4\pi p_b)^{1/3} \) and redshift \( z \) is given by the standard relation:
\[ \sigma^2(R, z) = D^2(\Omega_0, z) \int k^2 dk P(k) \left( \frac{3j_1(kR)}{kR} \right)^2 \] (38)
[69], where \( D \) is the growth factor of linear density fluctuations in a given cosmology and \( j_1(x) \) is a spherical Bessel function. If we normalize the fluctuation spectrum, \( P(k) \), by requiring \( \sigma(8 h^{-1} \text{Mpc}, z = 0) = b^{-1} \), then the MF depends uniquely on the product \( b\delta_v \), for a given cosmological model.

Assuming a power-law power spectrum for the initial fluctuation field, \( P(k) = A k^n \), the MF attains the following analytical expression:
\[ N(M, z) = \sqrt{\frac{2}{\pi}} \frac{p_b}{M_0^2} \frac{\delta_v b}{\sigma D(z)} \left( \frac{M}{M_0} \right)^{n-2} \exp \left[ -\frac{1}{2} \frac{\delta_v^2 b^2}{D^2(z)} \left( \frac{M}{M_0} \right)^{2n} \right], \] (39)
where \( \alpha = (n+3)/6 \) and \( M_0 \) is the mass contained in a \( 8 \, h^{-1} \text{Mpc} \) radius sphere at \( z = 0 \). The MF we will use here is normalized to the observed abundance of clusters at \( z \approx 0 \) by fitting the parameter \( \delta_v b \) (see [32] [29]). This fitting procedure is done by using the available data on the temperature function [67] and/or those on the X-ray luminosity function [70] [27]. The different data sets yield approximately the same best fit values for \( \delta_v b \) in the cosmological models we considered. Specifically, CMV [29] found \( \delta_v b = 2.5 \) for a standard CDM model and \( \delta_v b = 1.6 \) for a low density, vacuum dominated CDM model with \( \Omega_0 = 0.4 \) and \( \Omega_\Lambda = 0.6 \), which is the best fitting CDM model. For this class of CDM models, CMV found \( \delta_v b \approx 2.5 \Omega_0^{0.49} \), or equivalently \( \sigma_s \approx 0.67 \Omega_0^{-0.49} \) (if one assumes \( \delta_v = 1.68 \)).

In the case of power-law power spectra and for \( \Omega_0 = 1 \), eqs. (34), (36) and (39) give the following analytical result for the cluster contribution to the DGRB:

\[
I_\gamma = \sqrt{\frac{2}{\pi}} \frac{\rho_0}{4\pi} M_0^{-1/3} \left( \frac{n+3}{6} \right) (\delta_v b)^2 \sigma_{pp} \frac{c^2}{H_0} \frac{Q_\mu(E)}{D_{CR}(E)} \chi^2 \times \int_0^{z_{max}} dz (1+z)^{-\alpha_\mu+\epsilon+2-5/2} \int_y^\infty dy y^{\alpha_\mu+2-5/2} \exp \left\{ -\frac{1}{2} \left( \frac{\delta_v b}{D(z)} \right)^2 y^{2\alpha_\mu} \right\}, \tag{40}
\]

where \( \chi = r_v (1+z)/M_0^{1/3} \) has been obtained from eq. (4), \( \zeta \) is defined in eq. (18) and \( \epsilon \) is the power index of the energy dependence of \( D_{CR}(E) \): \( D_{CR}(E) \propto E^\epsilon \), where we found \( \epsilon = 1/3 \) (see eq.11).

A numerical calculation of the integrals in eq.(40) yields the immediate result that the main contribution to the \( \gamma \) (or neutrino) diffuse flux comes from low-z clusters, and actually from sources at \( z \lesssim 0.3 \). This result is confirmed by our numerical calculations of the redshift distribution, \( N(z) \), of \( \gamma \)-ray emitting clusters found in the different cosmological models we considered in this paper.

In Fig.5 we show the redshift distributions, \( N(z) \), as a function of \( z \), of \( \gamma \)-ray clusters expected in a flat (\( \Omega_0 = 1, h = 0.5 \)) CDM model in the case of an IGM evolving as \( f_\gamma \propto M^{0.2}(1+z)^{-1.8} \) (solid histogram) and in the case of absence of IGM evolution with redshift (dotted histogram). We considered clusters with fluxes \( F_\gamma(\gtrsim 100\,\text{MeV}) > 10^{-10} \text{photons}\,\text{s}^{-1}\text{cm}^{-2} \) with \( \gamma_g = 2.1 \). The presence of an IGM evolution modifies the cluster distribution mainly at \( z \gtrsim 0.2 \). Models with no IGM evolution (i.e. with \( s = 0 \)) predict \( \sim 10 \) times more clusters with \( F_\gamma(\gtrsim 100\,\text{MeV}) > 10^{-10} \text{photons}\,\text{s}^{-1}\text{cm}^{-2} \) at \( z \gtrsim 0.4 \) with respect to the case of a strong evolving IGM model with \( s = 1.8 \). However, such a difference tends to reduce to \( \sim 30\% \) at \( z \lesssim 0.2 \) and becomes negligible at \( z \lesssim 0.1 \). As the bulk (\( \gtrsim 90\% \)) of the bright \( \gamma \)-ray clusters is located at \( z \lesssim 0.15 \), the IGM evolution should play a little role on the cluster contribution to the DGRB.
The value of $z_{\text{max}}$ in eq.(40) is defined by two criteria: i) the largest cutoff in the redshift distribution of the cluster population; ii) the maximum energy of the i-secondaries. In fact, for $E \lesssim E_{\text{max}}$, the energy of production of the i-seconds is $E(1+z) \lesssim E_{\text{max}}$ (that is the maximum energy of the original CRs). If $E > E_{\text{max}}$, we must realize that the spectrum of the i-seconds is changed.

Note that in these reasonings, the diffusion coefficient $D_{\text{CR}}(E)$ depends on the typical homogeneity scale $d_0$ of the magnetic field in clusters (see Section 3). There are reasonable arguments to expect that this scale could change with redshift. However, we assume here that the only dependence with $z$ of the diffusion coefficient $D_{\text{CR}}$ is due to its dependence on the cosmic time (see eqs.11-13).

In Fig.6 we show the cluster contribution to the DGRB, $I_\gamma$, as a function of the $\gamma$-ray energy $E$, in the various cosmological models we consider in this paper (see Fig.6 caption for details). The expected contribution of galaxy clusters to the high energy DGRB [evaluated according to eqs.(36-40)] is found to be $\lesssim 5\%$ of the EGRET value [1] for all the considered cosmological models (see Fig.6). This result depends on different issues:

i) we choose $L_{\text{CR}} = 10^{44}$ erg/s for a $M_{15}$ cluster in each cosmological model. The value of $I_\gamma$ depends linearly on the choice of $L_{\text{CR}}$ so that any variation of this parameter reflects in a analogous variation of the predicted contribution to the DGRB;

ii) we normalize the local abundance of clusters to that observed in the X-rays. This estimate is the most precise and robust at the moment. Such a normalization yields a values of $\delta_{0,b}$ for each cosmological model, so that the evolution of galaxy clusters is completely determined;

iii) the distribution of the cluster IGM is self-consistently derived from the gravitational instability picture for the formation of galaxy clusters. The level of $I_\gamma$ depends linearly on the value of $f_{\delta}(M_{15}, z = 0)$ for which we choose the value 0.1. Changing this parameter in the viable range, $f_{\delta} \approx 0.05-0.3$ yield similar variations in $I_\gamma$;

iv) only low-$z$ clusters effectively contribute to the integral in eq.(36) as shown by the redshift distribution of clusters with $F_\gamma \geq 10^{-10} \, \text{cm}^{-2} \, \text{s}^{-1}$ (see Fig.5). Nonetheless, there is a mild dependence of the results from the considered cosmological model. The MDM models show the smallest value of the DGRB contributed by clusters due to the very recent formation epochs predicted for clusters in this model (see Fig.6). CDM models (flat and low density) show approximately the same level of DGRB: this is due to the normalization of their spectral amplitude to the observed abundance of nearby clusters observed in the X-rays [29].
In Fig. 7 we show the quantity $I_\gamma$ as a function of the $\gamma$-ray energy $E$, expected in a CDM+$\Lambda$ model ($\Omega_0 = 0.4, h = 0.6$) for the cases of the maximum rate of IGM evolution allowed by the present data, $f_g \propto M_{200}^{0.2} (1 + z)^{-2.2}$ (solid line) and for the case $f_g = \text{const}$ (dashed line). We note that the amount of DGRB contributed by galaxy clusters in different cosmological models depends slightly on the IGM evolution (see Fig. 7) as the clusters effectively contributing to the DGRB are those with $z \lesssim 0.2$. At these redshifts, the maximum change of $f_g$ (at fixed mass) is of the order of $\sim 20\%$ and this corresponds to a variation of $I_\gamma$ of the same order (see eqs. 18, 36 and 40).

Variations in the parameters of the cluster structure yield variations $\lesssim 5\%$ on the final result. Changing $\beta$ by $20\%$ yields variations in $\zeta$ of $\sim 8\%$ that give similar changes on $I_\gamma$. Variations in the primordial index $n$ of the perturbation spectra yield variations of order $\sim (n + 3)/6$ on $I_\gamma$.

Thus, while the level of the DGRB predicted for a specific choice of the cluster evolution models are at the level of $0.5 \div 0.8\%$ of the EGRET diffuse flux, more conservative estimates of $I_\gamma$ require to consider the theoretical uncertainties in the modelling of cluster structure and evolution.

Theoretical uncertainties in the description of the relevant quantities of cluster structure yield variations of the results contained within $\lesssim$ a few %. Let us be more specific.

Our results are based on the theoretical description of cluster evolution through the spherical top-hat model and the PS mass function. Despite the inherent uncertainties in the detailed models for cluster collapse, the final results depend on the combination $b z_0$, that is fixed by the fitting procedure to the local cluster abundance with uncertainties $\sim 5 \div 8\%$ [29]. Moreover, our results on $I_{\gamma,\nu}$ are weakly sensitive to the lower cutoff, $M_{\text{min}}$, in the MF (see eqs 35-39). This reduces the effects of possible changes of the slope of the MF in the efforts to go beyond the simple, first-order theory for the origin of the universal MF (see [48] for a discussion).

Uncertainties in the description of the IGM evolution (both its normalization, $f_{g0} \sim 0.1 \div 0.3$ at the scale of rich clusters, and its evolution with time parametrized in terms of the parameters $\eta \sim 0 \div 0.9$ and $s \sim 0 \div 1.5$) are discussed to point out the role of different, possible gas-phase phenomena in cluster evolution. Here we take into account the minimal and maximal levels of IGM evolution that are still consistent - given the observational uncertainties - with the current data ([65] [25] [26] [28]).

So, in Fig. 8 we show the possible range of $I_\gamma$ produced by a population of galaxy clusters considering reasonable uncertainties in the theoretical description of the clusters structure and of their distribution in mass for the considered cosmological models. According to our previous estimation of the uncer-
tain ties, it is reasonable to expect that galaxy clusters could provide up to a few \% of the EGRET [1] DGRB. Higher values of \( I_{\gamma} \) would require a choice \( L_{CR} \gg 10^{41} \text{ erg/s} \), or a large contribution from galaxy groups: both these possibilities seem to be quite unrealistic.

7 High energy neutrinos from clusters of galaxies

Gamma rays from galaxy clusters are always accompanied by neutrino production due to the decay of charged pions, according with eq.(2). In Fig.9 we show the range of cluster contribution to the diffuse neutrino background (DNB), \( I_{\nu} \), as a function of the neutrino energy \( E \), expected in the viable models for structure formation. We considered in the predictions of Fig.9 the level of theoretical uncertainties in the description of the cluster structure and evolution (see Sect.6 for a discussion). We considered the whole contributions due to the fluxes of \( \nu_\mu \) and \( \nu_e \), and we compare them to the level of the atmospheric neutrinos as given by Gondolo et al. [77]. The diffuse \( \nu \) flux produced by clusters is at a level of \( F_{\nu} \gtrsim 10^{-2} \text{ neutrinos km}^{-2} \text{ yr}^{-1} \text{ sr}^{-1} \text{ GeV}^{-1} \) at \( E \gtrsim 3 \cdot 10^{5} \text{ GeV} \), where the spectral shape of the cluster fluxes \( (\propto E^{-2^{1/2}3}) \) emerges from the \( E \) distribution of the atmospheric \( \nu \)'s (see Fig.9). However, such an estimate has to be considered quite optimistic because it refers to the specific choice of parameters that maximize \( I_{\gamma} \). In such a maximal model for \( I_{\gamma} \), at \( E = 10^{3} \text{ GeV} \) one could observe a sensitive diffuse flux from galaxy clusters at a level \( I_{\gamma} \sim 30 \div 2 \cdot 10^{3} \text{ neutrinos km}^{-2} \text{ yr}^{-1} \text{ sr}^{-1} \text{ GeV}^{-1} \). This diffuse emission could be one of the most intense neutrino backgrounds of cosmological origin at these energies, comparable only to that produced by unresolved AGN cores [71]. Single clusters could be detected with the next generation, large neutrino telescopes as the signals from rich, nearby Coma-like clusters are of the order \( F_{\nu} \approx 10^{3} \div 10^{4} \text{ neutrinos km}^{-2} \text{ yr}^{-1} \) (at \( z \lesssim 0.05 \)) at \( E \sim 10^{3} \text{ GeV} \). Much distant clusters are more difficult to be detected as their fluxes scale as \( \propto d_{L}^{-2} \) \( (z, \Omega_0) \) [see eqs.(34-35) and Fig.2].

8 Discussion and conclusions

In this paper we presented a detailed study of the diffuse emission of \( \gamma \)-rays and neutrinos from clusters of galaxies. Using realistic modelling of the cluster structure, of their formation history and of their evolution with cosmic time, we found that galaxy clusters can provide \( \lesssim 1\% \) of the DGRB measured by EGRET (in the first release by OWZ [1]).

Our estimate of \( I_{\gamma} \) is quite independent on the geometry of the universe, on the assumed cosmological model and on the amount of IGM evolution,
because most of their contribution to the DGRB comes from nearby, $z \lesssim 0.2$, clusters. In fact, at these redshifts the effects of curvature do not take place strongly in changing the perturbation growth factor, $\mathcal{D}(z, \Omega_0)$, (normalized at the present epoch), the difference in cluster evolution are small when the different models are normalized to the local abundance of clusters observed in X-rays and the available amount of IGM evolution - even if considered to be quite strong, $f_g \propto (1 + z)^{-1.5-2}$ - can provide only small variations to the cluster $\gamma$-ray luminosities, as $L_\gamma \propto f_g$ (see eq.18). On account of all these aspects, we consider that our results for the contribution of galaxy clusters to the DGRB are quite robust.

Our approach differs substantially from the previous ones in several (among others) aspects:

\(i\) we considered - differently from all the previous approaches - a self-consistent approach to the formation of clusters following the spherical collapse model [69] complemented with a realistic IGM density profile, consistent with the most recent determinations from X-ray observations. This fact has important effects on the CR confinement within cluster cores and hence on the relative $\gamma$-ray and neutrino emission rates;

\(ii\) we considered (as BBP did) here an energy dependent diffusion coefficient which results in a very general picture of the CR confinement within cluster cores;

\(iii\) we also considered here - at variance with the previous approaches - the effects of a possible evolution in the cluster IGM content. This is consistent with the present indications of a variation in the IGM content from groups to rich clusters in the local frame and with the X-ray, shock (or entropy) induced, luminosity evolution observed from numerical simulations [42] and predicted in analytical models (both shock and entropy models) for the evolution of X-ray clusters [48] [40];

\(iv\) we use the PS cluster MF that was found to be consistent with N-body simulations over a large dynamical range and up to $z \gtrsim 2$ [72]. We normalized it to the local abundance of clusters detected in X-rays. In the previous approaches average values for the overall cluster abundance, $n_{cl} \approx 4 \div 7 \cdot 10^{-5} \text{Mpc}^{-3}$, were used [11] [13] without considering the effect of an evolving cluster mass function;

\(v\) using a self-consistent modelling of the IGM we found an analytical correlation between $\gamma$-ray and X-ray emission for clusters. The predicted ratio $F_\gamma/F_X \propto f_g^{-1} r_e^{-1} T^{-1/2}$ [see eq.(32)] provides a behaviour of the $F_\gamma - F_X$ relation different from that obtained by Ensslin et al. [20], $F_\gamma/F_X \propto T^{1/2}$, because we did not assume any (partial) equipartition between IGM thermal energy
and relativistic jet particles. Our correlation results only from the basic electromagnetic and hadronic emission mechanisms in which the IGM protons are the targets for both the X-ray thermal bremsstrahlung emission and for the pp collisions responsible for γ-rays.

Using such a correlation we derived a sample of nearby clusters with predicted γ-ray fluxes observable with the next generation γ-ray telescopes. Incidentally, we found a γ-ray flux for Coma, \( F_{\gamma}(>100\,\text{MeV}) \approx 8.5 \times 10^{-9} \, \text{photon s}^{-1} \, \text{cm}^{-2} \) which is consistent with the EGRET upper limits for this cluster (previous specific predictions \([12, 20]\) seem to exceed the EGRET upper limit).

Our numerical results for \( I_{\gamma,\nu} \) are in reasonable agreement with those obtained by BBP, even though based on a quite different description of the cluster structure and evolution. This agreement is due to the fact that BBP considered a constant comoving density of clusters, \( n_{cl} \sim 5 \times 10^{-5} \, \text{Mpc}^{-3} \), assumed to be a fair sample of the baryons in the universe, and containing a fraction \( \Omega_b \approx 0.5 \Omega_{BBN} \) (where \( \Omega_{BBN} \) is the value of the baryon density predicted by Big Bang Nucleosynthesis). Under these assumptions, BBP obtained a value for \( I_{\gamma} \) higher by a factor \( \sim 3 \div 4 \) with respect to our result, based on values \( f_g \sim 0.1 \) (see Sect.2). Our refined calculations show why their assumption of considering \( n_{cl} \sim \text{const} \) was reasonable: the clusters effectively contributing to the DGRB are located at \( z \lesssim 0.2 \) (see Fig.5), where the effects of evolution do not have room to take definitely place (see Fig.7).

Because of the inherent uncertainties in the predictions of quantities whose calculation involve to set the values of parameters which are not known precisely, we also estimated the range spanned by \( I_{\gamma,\nu} \) for the combination of parameters allowed by the present observational ranges. In fact, the description of the cluster structure and evolution that we used in our analytic approach consider only ensemble averaged quantities. But we observe a whole distribution of the real cluster properties with respect to the average cluster moulding. Some amount of variance is needed to be considered in cluster modelling in order to ensure the predictive power of the viable models for structure formation. To explore the role of the uncertainties in the relevant quantities we considered several sources of uncertainties (see Sect.7).

From an inspection of Figs. 9 and 10 we note that the effects of the possible theoretical uncertainties in the description of the cluster and IGM evolution could change the predicted contributions for the DGRB and for the DNB by a factor \( \lesssim 3 \), setting the maximal level of \( I_{\gamma} \) to a few % of the EGRET value.

The DGRB seems to be mostly produced by AGNs (FSRQ and/or BLLacs) and/or blazars (we take here an estimate of \( \sim 60 \div 65\% \) [73] of the EGRET diffuse flux [1]). Diffuse γ-ray emission could be observed also from normal
galaxies yielding a contribution $\sim 5\%$ [73]. When added to the $\sim 10 \div 15\%$ of the DGRB contributed by their high-$E$ photons interacting with other existing backgrounds (e.g. IR, CMB [73]) one gets only $\sim 15 \div 20\%$ of the DGRB left for truly diffuse or extended sources. Of this amount, a fraction of the diffuse $\gamma$-ray flux $\sim 3 \div 5\%$ is predicted [74] to originate from decaying topological defects (see [75]) and interactions of UHE particles with the CMB. Note, however, that the amount and the spectral distribution of this possible diffuse background depend sensitively on the amplitude of the primordial magnetic field on scales larger than supercluster sizes. So, according to the previous estimates, the presence of all these sources of diffuse $\gamma$-ray emission (even though partially model dependent) determines an upper limit to the contribution of extended extragalactic sources to the DGRB, that is $\sim 10 \div 22\%$ of the OWZ EGRET level [1]. This sets rather weak constraints on the level of CR production in clusters and hence on the presence and activity of AGNs in clusters or on the formation and efficiency of accretion shocks around clusters. However, if we consider the revised level of the DGRB as derived by SWZ [2], then the previous upper limit reduces to $\lesssim 2\%$. Our predictions of the DGRB contributed by galaxy clusters $I_{\gamma,cl}/I_{EGRET} \sim 0.005 \pm 0.02$ is perfectly compatible with the presence of both a population of evolving FSRQ and AGNs dominating the $\gamma$-ray sky and with the presence of truly diffuse backgrounds like those previously discussed. Note, however, that the major source of uncertainty in the level of the extragalactic DGRB comes from the contribution of the AGNs. A fluctuation analysis of the EGRET data is needed to have more definite indications on the level of the DGRB contributed by discrete sources. If, on the other hand, CR acceleration will be found to be relevant in clusters (yielding $L_{CR}$ substantially larger than $10^{44}$ erg/s), then the predicted level of DGRB produced by galaxy clusters can set interesting limits to the space density and evolution of $\gamma$-ray AGNs.

The sensitivities and angular resolutions achievable by the next generation gamma-ray (INTEGRAL, GLAST, AMS) and neutrino (see [76] for a review) detectors will be able to shed a new light on the high energy phenomena occurring in large scale structures.

Acknowledgements We thank the Referee for useful comments and suggestions which improved substantially the presentation of the paper. We also acknowledge interesting and stimulating discussions with V.S. Berezinsky during a recent visit of S.C. at the LNGS. S.C. acknowledges also interesting discussions with G. Kanbach, A. Dar and F. Halzen, among others, at the 1997 Moriond Meeting High Energy Phenomena in the Universe. The research of P.B. is funded by a INFN Post-Doctoral Fellowship at the University of Chicago.
References


26
[73] Erlykin et al. 1996, A&A Suppl. 120, 623
Table 1. A list of predicted $\gamma$-ray fluxes from clusters of galaxies.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$z$</th>
<th>$n_c^{(a)}$</th>
<th>$T$ (keV)</th>
<th>$r_c$ (Mpc)</th>
<th>$\beta$</th>
<th>$F_X^{(b)}$</th>
<th>$F_\gamma^{(c)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1060</td>
<td>0.0111</td>
<td>4.55</td>
<td>3.9</td>
<td>0.1</td>
<td>0.67</td>
<td>0.488</td>
<td>0.457</td>
</tr>
<tr>
<td>A262</td>
<td>0.0168</td>
<td>4.45</td>
<td>2.4</td>
<td>0.095</td>
<td>0.55</td>
<td>0.224</td>
<td>0.149</td>
</tr>
<tr>
<td>A426</td>
<td>0.0183</td>
<td>4.55</td>
<td>6.3</td>
<td>0.285</td>
<td>0.57</td>
<td>7.50</td>
<td>1.136</td>
</tr>
<tr>
<td>A1367</td>
<td>0.0213</td>
<td>0.95</td>
<td>3.7</td>
<td>0.43</td>
<td>0.53</td>
<td>0.335</td>
<td>0.163</td>
</tr>
<tr>
<td>A400</td>
<td>0.0231</td>
<td>1.75</td>
<td>2.5</td>
<td>0.165</td>
<td>0.57</td>
<td>0.087</td>
<td>0.094</td>
</tr>
<tr>
<td>A1656</td>
<td>0.0232</td>
<td>2.89</td>
<td>8.3</td>
<td>0.42</td>
<td>0.75</td>
<td>2.51</td>
<td>0.85</td>
</tr>
<tr>
<td>A2199</td>
<td>0.0305</td>
<td>8.8</td>
<td>4.5</td>
<td>0.14</td>
<td>0.68</td>
<td>0.694</td>
<td>0.234</td>
</tr>
<tr>
<td>A2063</td>
<td>0.0337</td>
<td>4.15</td>
<td>4.1</td>
<td>0.175</td>
<td>0.62</td>
<td>0.246</td>
<td>0.110</td>
</tr>
<tr>
<td>A576</td>
<td>0.0392</td>
<td>4.05</td>
<td>4.3</td>
<td>0.115</td>
<td>0.49</td>
<td>0.197</td>
<td>0.059</td>
</tr>
<tr>
<td>A2657</td>
<td>0.0414</td>
<td>3.6</td>
<td>3.4</td>
<td>0.145</td>
<td>0.53</td>
<td>0.157</td>
<td>0.062</td>
</tr>
<tr>
<td>A2319</td>
<td>0.0529</td>
<td>3.1</td>
<td>9.9</td>
<td>0.41</td>
<td>0.68</td>
<td>1.21</td>
<td>0.267</td>
</tr>
<tr>
<td>A85</td>
<td>0.0556</td>
<td>5.0</td>
<td>6.2</td>
<td>0.225</td>
<td>0.62</td>
<td>0.622</td>
<td>0.145</td>
</tr>
<tr>
<td>A2256</td>
<td>0.0601</td>
<td>2.45</td>
<td>4.3</td>
<td>0.45</td>
<td>0.73</td>
<td>0.505</td>
<td>0.242</td>
</tr>
<tr>
<td>A1795</td>
<td>0.0621</td>
<td>5.8</td>
<td>5.8</td>
<td>0.3</td>
<td>0.72</td>
<td>0.519</td>
<td>0.130</td>
</tr>
<tr>
<td>A1775</td>
<td>0.0709</td>
<td>4.15</td>
<td>4.9</td>
<td>0.185</td>
<td>0.66</td>
<td>0.097</td>
<td>0.046</td>
</tr>
<tr>
<td>A399</td>
<td>0.0715</td>
<td>3.05</td>
<td>5.8</td>
<td>0.215</td>
<td>0.52</td>
<td>0.342</td>
<td>0.078</td>
</tr>
</tbody>
</table>

Table caption

\(^{(a)}\) $n_c$ in units $10^{-3}$ cm$^{-3}$.
\(^{(b)}\) X-ray fluxes in the $(2 - 10)$ keV band in units $10^{-10}$ erg cm$^{-2}$ s$^{-1}$.
\(^{(c)}\) Gamma ray fluxes $F_\gamma (> 100 \text{MeV})$ in units $10^{-8}$ photons s$^{-1}$ cm$^{-2}$. 
Fig. 1. Gamma ray luminosities for clusters in different cosmological scenarios: CDM ($\Omega_0 = 1$; dashed curve), CDM+$\Lambda$ ($\Omega_0 = 0.4$; continuous curve), open CDM ($\Omega_0 = 0.3$; long-dashes curve) and mixed DM ($\Omega_0 = 1$, $\Omega_0 = 0.3$; dotted curve).
Fig. 2. Gamma ray fluxes for clusters at $z = 0.023$, the redshift of the Coma cluster. The arrow indicates the EGRET upper limit for A1656 (Coma). Curves labelled as in Fig.1.
Fig. 3. Cluster $\gamma$-ray fluxes for the objects listed in Table 1 plotted against their IGM temperature (filled dots). The curves represent the $F_\gamma - T$ relationship in a CDM+$\Lambda$ ($\Omega_0 = 0.4, h = 0.6$) cosmology at redshift $z = 0.023$ (continuous curve) and at the maximum redshift of the clusters in our sample (dashed curve).
Fig. 4. The frequency of clusters with a predicted $\gamma$-ray flux in the sample of Table 1.
Fig. 5. The redshift distribution of clusters with fluxes $F_\gamma (> 100\text{MeV})$ in a CDM model with ($\eta = 0.2, s = 1.8$: continuous histogram) and without ($\eta = 0.2, s = 0$: dotted histogram) IGM evolution (see text for details).
Fig. 6. The contribution of galaxy clusters to the DGRB in different cosmological models: CDM ($\Omega_0 = 1$; dashed curve), CDM+$\Lambda$ ($\Omega_0 = 0.4$; continuous curve), open CDM ($\Omega_0 = 0.3$; long-dashes curve) and mixed DM ($\Omega_0 = 1$, $\Omega_\nu = 0.3$; dotted curve).
Fig. 7. The predicted level of DGRB from galaxy clusters in a CDM+Λ
($\Omega_0 = 0.4, h = 0.6$) model with ($\eta = 0.2, s = 2.2$: continuous curve) and without
($\eta = 0, s = 0$ dashed curve) IGM evolution (see text for details).
Fig. 8. The expected DGRB from clusters, considering various sources of theoretical uncertainties in the cluster modelling. A flat CDM ($\Omega_\Lambda = 1; h = 0.5; n = 1$) model is considered here.
Fig. 9. Same as Fig.8 but for the diffuse $\nu$ background. Heavy dashed line show the diffuse flux from atmospheric $\nu$'s (Gondolo et al. 1995).