Baryon Magnetic Moments and Quark Orbital Motion
in
the Chiral Quark model

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Abstract

Using the unified scheme for describing both quark spin and orbital angular momenta in the chiral quark model developed in the previous work, the magnetic moments of octet and decuplet baryons are calculated. The numerical result shows that the overall agreement with data is improved by including the orbital contributions.

13.40.Em, 13.88.+e, 12.39.Fe, 14.20.Dh
I. Introduction

Although the naive SU(6) constituent quark model (NQM) is attractively simple, it has had only limited quantitative success in accounting for the magnetic moments and semileptonic decays of the baryons. In the NQM all three quarks in the nucleon are assumed to be in the s-wave state, the nucleon spin is completely attributed to the quark spin and the orbital angular momentum (OAM) is zero \( <L_z^q> = 0 \). The EMC [1] and recent experiments [2–4] on deep inelastic lepton-nucleon scatterings (DIS) show that the quark spin in NQM cannot account for the proton spin and lead to ‘spin crisis’. In an earlier work [5], by relating the quark spin fractions, nucleon magnetic moments and the weak axial coupling constants of baryons, Sehgal shown that a large portion of nucleon spin arises from the orbital motion of the constituents. This idea has been generalized to all octet baryons to explain both the DIS quark spin fractions and baryon magnetic moments [6–8]. In these works, the quark orbital contributions are not explicitly included. In the past decade, the important role of the OAM in the nucleon has been discussed in different models and various forms, an incomplete list see [9–14]. In our previous paper [14], a unified scheme for describing both quark spin and orbital angular momenta in the symmetry breaking chiral quark model has been suggested. The spin and orbital angular momenta carried by the quarks and antiquarks are evaluated. In this paper, we use the results given in [14] to calculate the baryon magnetic moments and discuss the effects of the orbital contributions.

The magnetic moment of the baryon \( B \) can be written as

\[
\mu_B = \sum_q \mu_q ((\Delta q)^B - (\Delta \bar{q})^B) + <L_z^q>^B - <L_z^\bar{q}>^B
\]

where \( \mu_q \)s are the magnetic moments of quarks, \( \Delta q \equiv q_\uparrow - q_\downarrow \) and \( \Delta \bar{q} \equiv \bar{q}_\uparrow - \bar{q}_\downarrow \), and the notation \( B \) denotes the member of the baryon octet or decuplet. \( q_\uparrow, q_\downarrow (\bar{q}_\uparrow, \bar{q}_\downarrow) \) are quark (antiquark) numbers of spin parallel and antiparallel to the nucleon spin, or more precisely, quark numbers of positive and negative helicities, if the proton helicity is chosen to be \(+1/2\). \( <L_z^q>^B, <L_z^\bar{q}>^B \) denotes the total orbital angular momentum carried by quarks and antiquarks in the baryon \( B \). In the zeroth order approximation of the chiral quark model, the baryon is assumed to contain only three valence quarks. The antiquarks are produced from the first order chiral splitting processes \( q \to q' + GB(q \bar{q}) \), where GB denotes the Goldstone boson. We assume that the magnetic moment of the baryon is the sum of spin and orbital magnetic moments of individual charged particles (quarks or antiquarks). The assumption of additivity is commonly believed to be a good approximation for a loosely bound composite system, which is the basic description for the baryon in the effective chiral quark model. In reality the baryon may contains other neutral particles, such as gluons, as pointed out in [17]. Although the gluons do not make any contribution to the magnetic moment in (1a), the existence of intrinsic gluons would significantly change the valence quark structure of the baryon due to the spin and color couplings between the gluon and quarks. For example, the total color charge for the gluon-quark system must be zero, and the total angular momentum must be 1/2. We will discuss a hybrid quark-gluon mixing model of the octet baryon in Sec. IV.

Since in the chiral quark model all antiquark sea polarizations are zero, \( \Delta \bar{q}^B = 0 \) (the experiment [18] shown that the antiquark sea polarization is rather small), hence Eq.(1a)
can be approximately can be rewritten as

\[
\mu_B = \sum_{q=u,d,s} \mu_q [(\Delta q)^B + < L_z >_q^B - < L_z >_q^B] \tag{1b}
\]

where only three flavors of quark, \(u\), \(d\), and \(s\) are considered. To calculate the baryon magnetic moments (1b), we need to know not only the spin contents \(\Delta q\), but also the orbital contents \(< L_z >_q^B\) and \(< L_z >_q^B\) for all active quark (antiquark) flavors. All these quantities have been obtained in [14]. For the purpose of later use, we briefly review several key points and main results (Section II). The detail discussion can be found in [14].

II. Spin and Orbital Motions in Chiral Quark Model.

A. Chiral quark model.

The basic assumptions of the chiral quark model we used are: (i) the nucleon flavor, spin and orbital contents are determined by its valence quark structure and all possible chiral fluctuations \(q \rightarrow q' + GB\), (ii) the probabilities of the chiral splittings are rather small, one can treat the fluctuation \(q \rightarrow q' + GB\) as a small perturbation, for instance, the probability for the splitting \(u \rightarrow d + \pi^-\), or \(d \rightarrow u + \pi^+\) is about 0.10 – 0.15. The contributions from the higher order fluctuations can be neglected \((a^2 << 1)\).

The effective Lagrangian describing interaction between quarks and the octet Goldstone bosons and singlet \(\eta'\) is

\[
L_I = g_8 \bar{q} \begin{pmatrix} (GB)^0_+ & \pi^+ & \sqrt{\epsilon} K^+ \\ -\pi^- & (GB)^0_- & \sqrt{\epsilon} K^0 \\ -\sqrt{\epsilon} K^- & \sqrt{\epsilon} K^0 & (GB)^0_s \end{pmatrix} q, \tag{2a}
\]

where \((GB)^0_\pm = \pm \pi^0/\sqrt{2} + \sqrt{\epsilon} \eta^0/\sqrt{6} + \zeta' \eta^0/\sqrt{3}\), \((GB)^0_s = -\sqrt{\epsilon} \eta^0/\sqrt{6} + \zeta' \eta^0/\sqrt{3}\), and the symmetry breakings are explicitly included. The transition probability of chiral splitting \(u(d) \rightarrow d(u) + \pi^+(-)\) is \(a \equiv |g_8|^2\), and \(\epsilon a\) denotes the probability of \(u(d) \rightarrow s + K^-\). Similar definitions are used for \(\epsilon_\eta a\) and \(\zeta'^2 a\). Considering the mass suppression effect, one expects \(0 \leq \zeta'^2 \leq 1\), \(0 \leq \epsilon_\eta \leq 1\), and \(0 \leq \epsilon \leq 1\).

The important feature of the chiral fluctuation is that due to the coupling between the quarks and GB’s, a quark flips its spin or helicity and changes (or maintains) its flavor by emitting a charged (or neutral) Goldstone bosons. Therefore the flavor, spin, and orbital contents carried by the quark and antiquarks are significantly different from those obtained without the chiral fluctuations.

For spin-up or spin-down valence \(u\), \(d\), and \(s\) quarks, up to the first order fluctuation, the allowed processes are

\[
u_{\uparrow,\downarrow} \rightarrow d_{\downarrow,\uparrow} + \pi^+ , \quad u_{\downarrow,\uparrow} \rightarrow s_{\downarrow,\uparrow} + K^+ , \quad u_{\uparrow,\downarrow} \rightarrow u_{\downarrow,\uparrow} + (GB)^0_+ , \quad u_{\uparrow,\downarrow} \rightarrow u_{\uparrow,\downarrow} . \tag{3a} \]

\[
d_{\uparrow,\downarrow} \rightarrow u_{\downarrow,\uparrow} + \pi^- , \quad d_{\downarrow,\uparrow} \rightarrow s_{\downarrow,\uparrow} + K^0 , \quad d_{\uparrow,\downarrow} \rightarrow d_{\uparrow,\downarrow} + (GB)^0_-, \quad d_{\uparrow,\downarrow} \rightarrow d_{\uparrow,\downarrow} . \tag{3b} \]

\[
s_{\uparrow,\downarrow} \rightarrow u_{\downarrow,\uparrow} + K^- , \quad s_{\downarrow,\uparrow} \rightarrow d_{\downarrow,\uparrow} + K^0 , \quad s_{\uparrow,\downarrow} \rightarrow s_{\uparrow,\downarrow} + (GB)^0_s , \quad s_{\uparrow,\downarrow} \rightarrow s_{\uparrow,\downarrow} . \tag{3c} \]

The quark helicity flips in the chiral splitting processes \(q_{\uparrow,\downarrow} \rightarrow q_{\downarrow,\uparrow} + GB\), i.e. the first three processes in each of (3a), (3b), and (3c). The consequences are, (i) the total spin content carried by quarks and antiquarks would be smaller than that without considering the chiral splitting, (ii) most importantly, since the quark spin flips (or helicity sign changes)
in the fluctuations with GB emission, the quark spin component changes one unit of angular
momentum, \((s_z)_f - (s_z)_i = +1\) or \(-1\), the angular momentum conservation requires the same
amount change of the orbital angular momentum but with opposite sign, i.e. \((L_z)_f - (L_z)_i =
-1\) or \(+1\). This induced orbital motion distributes among the quarks and antiquarks after
the splitting. These two points are intimately related.

The following combinations of the parameters \(\epsilon, \epsilon_\eta, \) and \(\zeta'\) are useful in our formalism,

\[
A \equiv 1 - \zeta' + \frac{1 - \sqrt{3\epsilon_\eta}}{2}, \quad B \equiv \zeta - \sqrt{\epsilon_\eta} \quad C \equiv \zeta' + 2\sqrt{\epsilon_\eta} \tag{4a}
\]

\[
f \equiv \frac{1}{2} + \frac{\epsilon_\eta}{6} + \frac{\zeta'^2}{3}, \quad f_s \equiv \frac{2\epsilon_\eta}{3} + \frac{\zeta'^2}{3} \tag{4b}
\]

and

\[
\xi_1 \equiv 1 + \epsilon + f, \quad \xi_2 \equiv 2\epsilon + f_s \tag{4c}
\]

The special combinations \(A, B\) and \(C\) stem from the combinations of the octet and singlet
neutral bosons appeared in the effective chiral Lagrangian, while \(f\) and \(f_s\) stand for the
transition probabilities of the chiral splittings \(u_+(d^-) \rightarrow u_+(d^-) + (GB)^0_{s (+)}\) and \(s_+ \rightarrow s_+ +
(GB)^0_s\) respectively. It is easy to see that the total transition probability of the first three
processes in (3a), or (3b) is \(\xi_1 a\), and the corresponding probability in (3c) is \(\xi_2 a\).

B. Quark spin contents

Denoting the valence quark numbers in the baryon as \(n_{B}^{(v)}(u_+), n_{B}^{(v)}(u_-), n_{B}^{(v)}(d_+), n_{B}^{(v)}(d_-),
n_{B}^{(v)}(s_+), \) and \(n_{B}^{(v)}(s_-)\), the spin-up and spin-down quark (or antiquark) contents in the baryon
\(B\), up to the first order fluctuation, are

\[
n_B(q_{\uparrow, \downarrow}^\prime, or \bar{q}^\prime_{\uparrow, \downarrow}) = \sum_{q=u,d,s} \sum_{h=\uparrow, \downarrow} n_{B}^{(v)}(q_h) P_{q_h}(q_{\uparrow, \downarrow}^\prime, or \bar{q}_{\uparrow, \downarrow}^\prime) \tag{5}
\]

where \(P_{q_{\uparrow, \downarrow}^\prime}(q_{\uparrow, \downarrow}^\prime)\) and \(P_{q_{\uparrow, \downarrow}^\prime}(\bar{q}_{\uparrow, \downarrow}^\prime)\) are the probabilities of finding a quark \(q_{\uparrow, \downarrow}^\prime\) or an antiquark
\(\bar{q}_{\uparrow, \downarrow}^\prime\) arise from all chiral fluctuations of a valence quark \(q_{\uparrow, \downarrow}^\prime\). The probabilities, \(P_{q_{\uparrow, \downarrow}^\prime}(q_{\uparrow, \downarrow}^\prime)\) and
\(P_{q_{\uparrow, \downarrow}^\prime}(\bar{q}_{\uparrow, \downarrow}^\prime)\), depend on the effective interaction Lagrangian (2). They were given in Table I
in [14]. The spin-weighted quark contents are

\[
(\Delta q^B) = \sum_q [n_{B}^{(v)}(q_{\uparrow}) - n_{B}^{(v)}(q_{\downarrow})][P_{q_{\uparrow}}(q_{\uparrow}) - P_{q_{\downarrow}}(q_{\downarrow})] \tag{6a}
\]

while the spin-weighted antiquark contents are zero

\[
(\Delta \bar{q}^B) = 0, \tag{6b}
\]

due to \(P_{\bar{q}_{\uparrow}}(q_{\uparrow}) = P_{q_{\downarrow}}(q_{\downarrow}), P_{q_{\downarrow}}(q_{\downarrow}) = P_{q_{\uparrow}}(q_{\uparrow}),\) and \(P_{q_{\downarrow}}(q_{\downarrow}) = P_{q_{\uparrow}}(q_{\uparrow}) = P_{q_{\downarrow}}(q_{\uparrow}) = P_{q_{\downarrow}}(q_{\uparrow}).\) In
general the probabilities \(P_{q_{\uparrow, \downarrow}^\prime}(q_{\uparrow, \downarrow}^\prime)\) may vary with the baryons, because the suppression
effects may be different in different baryons. For the sake of simplicity, we assume that they
are universal for all baryons. Hence the \(B\)-dependence of the spin-weighted quark contents
appears only in the \(B\)-dependence of \(n_{B}^{(v)}(q_{\uparrow, \downarrow}^\prime)\).

C. Quark orbital angular momentum
In the quark splitting process, the flip of the quark spin will induce an orbital angular momentum. We assume that the induced orbital motion is equally shared by the quarks and antiquarks after splitting and introduce a partition factor $k$. Assuming the Goldstone boson has a simple quark structure, i.e. each boson consists of a quark and an antiquark, one has two quarks and one antiquark (total number is three) after each splitting $q \to q' + GB$. Hence up to first order splitting, one has $k = 1/3$. The last processes in (3a), (3b), and (3c), make no contributions to the orbital motion. Similar to the probabilities $P_{q\gamma \downarrow}(q'\uparrow)$ and $P_{q\gamma \uparrow}(q'\downarrow)$, we define $<L_z>_{q'/q\gamma}$ ($<L_z>_{q'/q\gamma}$) as the OAM carried by a specific quark $q'$ (antiquark $\bar{q}$), arises from a valence spin-up quark $q_c$ fluctuates into all allowed final states except for no emission case. The quantities $<L_z>_{q'/q\gamma}$ and $<L_z>_{q'/q\gamma}$ for $q = u, d, s$ also depend on the effective Lagrangian (2) and have been given in Table II in Ref. [14]. Again, we assume that the probabilities $<L_z>_{q'/q\gamma}$ and $<L_z>_{q'/q\gamma}$ are universal for all baryons.

The difference between the orbital angular momentum carried by quark $q$ and that carried by corresponding antiquark $\bar{q}$, for example for $u$-quark, is

$$<L_z>_B^B - <L_z>_B^u = \sum_q [n_B^{(v)}(q\uparrow) - n_B^{(v)}(q\downarrow)] [<L_z>_u/q\gamma - <L_z>_{\bar{u}/q\gamma}]$$

(7)

similar equations hold for $d$-quark and $s$-quark, and corresponding antiquarks. Where $\sum$ summed over all valence quarks in the baryon $B$.

Eqs.(1b), (6a), (6b) and (7) are main formulae to calculate the baryon magnetic moments. Since the quantities $P_{q\gamma}(q'\downarrow, \gamma\downarrow)$ and $<L_z>_q/q\gamma$ are known (Tables I and II in [14]) and universal for all baryons, we only need to know the valence quark numbers $n_B^{(v)}(q\uparrow)$ in a specific baryon $B$, and these numbers depend on the models of baryon. For our purpose of showing the effects of the orbital angular momentum, we only consider two special cases of quark valence structure: (1) static $SU(3) \otimes SU(2)$ model (Sec. III), and (2) hybrid quark-gluon mixing model (Sec. IV). We note that similar discussions on the baryon magnetic moments in the chiral quark model without considering the orbital contributions were given in [19,20]. We also note that a different version of including the orbital contribution in a simple $SU(3)$ symmetry chiral quark model was independently discussed in [13].

III. $SU(3)_f \otimes SU(2)_s$, Valence Structure (Model I).

A. Octet baryons.

Assuming the flavor-spin piece of the valence quark structure is $SU(3)_f \otimes SU(2)_s$, the valence quark numbers in the proton are

$$n_p^{(v)}(u\uparrow) = \frac{5}{3}, \quad n_p^{(v)}(u\downarrow) = \frac{1}{3}, \quad n_p^{(v)}(d\uparrow) = \frac{1}{3}, \quad n_p^{(v)}(d\downarrow) = \frac{2}{3}, \quad n_p^{(v)}(s\uparrow,\downarrow) = 0,$$

(8)

where $n_p^{(v)}(s\uparrow,\downarrow) = 0$ is due to no valence strange quarks exist in the proton. The quark spin contents $\Delta q$ and the difference between the quark and antiquark orbital angular momenta $<L_z>_u/d/s - <L_z>_{\bar{u}/d/s}$ in the octet baryons are listed in Table I. Using (1b) and Table I, the magnetic moments of octet baryons can be obtained. We make some remarks before going to the numerical results.

It is easy to verify that the magnetic moments of the octet baryon satisfy the following sum rules

$$(4.70) \quad \mu_p - \mu_n = \mu_{\Sigma^+} - \mu_{\Sigma^-} - (\mu_{\Xi^0} - \mu_{\Xi^-})$$

(4.22)
where the values of the two sides taken from the data [21] are shown in parentheses. The relations (9a) and (9b) were first given by Franklin in [15]. One can also show that our nonlinear sum rule (9c) is equivalent to Eq. (16) given in [16]. The relations (9a), (9b), and (9c) are not new and violated at about 10−15% level. They have been discussed in many works, for instance [6–8]. However, the new relation (9d) is very well satisfied. All these sum rules (9a)-(9d) also hold in the simple quark model. Our result shows that if the SU(3) ⊗ SU(2) valence quark structure is used, the chiral fluctuations cannot change these sum rules even the orbital angular momenta are included. Furthermore, we have shown in [8] that the sum rules (9a)-(9c) also hold for more general case.

Explicitly, (9a)-(9d) can be written as

\[ \mu_p - \mu_n = \frac{5}{3} \delta_1 (\mu_u - \mu_d) \]  
(10a)

\[ \mu_\Lambda = -a \epsilon (\mu_u + \mu_d) + \delta_2 \mu_s \]  
(10b)

\[ \mu_p^2 - \mu_n^2 = \frac{5}{3} \delta_1 (\mu_u - \mu_d) [(\delta_1 + 2a) (\mu_u + \mu_d) - 2a \epsilon \mu_s] \]  
(10c)

\[ \mu_p - \mu_{\Sigma^+} = -\frac{1}{3} \delta_3 (\mu_d - \mu_s) \]  
(10d)

where \( \delta_1, \delta_2, \) and \( \delta_3 \) are defined as

\[ \delta_1 = 1 - a(2 \xi_1' - \epsilon - 2) \]  
(11a)

\[ \delta_2 = 1 - 2a(\xi_1' - \epsilon) \]  
(11b)

\[ \delta_3 = 1 - a[(1 - \epsilon) + r_d (2 \xi_1' - 2 \epsilon - 1) - r_s (2 \xi_2' - 3 \epsilon)]/(r_d - r_s) \]  
(11c)

where \( r_d,s \equiv \mu_{d,s}/\mu_u. \) If there are no chiral fluctuations, then \( a = 0, \delta_{1,2,3} \rightarrow 1, \) and (10a)-(10d) reduce to the simple quark model results

\[ \mu_p - \mu_n = \frac{5}{3} (\mu_u - \mu_d), \quad \mu_\Lambda = \mu_s, \quad \mu_p + \mu_n = \mu_u + \mu_d, \quad \mu_p - \mu_{\Sigma^+} = -\frac{1}{3} (\mu_d - \mu_s). \]  
(12)

As we discussed in [22], an ‘one-parameter’ scheme of the chiral quark model gives a good description to most existing spin and flavor observables. Here we will use the similar scheme, where the chiral parameters \( a, \epsilon \) and \( \xi' \) are determined by \( \Delta u - \Delta d = 1.258, \bar{d} - \bar{u} = 0.130, \) and \( \Delta s = -0.07. \) To predict the magnetic moments of the octet and decuplet baryons, we have adjusted \( \mu_u \) as only one free parameter with two constraints of \( \mu_s/\mu_d = 2/3 \) and \( \mu_d/\mu_u = -0.45. \) The numerical results for \( k = 1/3 \) and \( k = 0 \) are given in Table II, where the simple SU(6) quark model (NQM) results are also listed. One can see that the agreement between the chiral quark model prediction and data is improved by including the OAM contributions \( (k = 1/3). \) The flavor and spin contents, which are not directly related to the orbital motions, are listed in Table VII.
B. Decuplet baryons

Similar to the octet baryons, the quark spin contents $\Delta q$ and the difference between the quark and antiquark orbital angular momenta $<L_z>_{u,d,s} - <L_z>_{\bar u,\bar d,\bar s}$ in the decuplet baryons are given in Table III. Using (1b) and Table III, the decuplet magnetic moments are obtained. The numerical results and comparison with the data are listed in Table IV.

It is easy to verify that the following \textit{equal spacing} rules hold for the decuplet baryons

\begin{align}
\mu_{\Delta^+} &= 2\mu_u + \mu_d, \quad \mu_{\Delta^0} = \mu_u + 2\mu_d, \quad \mu_{\Delta^-} = 3\mu_d, \quad \delta_1 = 3 \\
\mu_{\Sigma^+} &= 3\mu_u, \quad \mu_{\Sigma^0} = \mu_u + \mu_d + \mu_s, \quad \mu_{\Sigma^+} = \mu_u + 2\mu_s, \\
\mu_{\Xi^0} &= \mu_u + 2\mu_s, \quad \mu_{\Xi^-} = \mu_d + 2\mu_s, \\
\mu_{\Omega^-} &= 3\mu_s.
\end{align}

Since only two data, $\mu_{\Delta^+}$ and $\mu_{\Omega^-}$, are available, and theoretical predictions given by different models are quite similar as shown in Table V, hence one cannot make definite conclusion on decuplet baryon magnetic moments.

IV. Quark Gluon Mixing (model II).

In [23], Lipkin suggested a hybrid model with the quark gluon mixing model of the nucleon. The proton is described by

\begin{equation}
[p, J = J_z = 1/2] = \cos \theta|[3q]_{J_z=1/2} > + \sin \theta|[(3q)^{(8)}]_{J_{qg}=1/2} \otimes (G)^{(8)}]_{J=1}^{(0)} > \quad (15)
\end{equation}

where $\theta$ is the mixing angle, $[3q]_{J_z=1/2} >$ is the spin-up ground state $SU(3)^f \otimes SU(2)^s$ proton wave function, in which the three valence quarks are coupled to a color singlet (see the notation (0) superscripted in the first term on the right-hand side of (15)) with the total angular momentum $J = J_z = 1/2$. The wave function $|[(3q)^{(8)}]_{J_{qg}=1/2} \otimes (G)^{(8)}]_{J=1}^{(0)} >$ is also a spin up ground state proton wave function, but consists of three valence quarks and a gluon. Since the gluon is a color-octet object, the three valence quarks in the $(3q + G)$ bound state must be coupled to a color-octet (shown by superscript notation (8) in the second term on the right-hand side of (15)). In addition, the three quarks must be coupled to the total angular momentum $J_{qg} = 1/2$. They coupled with a gluon (spin 1, color octet) to make a color singlet with total angular momentum $J_z = 1/2$.

We now apply the chiral dynamical mechanism to the model wave function (15). For the first term in Eq.(15), all discussions given in the last section can be used. However, they should be modified for the second term, i.e. the $3q + G$ piece. The valence quark numbers given in (8) for the standard $SU(3) \otimes SU(2)$ valence quark structure can no longer be used

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for the $3q + G$ piece, $\langle[(3q)]_{J_{sg}=1/2} \otimes (G)_{J_{g}=1}^{(8)}\rangle_{J_s=1/2}$. Instead, the valence quark numbers are

$$n_p^{3q+G,(v)}(u_\uparrow) = \frac{8}{9}, \quad n_p^{3q+G,(v)}(u_\downarrow) = \frac{4}{9}, \quad n_p^{3q+G,(v)}(d_\uparrow) = \frac{5}{9}, \quad n_p^{3q+G,(v)}(d_\downarrow) = \frac{5}{9}, \quad (16)$$

where $n_p^{3q+G,(v)}(s_{u,d}) = 0$ still hold for the $3q + G$ piece in the proton.

Assuming the mechanism of chiral fluctuation and the strength of these fluctuations are the same as before, i.e. the chiral splitting processes do not depend on whether the three quarks coupled to a gluon or not. Under this approximation, the spin-up and spin-down quark (antiquark) contents in the proton can be written as

$$n_p(q'^{\uparrow,\downarrow}_{\uparrow,\downarrow}, \text{or } \bar{q}'^{\uparrow,\downarrow}_{\uparrow,\downarrow}) = \cos^2 \theta \cdot n_p^{(3q)}(q'^{\uparrow,\downarrow}_{\uparrow,\downarrow}, \text{or } \bar{q}'^{\uparrow,\downarrow}_{\uparrow,\downarrow}) + \sin^2 \theta \cdot n_p^{(3q+G)}(q'^{\uparrow,\downarrow}_{\uparrow,\downarrow}, \text{or } \bar{q}'^{\uparrow,\downarrow}_{\uparrow,\downarrow}) \quad (17)$$

where

$$n_p^{(3q)}(q'^{\uparrow,\downarrow}_{\uparrow,\downarrow}, \text{or } \bar{q}'^{\uparrow,\downarrow}_{\uparrow,\downarrow}) = \sum_{q=u,d,h} \sum_{h=\uparrow,\downarrow} n_p^{(3q),(v)}(q_h) P_{hq}(q'^{\uparrow,\downarrow}_{\uparrow,\downarrow}, \text{or } \bar{q}'^{\uparrow,\downarrow}_{\uparrow,\downarrow}) \quad (18a)$$

$$n_p^{(3q+G)}(q'^{\uparrow,\downarrow}_{\uparrow,\downarrow}, \text{or } \bar{q}'^{\uparrow,\downarrow}_{\uparrow,\downarrow}) = \sum_{q=u,d,h} \sum_{h=\uparrow,\downarrow} n_p^{(3q+G),(v)}(q_h) P_{hq}(q'^{\uparrow,\downarrow}_{\uparrow,\downarrow}, \text{or } \bar{q}'^{\uparrow,\downarrow}_{\uparrow,\downarrow}) \quad (18b)$$

Eq. (18a) is the same as that given in (5), hence the results given in Sec. III can be directly used here. The only difference is that the first term on the right-hand side (RHS) of Eq.(17) has a factor $\cos^2 \theta$.

For the second term in Eq.(17), one needs to calculate $n_p^{(3q+G)}(q'^{\uparrow,\downarrow}_{\uparrow,\downarrow}, \text{or } \bar{q}'^{\uparrow,\downarrow}_{\uparrow,\downarrow})$ via Eq.(18b), where the valence quark numbers $n_p^{(3q+G),(v)}(q'^{\uparrow,\downarrow}_{\uparrow,\downarrow})$ are given in (16). Using (6) and (7), one obtains the spin and orbital contents for the $3q + G$ piece of the proton. Similar results can be obtained for the neutron and other octet baryons. All results are listed in Table V. Two remarks on the nucleon sector are in order.

1. The total spin contents for $3q + G$ piece is

$$\Delta \Sigma_p^{(3q+G)} = -\frac{1}{3} + \frac{2a}{3} \xi_1 = -\frac{1}{3} [1 - 2a \xi_1] = -\frac{1}{3} \Delta \Sigma_p^{(3q)} \quad (19a)$$

and the total orbital angular momenta of quarks and antiquarks are

$$< L_z >_{3q+\bar{q}}^{(3q+G)} = -ak \xi_1 = -\frac{1}{3} < L_z >_{3q+\bar{q}}^{(3q)} \quad (19b)$$

Eq. (19a) shows that total quark spin in the $3q + G$ state is one third of that in the $3q$ state. Using (17), one has

$$\Delta \Sigma_p^{[(3q)+(3q+G)]} = (\cos^2 \theta - \frac{1}{3} \sin^2 \theta) (\Delta \Sigma_p^{(3q)}) \quad (20a)$$

and

$$< L_z >_{p,(q+\bar{q})}^{[(3q)+(3q+G)]} = (\cos^2 \theta - \frac{1}{3} \sin^2 \theta) < L_z >_{p,3q}^{(3q)}. \quad (20b)$$

Hence we have

$$< J_z >_{p,(q+\bar{q})}^{[(3q)+(3q+G)]} = (\cos^2 \theta - \frac{1}{3} \sin^2 \theta)(\frac{1}{2} - (1 - 3k) a \xi_1), \quad (21)$$
where \(< J_z >_p^{(3q)+(3q+G)}\) is the total angular momentum carried by the quarks and antiquarks in the gluon mixing model.

Taking \(\theta \to 0\), \(< J_z >_G = 0\), one obtains the chiral quark model result without the gluon mixing effect (see Eq.(26a) in [14]). For the limit \(a \to 0\), (21) reduces to the result given in [23]. It is easy to see from (20a) that since the total quark and antiquark spin in the proton has already been reduced by the chiral fluctuation mechanism (\(\frac{1}{2} \Delta \Sigma_p^{(3q)} = \frac{1}{2} - a \xi_1 < \frac{1}{2}\)), to fit the smallness of \(\Delta \Sigma\) indicated by DIS data, one does not necessarily resort to a large mixing angle as shown in [23].

(2) From (21) and the proton spin sum rule
\[
\frac{1}{2} \Delta \Sigma^+ < L_z >_{q+\bar{q}} + < J_z >_G = \frac{1}{2}
\] (22a)

one obtains
\[
(\cos^2 \theta - \frac{1}{3} \sin^2 \theta)\left(\frac{1}{2} - (1 - 3k)a \xi_1\right) + < J_z >_G = \frac{1}{2}
\] (22b)
in the gluon mixing model. If we assume that the induced OAM arising from the chiral splittings is entirely and equally shared among quarks and antiquarks, and not shared by the gluons, then \(k = 1/3\). From (22), one has \(< J_z >_G = \frac{3}{2} \sin^2 \theta\). Assuming the gluon angular momentum is about \(0.20 \pm 0.10\) [24], then \(\sin^2 \theta \approx 0.30 \pm 0.15\), which implies there is a large gluon mixing in the proton. However, if the gluon also shares the induced OAM, then \(k < 1/3\), and
\[
< J_z >_G = \frac{2}{3} \sin^2 \theta + (1 - 3k)a \xi_1 (\cos^2 \theta - \frac{1}{3} \sin^2 \theta).
\] (23)

Assuming \(< J_z >_G \approx 0.15\), one obtains \(\sin^2 \theta \approx 0.05\) for \(k \approx 1/5\) and \(\sin^2 \theta \approx 0.13\) for \(k \approx 1/4\). If \(< J_z >_G \approx 0.20\), one obtains \(\sin^2 \theta \approx 0.15\) for \(k \approx 1/5\) and \(\sin^2 \theta \approx 0.21\) for \(k \approx 1/4\).

To maintain the consistency, we have used the same constraints of \(\mu_s/\mu_d = 2/3\) and \(\mu_d = -0.45 \mu_u\), and adjusted \(\mu_u\) as a free parameter. For the gluon mixing, we fix \(< J_z >_G = 0.15\), and choose \(k\) as adjusted parameter, then the mixing angle \(\theta\) is not independent and determined by Eq.(23). The magnetic moments of the octet baryons in the hybrid gluon mixing model (model II) are given in Table VI. The corresponding predictions on the spin and flavor observables in the nucleon for both model I and model II are listed in Table VII.

Summary

(1) In the model I, the agreement between the magnetic moments of the octet baryons and data is improved by including the OAM contributions (see Table II).

(2) The numerical result of baryon magnetic moments in the model II is almost the same as that given in the model I. For the sake of simplicity, the gluon mixing angle has been assumed to be universal (i.e. only one mixing angle for all octet baryons). If we would have been introduced three different mixing angles for \(N, \Sigma\) and \(\Xi\) isomultiplets as done in [20] for \(k = 0\) case, where the OAM contributions were not included, we would have a better agreement with data.

(3) For the decuplet baryons, the agreement with two existing data (\(\Delta^{++}\) and \(\Omega^-\)) looks very good for both \(k = 0\) and \(k = 1/3\) (Table IV). Since there is no significant difference between the predictions for \(k = 1/3\) and \(k = 0\), to test the OAM effects, more precise data
in the decuplet sector is needed. Note that the same parameter $\mu_u$ and same constraints $\mu_s/\mu_d = 2/3$ and $\mu_d = -0.45\mu_u$ are used as in the octet sector.

(4) One can see from Table VII that the flavor and spin fractions given in both models I and II are in good agreement with data. It should be noted that compared to the NMC data ($\bar{d} - \bar{u} = 0.147 \pm 0.039$) [28] and NA51 data ($[\bar{u}(x)/\bar{d}(x)]_{x=0.18} = 0.51 \pm 0.06$) [29], a smaller value of $\bar{d} - \bar{u}$ and a larger value of $\bar{u}(x)/\bar{d}(x)$ (or lower data points of $\bar{d}(x)/\bar{u}(x)$) have been reported [30]. All these data are quoted in Table VII. For the comparison of the chiral quark model predictions on the flavor and spin observables with data, see discussion given in [22].

To summary, The symmetry breaking chiral quark model have been quite successful in explaining many puzzles of the nucleon structure. By including the orbital angular momentum contributions, a better agreement with data for the baryon magnetic moments are also obtained.

Acknowledgments

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# REFERENCES

TABLES

TABLE I. The quark spin and orbital angular momenta in the octet baryons in model I, where \( \xi_1 = 1 + \epsilon + f \) and \( \xi_2 = 2\epsilon + f_s \).

<table>
<thead>
<tr>
<th>Baryon ∆</th>
<th>( \Delta u^B )</th>
<th>( \Delta d^B )</th>
<th>( \Delta s^B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>( \frac{4}{5} - \frac{9}{3}(8\xi_1 - 4\epsilon - 5) )</td>
<td>( -\frac{1}{5} - \frac{9}{3}(-2\xi_1 + \epsilon + 5) )</td>
<td>( -ae )</td>
</tr>
<tr>
<td>( \Sigma^+ )</td>
<td>( \frac{2}{3} - \frac{9}{3}(8\xi_1 - 5\epsilon - 4) )</td>
<td>( -\frac{2}{3}(4 - \epsilon) )</td>
<td>( -\frac{1}{3} - \frac{2a}{3}(-\xi_2 + 3\epsilon) )</td>
</tr>
<tr>
<td>( \Sigma^0 )</td>
<td>( \frac{2}{3} - \frac{9}{3}(4\xi_1 - 3\epsilon) )</td>
<td>( \frac{2}{3} - \frac{9}{3}(4\xi_1 - 3\epsilon) )</td>
<td>( -\frac{1}{3} - \frac{2a}{3}(-\xi_2 + 3\epsilon) )</td>
</tr>
<tr>
<td>( \Lambda^0 )</td>
<td>( -ae )</td>
<td>( -ae )</td>
<td>( 1 - 2a(x_2 - \epsilon) )</td>
</tr>
<tr>
<td>( \Xi^0 )</td>
<td>( -\frac{1}{3} - \frac{9}{3}(-2\xi_1 + 5\epsilon + 1) )</td>
<td>( -\frac{9}{3}(4\epsilon - 1) )</td>
<td>( \frac{4}{3} - \frac{2}{3}(8\xi_2 - 9\epsilon) )</td>
</tr>
</tbody>
</table>

\[ <L_z>_{u-d} \quad <L_z>_{d-\bar{d}} \quad <L_z>_{d-\bar{d}} \]

| \( \mu_u = \frac{2}{3}\mu_d \) | \( \mu_s = \frac{1}{3}\mu_d \) | \( \mu_s = \frac{1}{3}\mu_d \) |
| \( \mu_d = -0.45\mu_u \) | \( \mu_d = -0.45\mu_u \) | \( \mu_d = -0.50\mu_u \) |
| \( \mu_u = 2.22\mu_N \) | \( \mu_u = 2.49\mu_N \) | \( \mu_u = 1.91\mu_N \) |
TABLE III. The quark spin and orbital angular momenta in the decuplet baryons in model I.

<table>
<thead>
<tr>
<th>Baryon</th>
<th>$\Delta u^{B^*}$</th>
<th>$\Delta d^{B^*}$</th>
<th>$\Delta s^{B^*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^{++}$</td>
<td>$3 - 3a(2\xi_1 - \epsilon - 1)$</td>
<td>$-3a$</td>
<td>$-3ae$</td>
</tr>
<tr>
<td>$\Delta^+$</td>
<td>$2 - a(4\xi_1 - 2\epsilon - 1)$</td>
<td>$1 - a(2\xi_1 - \epsilon + 1)$</td>
<td>$-3ae$</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>$1 - 2a\xi_1$</td>
<td>$1 - 2a\xi_2$</td>
<td>$1 - 2a\xi_2$</td>
</tr>
<tr>
<td>$\Xi^+$</td>
<td>$2 - a(4\xi_1 - \epsilon - 2)$</td>
<td>$-a(\epsilon + 2)$</td>
<td>$1 - 2a\xi_2$</td>
</tr>
<tr>
<td>$\Omega^-$</td>
<td>$1 - a(2\xi_1 + \epsilon - 1)$</td>
<td>$-a(2\epsilon + 1)$</td>
<td>$2 - a(4\xi_2 - 3\epsilon)$</td>
</tr>
</tbody>
</table>

TABLE IV. Comparison of our predictions with data for the decuplet baryon magnetic moments in the model I. The naive quark model (NQM) results are also listed.

<table>
<thead>
<tr>
<th>Baryon</th>
<th>data</th>
<th>k=1/3</th>
<th>k=0</th>
<th>NQM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^{++}$</td>
<td>$4.52 \pm 0.50 \pm 0.45^a$</td>
<td>5.30</td>
<td>5.17</td>
<td>5.73</td>
</tr>
<tr>
<td></td>
<td>$3.7 &lt; \mu_{\Delta^{++}} &lt; 7.5^b$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta^+$</td>
<td>$2.54$</td>
<td>$2.45$</td>
<td>$2.87$</td>
<td>$2.87$</td>
</tr>
<tr>
<td>$\Delta^0$</td>
<td>$-0.22$</td>
<td>$-0.27$</td>
<td>$0.00$</td>
<td>$0.00$</td>
</tr>
<tr>
<td>$\Delta^-$</td>
<td>$-2.98$</td>
<td>$-2.99$</td>
<td>$-2.87$</td>
<td>$-2.87$</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>$2.94$</td>
<td>$2.78$</td>
<td>$3.18$</td>
<td>$3.18$</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
<td>$0.18$</td>
<td>$0.06$</td>
<td>$0.32$</td>
<td>$0.32$</td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>$-2.58$</td>
<td>$-2.66$</td>
<td>$-2.55$</td>
<td>$-2.55$</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>$0.49$</td>
<td>$0.39$</td>
<td>$0.64$</td>
<td>$0.64$</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>$-2.27$</td>
<td>$-2.33$</td>
<td>$-2.23$</td>
<td>$-2.23$</td>
</tr>
<tr>
<td>$\Omega^-$</td>
<td>$-2.02 \pm 0.05^b$</td>
<td>$-1.91$</td>
<td>$-2.00$</td>
<td>$-1.91$</td>
</tr>
<tr>
<td></td>
<td>$-2.024 \pm 0.056^c$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-1.94 \pm 0.17 \pm 0.14^d$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$a$ - [25], $b$ - [21]
$c$ - [26], $d$ - [27]

$\mu_s = \frac{2}{3}\mu_d, \mu_s = \frac{2}{3}\mu_d, \mu_s = \frac{2}{3}\mu_d$

$\mu_d = -0.45\mu_u, \mu_d = -0.45\mu_u, \mu_d = -0.50\mu_u$

$\mu_u = 2.22\mu_N, \mu_u = 2.49\mu_N, \mu_u = 1.91\mu_N$
TABLE V. The quark spin and orbital angular momenta in the octet baryons for \((3q + G)\) piece in the model II.

<table>
<thead>
<tr>
<th>Baryon</th>
<th>(\Delta u^B)</th>
<th>(\Delta d^B)</th>
<th>(\Delta s^B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p)</td>
<td>(-\frac{4}{9} + \frac{a_0}{9}(4\xi_1 - 2\epsilon - 1))</td>
<td>(-\frac{4}{9} + \frac{a_0}{9}(2\xi_1 - \epsilon + 1))</td>
<td>(\frac{2a_0}{9}\xi_2)</td>
</tr>
<tr>
<td>(\Sigma^+)</td>
<td>(-\frac{4}{9} + \frac{a_0}{9}(4\xi_1 - \epsilon - 2))</td>
<td>(\frac{a_0}{9}(\epsilon + 2))</td>
<td>(-\frac{1}{9} + \frac{2a_0}{9}\xi_2)</td>
</tr>
<tr>
<td>(\Sigma^0)</td>
<td>(-\frac{1}{3} + \frac{2a_0}{9}\xi_1)</td>
<td>(-\frac{1}{9} + \frac{2a_0}{9}\xi_2)</td>
<td>(-\frac{1}{9} + \frac{2a_0}{9}\xi_2)</td>
</tr>
<tr>
<td>(\Lambda^0)</td>
<td>(-\frac{1}{3} + \frac{a_0}{9}(\epsilon - 1))</td>
<td>(-\frac{1}{3} + \frac{2a_0}{9}(2\xi_1 - 3\epsilon))</td>
<td>(-\frac{1}{3} + \frac{2a_0}{9}(2\xi_1 - 3\epsilon))</td>
</tr>
</tbody>
</table>

\(<L_z>^B_{u-d}\) \(<L_z>^B_{d-d}\) \(<L_z>^B_{s-s}\)

<table>
<thead>
<tr>
<th>Baryon</th>
<th>data</th>
<th>k=1/5 ((\sin^2\theta=0.05))</th>
<th>k=1/4 ((\sin^2\theta=0.13))</th>
<th>NQM</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p)</td>
<td>2.79±0.00</td>
<td>2.68</td>
<td>2.65</td>
<td>2.87</td>
</tr>
<tr>
<td>(n)</td>
<td>-1.91±0.00</td>
<td>-1.91</td>
<td>-1.91</td>
<td>-1.91</td>
</tr>
<tr>
<td>(\Sigma^+)</td>
<td>2.46±0.01</td>
<td>2.55</td>
<td>2.54</td>
<td>2.62</td>
</tr>
<tr>
<td>(\Sigma^-)</td>
<td>-1.16±0.03</td>
<td>-1.09</td>
<td>-1.06</td>
<td>-1.20</td>
</tr>
<tr>
<td>(\Lambda^0)</td>
<td>-0.61±0.00</td>
<td>-0.63</td>
<td>-0.62</td>
<td>-0.63</td>
</tr>
<tr>
<td>(\Xi^0)</td>
<td>-1.25±0.01</td>
<td>-1.44</td>
<td>-1.47</td>
<td>-1.49</td>
</tr>
<tr>
<td>(\Xi^-)</td>
<td>-0.65±0.00</td>
<td>-0.52</td>
<td>-0.51</td>
<td>-0.53</td>
</tr>
<tr>
<td>(\Sigma^0)</td>
<td>-0.72</td>
<td>0.73</td>
<td>0.73</td>
<td>0.71</td>
</tr>
</tbody>
</table>

\[\mu_s = \frac{2}{3}\mu_d, \mu_s = \frac{2}{3}\mu_d, \mu_s = \frac{2}{3}\mu_d\]
\[\mu_d = -0.45\mu_u, \mu_d = -0.45\mu_u, \mu_d = -0.50\mu_u\]
\[\mu_u = 2.46\mu_N, \mu_u = 2.69\mu_N, \mu_u = 1.91\mu_N\]
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Data</th>
<th>Model I</th>
<th>Model II</th>
<th>NQM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{d} - \bar{u}$</td>
<td>$0.147 \pm 0.039^a$</td>
<td>0.130$^*$</td>
<td>0.130$^*$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$0.100 \pm 0.018^b$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{u}/\bar{d}$</td>
<td>$\left[ \frac{\bar{u}(x)}{\bar{d}(x)} \right]_{x=0.18} = 0.51 \pm 0.06^c$</td>
<td>0.68</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\left[ \frac{\bar{u}(x)}{\bar{d}(x)} \right]_{0.1&lt;x&lt;0.2} = 0.67 \pm 0.06^b$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2\bar{s}/(\bar{u} + \bar{d})$</td>
<td>$\frac{\bar{s}(x)}{\bar{u}(x) + \bar{d}(x)} = 0.477 \pm 0.051^d$</td>
<td>0.72</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>$2\bar{s}/(u + d)$</td>
<td>$\frac{\bar{s}(x)}{u(x) + d(x)} = 0.099 \pm 0.009^d$</td>
<td>0.13</td>
<td>0.14</td>
<td>0</td>
</tr>
<tr>
<td>$\sum \bar{q}/\sum q$</td>
<td>$\frac{\sum \bar{q}(x)}{\sum q(x)} = 0.245 \pm 0.005^d$</td>
<td>0.24</td>
<td>0.24</td>
<td>0</td>
</tr>
<tr>
<td>$f_s$</td>
<td>$0.10 \pm 0.06^e$</td>
<td>0.10</td>
<td>0.11</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$0.15 \pm 0.03^f$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{f_3}{f_8}$</td>
<td>$\frac{\sum \bar{q}(x)}{\sum q(x)} = 0.076 \pm 0.022^d$</td>
<td>0.22</td>
<td>0.22</td>
<td>1/3</td>
</tr>
<tr>
<td>$\Delta u$</td>
<td>$0.85 \pm 0.05^h$</td>
<td>0.86</td>
<td>0.80</td>
<td>4/3</td>
</tr>
<tr>
<td>$\Delta d$</td>
<td>$-0.41 \pm 0.05^h$</td>
<td>-0.40</td>
<td>-0.38</td>
<td>-1/3</td>
</tr>
<tr>
<td>$\Delta s$</td>
<td>$-0.07 \pm 0.05^h$</td>
<td>-0.07$^*$</td>
<td>-0.07$^*$</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta \bar{u}, \Delta \bar{d}$</td>
<td>$-0.02 \pm 0.11^i$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta_3/\Delta_8$</td>
<td>$2.17 \pm 0.10^j$</td>
<td>2.12</td>
<td>2.11</td>
<td>5/3</td>
</tr>
</tbody>
</table>

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a – [28], b – [30]
c – [29], d – [31]
e – [32], f – [33]
g – [34], h – [35]
i – [18], j – [21]