Proliferation of de Sitter Space

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Abstract

I show that de Sitter space disintegrates into an infinite number of copies of itself. This occurs iteratively through a quantum process involving two types of topology change. First a handle is created semiclassically, on which multiple black hole horizons form. Then the black holes evaporate and disappear, splitting the spatial hypersurfaces into large parts. Applied to cosmology, this process leads to the production of a large or infinite number of universes in most models of inflation and yields a new picture of global structure.

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1 Introduction

1.1 Semi-classical Perdurance of de Sitter Space

de Sitter space is the maximally symmetric solution of the vacuum Einstein equations with a cosmological constant $\Lambda$. It may be visualized as a (3,1)-hyperboloid embedded in (4,1) Minkowski space. In this spacetime, geodesic observers find themselves immersed in a bath of thermal radiation of temperature $T = \sqrt{\Lambda/3/(2\pi)}$. This raises the question of stability, which was investigated in 1983 by Ginsparg and Perry. They showed that de Sitter space does not possess the classical Jeans instability found in hot flat space. It does, however, possess a semiclassical instability to a spontaneous topology change corresponding to the nucleation of a black hole. Geometrically this process corresponds to the creation of a handle; it occurs at a rate of $e^{-\pi/\Lambda}$ per horizon four-volume of size $9/\Lambda^2$.

When the black hole appears, it will typically be degenerate; that is, it will have the same size as the cosmological horizon, and will be in thermal equilibrium with it. Ginsparg and Perry argued on an intuitive basis that this equilibrium would be unstable, with quantum fluctuations causing the black hole to be slightly smaller, and hotter, than the cosmological horizon. Then, presumably, the black hole would start to evaporate and eventually disappear. They showed that the time scale between black hole nucleations is vastly larger than the time needed for evaporation (see also Ref. []). In this sense, de Sitter space would “perdure”.

In a collaboration with S. Hawking, this argument was recently confirmed[]. We used a model that includes the quantum radiation in the s-wave and large $N$ limit, at one loop. We found, to our surprise, that nearly degenerate Schwarzschild-de Sitter black holes anti-evaporate. But there is a different way of perturbing the degenerate solution which leads to evaporation, and we found that this mode would always be excited when black holes nucleate spontaneously.

The process of black hole creation and subsequent evaporation is shown in Fig. 1 (evolution of the spacelike sections) and Fig. 2 (causal structure).

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1 The process really describes a pair of black holes, in the sense that there will be two separate horizons. There will, however, be only one black hole interior. Later in this paper, situations with multiple black hole interiors will arise. Therefore any individual black hole interior, bounded by a pair of horizons, will be referred to here as a single black hole.
Figure 1: Evolution of spacelike hypersurfaces of de Sitter space during the creation and subsequent evaporation of a single black hole. The spontaneous creation of a handle changes the spatial topology from spherical ($S^3$) to toroidal ($S^1 \times S^2$) with constant two-sphere radius. (The double-headed arrow indicates that opposite ends of the middle picture should be identified, closing the $S^1$.) If the quantum fluctuations are dominated by the lowest Fourier mode on the $S^1$, there will be one minimum and one maximum two-sphere radius, corresponding to a black hole (b) and a cosmological horizon (c). This resembles a ‘wobbly doughnut’ with cross-sections of varying thickness. As the black hole evaporates, the thinnest cross-section decreases in size. Finally the black hole disappears, i.e. the doughnut is pinched at its thinnest place and reverts to the original spherical topology.

Crucially, for the case of a single black hole, the topology reverts to the original de Sitter space after the process is completed.

1.2 Proliferation

In this paper, I consider different perturbations of the degenerate black hole solution, which correspond to higher quantum fluctuation modes. Occasionally, one such mode will dominate over lower modes, leading to the presence of multiple pairs of apparent black hole horizons in a single nucleation event, and eventually to the formation of several black hole interiors. I will show that they evaporate, and that space will disconnect at the event when a black hole finally disappears. If more than one black hole is present, this leads to the disintegration of the universe.
This process is depicted in Fig. 3, where the evolution of the space-like sections in a process of multiple black hole creation and subsequent evaporation is shown. The corresponding causal diagram is given in Fig. 4. These pictures should be compared to the similar, yet much less dramatic evolution in the case where only one black hole is created, Figs. 1 and 2.

The decay described here can be viewed as a sequence of topology changes. The first topology change is the semiclassical creation of a handle with multiple black hole horizons. It corresponds to a local non-perturbative quantum fluctuation of the metric on the scale of a single horizon volume. It will therefore happen independently in widely separated horizon volumes of de Sitter space. While the black holes evaporate, the asymptotically de Sitter regions between the black holes grow exponentially. Thus, the second topology change, corresponding to the final disappearance of the black holes, yields a number of separate de Sitter spaces which already contain exponentially large spacelike sections. Because it occurs at such large scales, the effect differs from many other considerations of baby universes (see, e.g., Ref. [?]). Inside the daughter universes, the handle-creation process will again occur lo-
Figure 3: Evolution of spacelike hypersurfaces of de Sitter space during the creation of a handle yielding multiple black holes ($n = 2$) and their subsequent evaporation. This should be compared to Fig. 1. If the quantum fluctuations on the $S^1 \times S^2$ handle are dominated by the second Fourier mode on the $S^1$, there will be two minima and two maxima of the two-sphere radius, seeding to two black hole interiors (b) and two asymptotically de Sitter regions (c). This resembles a 'wobbly doughnut' on which the thickness of the cross-sections oscillates twice. As the black holes evaporate, the minimal cross-sections decrease. When the black holes disappear, the doughnut is pinched at two places, yielding two disjoint spaces of spherical topology, the daughter universes.

2If one starts with a single de Sitter universe, the geometry resulting from the iterative disconnections will of course be connected if viewed as a four-dimensional manifold: at sufficiently early times, any two observers would have been in the same universe. Only after the handles are nucleated, and the black holes evaporate, do the spatial sections fall apart. Spacetime decomposition can be defined in the following coordinate-independent way: For a geodesically complete Lorentzian four-manifold $\mathcal{M}$, consider the set $S$ of all spacelike sections $S$ which have no boundaries (other than spatial infinity in the case of non-compact spaces). If there exists a pair $S_1, S_2 \in S$ such that $J^+(S_1) \cap J^+(S_2) = \emptyset$, the spacetime decomposes, and $S_1$ and $S_2$ are sections of different universes. Here $J^+(U)$ denotes the causal future of a set $U$. 
1.3 Outline

The geometry of black holes in de Sitter space, and their spontaneous nucleation, will be reviewed in Sec. 2; this requires a brief derivation of the Nariai solution, which corresponds to a degenerate unstable handle attached to de Sitter space.

In Sec. 3, a model will be introduced that includes quantum radiation at one loop. It was used to investigate the behavior of different perturbations of the Nariai solution, in Ref. [?]. There we considered only perturbations of the two-sphere radius that corresponded to the lowest Fourier mode. But the quantum model is general enough to deal also with higher mode perturbations, for which the two-sphere size has \( n \) maxima and \( n \) minima around the \( S^1 \), with \( n > 1 \).

In Sec. 4, I will present solutions for the amplitude of such perturbations which are regular on the Euclidean sector of the nucleation geometry. I
will show that the condition of regularity leads to the following Lorentzian evolution: The perturbation oscillates while its wavelength is within a horizon volume. When it leaves the horizon, it freezes, leading to the formation of \( n \) black holes, which are shown to evaporate. When a black hole finally disappears, the spacelike section disconnects at the evaporation point. The \( S^1 \times S^2 \) topology thus dissociates into \( n \) disjoint three-spheres.

Finally, in Sec. 5, I discuss the cosmological implications of this instability. During the inflationary era, it causes the production of large new universes. In models of eternal inflation, there will be an infinite number of separate universes.

## 2 Schwarzscild-de Sitter Black Holes

The neutral, static, spherically symmetric solutions of the vacuum Einstein equations with a cosmological constant \( \Lambda \) are given by the Schwarzschild-de Sitter metric

\[
\text{ds}^2 = -V(r)dt^2 + V(r)^{-1}dr^2 + r^2d\Omega^2,
\]

where

\[
V(r) = 1 - \frac{2\mu}{r} - \frac{\Lambda}{3}r^2; \quad (2.2)
\]

\( d\Omega^2 \) is the metric on a unit two-sphere and \( \mu \) is a mass parameter. For \( 0 < \mu < \frac{1}{2}\Lambda^{-1/2} \), \( V \) has two positive roots \( r_c \) and \( r_b \), corresponding to the cosmological and the black hole horizons, respectively. For \( \mu = 0 \) there will be no black hole horizon and one obtains the de Sitter solution.

In the limit \( \mu \to \frac{1}{2}\Lambda^{-1/2} \) the size of the black hole horizon approaches the size of the cosmological horizon, and the above coordinates become inadequate, since \( V(r) \to 0 \) between the horizons. One may define the parameter \( \epsilon \) by

\[
9\mu^2\Lambda = 1 - 3\epsilon^2, \quad 0 \leq \epsilon \ll 1. \quad (2.3)
\]

Then the degenerate case corresponds to \( \epsilon \to 0 \). New time and radial coordinates, \( \psi \) and \( \chi \), will be given by

\[
\tau = \frac{1}{\epsilon\sqrt{\Lambda}}\psi; \quad r = \frac{1}{\sqrt{\Lambda}}\left[1 - \epsilon\cos\chi - \frac{1}{6}\epsilon^2\right]. \quad (2.4)
\]
The black hole horizon lies at $\chi = 0$ and the cosmological horizon at $\chi = \pi$. The new metric obtained from the transformations is, to first order in $\epsilon$,

$$
\begin{align*}
\text{ds}^2 &= -\frac{1}{\Lambda} \left( 1 + \frac{2}{3} \epsilon \cos \chi \right) \sin^2 \chi \, d\psi^2 + \frac{1}{\Lambda} \left( 1 - \frac{2}{3} \epsilon \cos \chi \right) \, d\chi^2 \\
&\quad + \frac{1}{\Lambda} \left( 1 - 2 \epsilon \cos \chi \right) \, d\Omega^2_2.
\end{align*}
$$

It describes Schwarzschild-de Sitter solutions of nearly maximal black hole size.

In these coordinates the topology of the spacelike sections of Schwarzschild-de Sitter becomes manifest: $S^1 \times S^2$. (This is why one can speak of handle-creation.) In these solutions, the radius of the two-spheres, $r$, varies along the $S^1$ coordinate, $\chi$, with the minimal (maximal) two-sphere corresponding to the black hole (cosmological) horizon. In the degenerate case, for $\epsilon = 0$, the two-spheres all have the same radius. The degenerate metric is is called the “Nariai” solution; it is simply the direct product of (1+1)-dimensional de Sitter space and a two-sphere of constant radius.

The Euclidean de Sitter solution can be obtained from the Lorentzian metric, Eq. (2.1), with $\mu = 0$, by Wick-rotating the time variable: $\tau = it$. This yields a Euclidean four-sphere of radius $(\Lambda/3)^{-1/2}$. Its action is $-3\pi/\Lambda$. A regular Euclidean sector also exists for the Nariai solution: With $\epsilon = 0$ and $\xi = i\psi$, Eq. (2.6) describes the Euclidean metric corresponding to the direct product of two round two-spheres of radius $\Lambda^{-1/2}$. This solution has a Euclidean action of $-2\pi/\Lambda$.

To obtain the creation rate of the degenerate Schwarzschild-de Sitter solution on a de Sitter background, one has to subtract the background action and exponentiate minus the difference [\text{?}, \text{?}, \text{?}]. This yields the rate $e^{-\pi/\Lambda}$. Clearly, the topological transition is strongly suppressed unless the cosmological constant is of order the Planck value. But this does not mean the process can be neglected; after all, de Sitter space, and many inflationary scenarios, contain an exponentially large or infinite number of Hubble volumes.
3 One-loop Model

The four-dimensional Lorentzian Einstein-Hilbert action with a cosmological constant is given by:

$$S = \frac{1}{16\pi} \int d^4x (-g^{IV})^{1/2} \left[ R^{IV} - 2\Lambda - \frac{1}{2} \sum_{i=1}^{N} (\nabla^{IV} f_i)^2 \right],$$  \hspace{1cm} (3.1)

where $R^{IV}$ and $g^{IV}$ are the four-dimensional Ricci scalar and metric determinant. The scalar fields $f_i$ are included to carry the quantum radiation.

I shall consider only spherically symmetric fields and quantum fluctuations. Thus the metric may be written as

$$ds^2 = e^{2\phi} \left( -dt^2 + dx^2 \right) + e^{-2\phi} d\Omega^2,$$  \hspace{1cm} (3.2)

where $x$ is the coordinate on the $S^1$, with period $2\pi$. The angular coordinates can be integrated out, which reduces the action to

$$S = \frac{1}{16\pi} \int d^2x (-g)^{1/2} e^{-2\phi} \left[ R + 2(\nabla\phi)^2 + 2e^{2\phi} - 2\Lambda - \sum_{i=1}^{N} (\nabla f_i)^2 \right],$$  \hspace{1cm} (3.3)


$$W^* = -\frac{1}{48\pi} \int d^2x (-g)^{1/2} \left[ \frac{1}{2} \nabla^2 R - 6(\nabla\phi)^2 \nabla^2 R + 6\phi R \right].$$  \hspace{1cm} (3.4)

As in Ref. [?], the $(\nabla\phi)^2$ term may be neglected.

This action can be made local by introducing an independent scalar field $Z$ which mimics the trace anomaly. With the classical solution $f_i = 0$ the $f$ fields can be integrated out. This leads to the action

$$S = \frac{1}{16\pi} \int d^2x (-g)^{1/2} \left[ \left( e^{-2\phi} + \frac{N}{3}(Z - 6\phi) \right) R 
- \frac{N}{6} (\nabla Z)^2 + 2e^{-2\phi} (\nabla\phi)^2 - 2e^{-2\phi} \Lambda \right].$$  \hspace{1cm} (3.5)

In the large $N$ limit, the contribution from the quantum fluctuations of the scalars dominates over that from the metric fluctuations. In order for quantum corrections to be small, one should take $N\Lambda \ll 1$. 

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Differentiation with respect to $t (x)$ will be denoted by an overdot (a prime). For any functions $f$ and $g$, define:

$$\partial f \partial g \equiv -\dot{f}g + f'g', \quad \partial^2 g \equiv -\ddot{g} + g''$$  \hspace{1cm} (3.6)

$$\delta f \delta g \equiv \dot{f}\dot{g} + f'g', \quad \delta^2 g \equiv \ddot{g} + g''$$  \hspace{1cm} (3.7)

Variation with respect to $\rho$, $\phi$ and $Z$ yields the following equations of motion:

$$-\left(1 + N e^{2\phi}\right) \partial^2 \phi + 2(\partial\phi)^2 + \frac{N}{6} e^{2\phi} \partial^2 Z + e^{2\rho + 2\phi} \left(\Lambda e^{-2\phi} - 1\right) = 0; \quad (3.8)$$

$$\left(1 + N e^{2\phi}\right) \partial^2 \rho - \partial^2 \phi + (\partial\phi)^2 + \Lambda e^{2\rho} = 0; \quad (3.9)$$

$$\partial^2 Z - 2\partial^2 \rho = 0. \quad (3.10)$$

There are two equations of constraint:

$$\left(1 + N e^{2\phi}\right) \left(\delta^2 \phi - 2\delta \phi \delta \rho\right) - (\delta \phi)^2 = \frac{N}{12} e^{2\phi} \left[(\delta Z)^2 + 2\delta^2 Z - 4\delta Z \delta \rho\right]; \quad (3.11)$$

$$\left(1 + N e^{2\phi}\right) \left(\dot{\phi}' - \dot{\rho} \phi' - \rho' \phi\right) - \dot{\phi} \phi' = \frac{N}{12} e^{2\phi} \left[\dot{Z} Z' + 2\dot{Z}' - 2 \left(\dot{\rho} Z' + \rho' \dot{Z}\right)\right]. \quad (3.12)$$

From Eq. (3.10), it follows that $Z = 2\rho + \eta$, where $\eta$ satisfies $\partial^2 \eta = 0$. It was shown in Ref. [?] that the remaining freedom in $\eta$ can be used to satisfy the constraint equations for any choice of $\rho$, $\dot{\rho}$, $\phi$ and $\dot{\phi}$ on an initial spacelike section.

### 4 Multi-Black Hole Solutions

#### 4.1 Perturbation Ansatz

In Sec. 2 it was described how one can describe the process of handle-formation in de Sitter space using instantons. It was found that typically one obtains a degenerate Nariai solution as a result of this topology change. In the coordinates of Eq. (3.2), it is given by

$$e^{2\rho} = \frac{1}{\Lambda_1 \cos^2 t}, \quad e^{2\phi} = \Lambda_2,$$  \hspace{1cm} (4.1)
Because of the presence of the quantum radiation, one no longer has exactly \( \Lambda_1 = \Lambda_2 = \Lambda \), but, to first order in \( N \Lambda \),

\[
\frac{1}{\Lambda_1} = \frac{1}{\Lambda} (1 + N \Lambda), \quad \Lambda_2 = \Lambda \left(1 - \frac{N \Lambda}{3}\right)
\]  

(4.2)

(see Ref. [?] for more details).

Quantum fluctuations will perturb this solution, so that the two-sphere radius, \( e^{-\phi} \), will vary slightly along the one-sphere coordinate, \( x \). Decomposition into Fourier modes on the \( S^1 \) yields the perturbation ansatz

\[
e^{2\phi} = \Lambda_2 \left[ 1 + 2\epsilon \sum_n (\sigma_n(t) \cos nx + \tilde{\sigma}_n(t) \sin nx) \right],
\]  

(4.3)

where \( \epsilon \) is taken to be small.

One does not lose much generality by assuming that the amplitude of one mode dominates over all others.\footnote{This just means that the horizons will be evenly, rather than irregularly, spaced on the \( S^1 \). The constant mode \( (n = 0) \) will be ignored here, since it does not lead to the formation of any black hole horizons, but to a topologically non-trivial, locally de Sitter spacetime.} This will most likely be the first Fourier mode \( (n = 1) \), in which case one obtains the usual single black hole investigated in Ref. [?], and no spatial decomposition takes place when it evaporates. But occasionally, a higher mode will have a dominating amplitude. It is reasonable to estimate the likelihood of such an event to be of order \( e^{-n^2} \). For example, the second mode will dominate in a few percent of handle-nucleation events. In any case, the suppression of this condition is negligible compared to the rarity of nucleating a handle in the first place. With this assumption, and a trivial shift in the \( S^1 \) coordinate \( x \), the perturbation ansatz simplifies to

\[
e^{2\phi} = \Lambda_2 \left[ 1 + 2\epsilon \sigma_n(t) \cos nx \right],
\]  

(4.4)

Following Ref. [?], \( \sigma_n \) will be called the metric perturbation. A similar perturbation could be introduced for \( e^{2\rho} \), but it does not enter the equation of motion for \( \sigma_n \) at first order in \( \epsilon \). This equation is obtained from the equations of motion for \( \phi, \rho, \) and \( Z \), yielding

\[
\frac{\ddot{\sigma}_n}{\sigma_n} = \frac{c(c+1)}{\cos^2 t} - n^2,
\]  

(4.5)

where \( c = 1 + 2N\Lambda/9 \) to first order in \( N\Lambda \).
4.2 Horizon Tracing

In order to describe the evolution of the black hole, one must know where the horizon is located. The condition for a horizon is $(\nabla \phi)^2 = 0$. Eq. (4.4) yields

$$\frac{\partial \phi}{\partial t} = \epsilon \sigma_n \cos nx, \quad \frac{\partial \phi}{\partial x} = -\epsilon \sigma_n n \sin nx.$$  

Therefore, there will be $2n$ black hole horizons, and $2n$ cosmological horizons, located at

$$x_b^{(k)}(t) = \frac{1}{n} \left( 2\pi k + \arctan \left| \frac{\dot{\sigma}_n}{n\sigma_n} \right| \right),$$  

$$x_b^{(n+k)}(t) = \frac{1}{n} \left( -2\pi k + \arctan \left| \frac{\dot{\sigma}_n}{n\sigma_n} \right| \right),$$  

$$x_c^{(l)}(t) = x_b^{(l)}(t) + \frac{\pi}{n},$$

where $k = 0 \ldots n - 1$ and $l = 0 \ldots 2n - 1$.

To first order in $\epsilon$, the size of the black hole horizons, $r_b$, is given by

$$r_b(t)^{-2} = e^{2\phi[t,x_b^{(l)}(t)]} = \Lambda_2 [1 + 2\epsilon \delta(t)],$$

where the horizon perturbation is defined to be

$$\delta \equiv \sigma_n \cos nx_b^{(l)} = \sigma_n \left( 1 + \frac{\dot{\sigma}_n^2}{n^2 \sigma_n^2} \right)^{-1/2}. $$

To obtain explicitly the evolution of the black hole horizon radius, $r_b(t)$, one must solve Eq. (4.5) for $\sigma_n(t)$, and use the result in Eq. (4.11) to evaluate Eq. (4.10). If the horizon perturbation grows, the black hole is shrinking: this corresponds to evaporation. It will be shown below that evaporation is indeed realized for perturbation solutions $\sigma_n$ satisfying the condition of regularity on the Euclidean instanton (the “no-boundary condition”).

4.3 Regular Solutions

The metric of the Euclidean Nariai solution can be obtained from Eq. (4.1) by taking $\tau = it$. After rescaling Euclidean time as $\sin u = 1/\cosh \tau$, it takes the form

$$ds^2 = \frac{1}{\Lambda_1} \left( du^2 + \sin^2 u \, dx^2 \right) + \frac{1}{\Lambda_2} d\Omega^2.$$
This $S^2 \times S^2$ instanton describes the spontaneous nucleation of a degenerate handle in de Sitter space. The nucleation path runs from the South pole of the first two-sphere, at $u = 0$, to $u = \pi/2$, and then parallel to the imaginary time axis ($u = \pi/2 + iv$) from $v = 0$ to $v = \infty$. This can be visualized geometrically by cutting the first two sphere in half, and joining to it a Lorentzian $1 + 1$-dimensional de Sitter hyperboloid.

In order to see the effect of quantum fluctuations, one must find solutions to the equation of motion for the perturbation amplitude, Eq. (4.5). The solutions have to be regular everywhere on the nucleation geometry. In particular, they must vanish on the south pole. This condition selects a one-dimensional subspace of the two-dimensional solution space of the second-order equation. For any $n \geq 1$, the family of solutions, parametrized by a real prefactor $A$, is given by

$$\sigma_n(u) = Ae^{i(c-n)\pi/2} \left( \tan \frac{u}{2} \right)^n (n + \cos cu). \quad (4.13)$$

The phase is chosen such that $\sigma_n$ will be real at late Lorentzian times, when measurements can be made. The solution is exact for $c = 1$ (which corresponds to no quantum matter) and is an excellent approximation for $NA \ll 1$, $n \geq 2$.

The Lorentzian behavior of this solution can be obtained by substituting $u = \pi/2 + iv$. While the $n^2$ term dominates on the right hand side of Eq. (4.5), one would expect the perturbation to oscillate. Indeed, it is easy to show that the above solution oscillates until $v \approx \text{arcsinh} \ n$, during which time it undergoes approximately $(n \arctan n)/2\pi - 1/8$ cycles.

The Lorentzian evolution of the background metric is characterized by the exponential growth of the $S^1$ radius, while both the $S^2$ size and the horizon radius are practically constant. Perturbations with $n \geq 2$ initially have a wavelength smaller than the horizon and therefore represent not black holes, but mere ripples in the metric. This explains the initial oscillatory behavior. But the wavelength grows with the $S^1$ radius until the perturbations leave the horizon. Then they freeze, seeding the gravitational collapse of the two-spheres in the $n$ regions between the $2n$ black hole horizons, and the exponential growth of the two-spheres in the $n$ regions between the $2n$ cosmological horizons.

Due to the exchange of quantum radiation between the horizons, the black holes evaporate and shrink, while the cosmological horizons become larger. This follows from the behavior of $\delta(v)$ at late Lorentzian times, when $\sigma_n(v) =$

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Indeed, from Eq. (4.11) one finds that the horizon perturbation grows:

$$\delta(v) = \frac{A n}{c} \exp\left(\frac{2N\Lambda}{9}v\right).$$

The behavior of the multiple black holes is thus similar to that of the single black hole investigated in Ref. [?]. This result was confirmed for conformal scalars in Ref. [?]. Recently it was claimed that for different types of quantum matter, some of the spontaneously created handles may be stable at least initially [?]. For the arguments given here, however, it is sufficient that a fraction of handles develop multiple evaporating black holes.

The perturbative description breaks down when the black holes become much smaller than the cosmological horizons. But then the cosmological horizon will have a negligible influence on the black hole it surrounds, which will behave like a neutral black hole immersed in asymptotically flat space, evaporating at an ever-increasing rate. While standard physics breaks down at the endpoint of the evaporation of an uncharged black hole, it seems reasonable to assume that it will disappear altogether in a final Planckian flash of radiation.

Each black hole on the $S^1$ is surrounded by two cosmological horizons, beyond which lie intermediate regions bounded by cosmological horizons surrounding the two neighboring black holes (see Fig. 4). As the regions between the black hole horizons undergo gravitational collapse, these regions between pairs of cosmological horizons undergo a similar run-away, corresponding to the exponential expansion into asymptotically de Sitter universes. While the black holes evaporate, they grow extremely large. When a black hole finally disappears, the two asymptotic de Sitter regions on either side of it will disconnect. If there is only one black hole on the entire $S^1$, these “two” asymptotic de Sitter regions are merely opposite ends of the same region. Then the universe does not actually decompose into two parts, but merely reverts to its original, trivial $S^3$ topology. If, however, there are $n \geq 2$ black holes on the $S^1$, there will be $n$ distinct asymptotic de Sitter regions. As all the black holes evaporate, these large regions will pinch off, and become $n$ separate de Sitter universes. Each of the daughter universes will themselves harbor handle-creation events, so the disintegration process continues ad infinitum.

In this sense, de Sitter space proliferates.
5 Cosmological Implications

In most models of inflation, the universe undergoes a period of exponential expansion, driven by the vacuum energy $V(\phi)$ of a scalar field. While the evolution is vacuum dominated, the universe behaves like de Sitter space with an effective cosmological constant proportional to $V$. Typically, the field $\phi$ rolls down slowly, corresponding to a slow decrease of the effective cosmological constant, until it reaches the minimum of its potential. There $V = 0$, and inflation ends. Because of the slowness of this change, the universe looks locally de Sitter during inflation.

The handle-creation effect is a local event taking place on the scale of a horizon volume; on this scale, the change in $\phi$ is usually negligible. This is true for open as well as closed models. Therefore handle creation takes place during inflation and can be described by the same methods that were used above for the case of a fixed cosmological constant. Then multiple black holes can form and the universe will disintegrate when they evaporate.

It is easy to check that for most models of chaotic inflation, with power-law inflaton potentials, the total number of handle-creation events during inflation will be exponentially large. A non-negligible fraction of these handles will evolve into multiple black holes and eventually induce the disintegration of the inflating universe. Thus the inflationary era starts in a single universe but ends in an exponentially large number of disconnected universes, each of which enters into a radiation dominated phase.

According to most inflationary models, the universe is vastly larger than the present horizon. The proliferation effect I have discussed renders our position even more humble: Not only do we live in an exponentially small part of the universe, compared to its global size; but our universe is only one of exponentially many disconnected universes, all originating from the same small region in which inflation started.

So far it has been assumed that the inflaton field rolls down slowly according to its classical equations of motion. Linde has shown, however, that for sufficiently large values of the inflaton field, its random quantum fluctuations will dominate over the classical decrease. Every Hubble time, $e^3 \approx 20$ new horizon volumes are produced. If the quantum jumps of $\phi$ dominate, the effective cosmological constant will increase in about 10 of these new volumes, and decrease in the other 10. Thus inflation continues forever in some regions, and ends only where the inflaton field happens to jump down into the regime in which it must decrease classically.
This idea, called “eternal inflation”, has profound consequences for the global structure of the universe. It implies that inflation never ends globally and leads to a stationary overall stochastic state of the universe in some models. Crucially, in models allowing eternal inflation, there will be an infinite number of handle-creation events, and correspondingly, the universe will disintegrate into infinitely many parts. Inflation continues forever, and the production of new universes never ceases. The combination of eternal inflation with the disintegration effect, then, has led to a picture of global structure in which the number of separate universes is unbounded.

6 Conclusions

de Sitter space proliferates by disintegrating into separate, large copies of itself. This occurs through a decay process involving the formation and evaporation of multiple black holes. The process consists of several steps:

- The spontaneous creation of a handle, changing the spatial topology from $S^3$ to $S^1 \times S^2$. This process is suppressed by a factor of $e^{-\pi/\Lambda}$ and can be described semiclassically using gravitational instantons.

- The breaking of the degeneracy of the handle geometry by random quantum fluctuations. In the saddlepoint solution, the radius of the two-spheres is independent of the $S^1$ coordinate. If the $n$-th Fourier mode perturbation dominates, $n$ black hole interiors will form after the $S^1$ has expanded sufficiently.

- The evaporation of the $n$ black holes. By including one-loop quantum radiation in the s-wave and large $N$ approximation, one finds that black holes nucleated semi-classically in de Sitter space evaporate. Meanwhile, exponentially large de Sitter regions develop beyond the cosmological horizons.

- The disconnection of the spacelike hypersurfaces. When the black holes finally disappear, the toroidal $S^1 \times S^2$ topology is pinched at the evaporation endpoint: The $S^2$ radius becomes zero there and the hypersurface dissociates. After all the black holes evaporate, this leaves $n$ exponentially expanding three-spheres, corresponding to $n$ separate de Sitter universes.
The process repeats in the resulting fragments and continues indefinitely, producing infinitely many distinct universes.

According to the inflationary paradigm, the most successful theory of primordial cosmology, our universe went through a period of de Sitter-like expansion before the onset of the radiation and matter dominated eras. Therefore the process described in this paper is relevant to the global structure of the universe: It means that we live in one of many universes that originated in the same primordial inflationary region. There is a large class of models that lead to eternal inflation; in this case, we inhabit one of an infinite number of separate universes produced from a single region.

The process I have described lends itself to various generalizations, further enriching our picture of the global structure of the universe. One could consider, for example, the creation of handles carrying magnetic flux \([?, ?]\). The resulting black holes could not evaporate completely, resulting in a network of large de Sitter bubbles connected through thin extremal black hole throats.

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