CP Violation in Top Quark Pair Production at Hadron Colliders

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Abstract

CP violating effects in top quark pair production at the future 2 TeV $p\bar{p}$ Tevatron and 14 TeV $pp$ LHC colliders are investigated. We study three kinds of CP violating sources: the supersymmetric CP-odd phase of the stop trilinear soft breaking term: $\arg(A_t)$, the CP-odd parameter in two-Higgs doublet extensions of the standard model(2HDM), and the model-independent top quark chromoelectric dipole moment(CEDM), respectively. Optimal observables as well as simple observables are used. We find that it is possible to observe CP violating effects from $\arg(A_t)$ in top quark pair production at the 2 TeV Tevatron with $\sim 30 fb^{-1}$ integrated luminosity when $m_{\tilde{g}} \sim 200$ GeV. If the experimental systematic errors are sufficient small, the LHC with $\sim 150 fb^{-1}$ can put a limit of order $10^{-1}$ on the phase $\arg(A_t)$ and the CP-odd parameter in 2HDM by using optimal observables. The CEDM of the top quark can be measured to an accuracy of $10^{-18}$ cm g s at the Tevatron and few $\times 10^{-20}$ cm g s at the LHC.

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I. INTRODUCTION

At the future hadron colliders such as the upgraded Tevatron with $\sqrt{s} = 2$ TeV and the CERN LHC with $\sqrt{s} = 14$ TeV, the annual top quark pair yields are about $7 \times 10^4$ (with integrated luminosity $\int \mathcal{L} = 10 fb^{-1}$) and $7 \times 10^7$ (with $\int \mathcal{L} = 100 fb^{-1}$), respectively. The large numbers of top quark pairs allow us to do precise measurements on physical quantities associated with the top quark. Among them are the production cross sections, the top quark mass and the discrete symmetry properties. Due to the QCD uncertainties and the experimental systematic errors, the precision of cross section measurement is only about $5\% - 6\%$ [1]. The discrete symmetry properties such as parity nonconservation and CP violation do not suffer from QCD background uncertainty and their accuracies of measurements depends mainly on the statistical errors provided the experimental systematic errors are sufficient small. Therefore the discrete symmetry properties can be measured more precisely than the cross sections. New physics which has no observable contributions to cross section may have observable effects in parity or CP violation. Since the Standard Model(SM) contributions to parity violation [2] and CP violation are small, possible observed large effect of them will reveal new physics.

In a previous work [3], we have shown that the minimal supersymmetric(SUSY) extension of the standard model (MSSM) [4] gives observable parity violating effects at the Tevatron while its corrections to the production cross section are within the QCD theoretical uncertainties. In this paper, we shall concentrate on the CP violating effects induced by non-standard model interactions. We investigate three kinds of CP violating sources: the supersymmetric CP-odd phase of the stop trilinear soft breaking term: $\text{arg}(A_t)$, the CP-odd parameter in two-Higgs doublet extensions of the standard model(2HDM), and the model-independent top quark chromoelectric dipole moment(CEDM), respectively. In the MSSM, CP violation exists in strong interaction and in the 2HDM, CP violation can have strong Yukawa couplings due to the heavy top quark mass. Therefore, both can produce possible large effects. It is also useful to study CP violation in a model-independent way when we do not know what is the new physics. CP violation in top quark pair production at hadron colliders is also studied in Refs. [5]- [14]. In Ref. [7], SUSY QCD CP violating effects are studied in the $gg \rightarrow t\bar{t}$ process by using a charge energy asymmetry observable which is only sensitive to the imaginary part of the loop integrals. In this work, we use the optimal observables as well as naive observables constructed from the final state momenta.
Possible large CP violating effects in 2HDM and the methods of observing them in top quark pair production at hadron colliders are studied in Refs. [6] [12]. We extend those studies by applying optimal observables. In Ref. [8], the method of extracting real top quark CEDM in the reaction $gg \rightarrow t\bar{t}$ is studied. It is found that the optimal observables are particularly effective. We include here an imaginary part of CEDM and the reaction $q\bar{q} \rightarrow t\bar{t}$ at the Tevatron. Furthermore, we use the exact amplitudes of $gg(q\bar{q}) \rightarrow t\bar{t} \rightarrow b\bar{l}_1^+\nu_1\bar{b}l_2^-\nu_2$ ($b\bar{q}_1^c\bar{q}_2\bar{q}_3^c$). In Ref. [8], the top quark spin in its rest frame is taken to be in the direction of the lepton. This is a kind of approximation, although the lepton is a good analyzer of the top quark spin, because in the top rest frame, the lepton momentum has the angular distribution proportional to $1 + \cos \psi$ with $\psi$ being the angle between the top spin and the lepton momentum. The possibility of using polarized proton was studied in Ref. [14].

In Sec II, we describe the models and the calculations. The methods of extracting CP violating effects are given in Sec. III. In Sec IV, we present our results, discussions and conclusions.

II. MODELS AND CALCULATIONS

A. CP violation in MSSM

In the MSSM, two possibilities can induce CP violation in top quark interactions: the complex phase in Higgs mass parameter $\mu$, and the complex phase in scalar top supersymmetric soft breaking trilinear coupling $A_t$. The experimental limit on the Neutron Electric Dipole Moment (NEDM), $d_n \leq 1.1 \times 10^{-25} e\text{cm}$ [15], places a severe constraint on the phase of $\mu$. Therefore, the only significant SUSY CP-odd phase in association with the top quark is $\text{arg}(A_t)$. In Ref. [16], it is argued that the phase $\text{arg}(A_t)$ is not strongly constrained by current experiments and the effects in single top quark production and decay are studied. In this work, we shall assume $\text{arg}(\mu) = 0$ and let $\text{arg}(A_t)$ to be a free parameter of no a priori constraints.

The parameter $\text{arg}(A_t)$ enters in the scalar top quark mixing. The mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$ of scalar top quark are related to the current eigenstates $\tilde{t}_L$ and $\tilde{t}_R$ by

$$\tilde{t}_1 = \tilde{t}_L \cos \theta_t + \tilde{t}_R \sin \theta_t e^{-i\beta_t}, \quad \tilde{t}_2 = -\tilde{t}_L \sin \theta_t e^{i\beta_t} + \tilde{t}_R \cos \theta_t$$

(1)

The mixing angle $\theta_t$, phase $\beta_t$ as well as the masses $m_{\tilde{t}_{1,2}}$ can be calculated by diagonalizing the following mass matrix [4].
\[ M_t^2 = \begin{pmatrix} M_{i_L}^2 & m_t m_{LR} \\ m_t m_{LR} & M_{i_R}^2 \end{pmatrix}, \]

\[ M_{i_L}^2 = m_{i_L}^2 + m_t^2 + \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \cos(2\beta) m_Z^2, \]

\[ M_{i_R}^2 = m_{i_R}^2 + m_t^2 + \frac{2}{3} \sin^2 \theta_W \cos(2\beta) m_Z^2, \]

\[ m_{LR} = -\mu \cot \beta - A_t, \quad (2) \]

where \( m_{i_L}^2, m_{i_R}^2 \) are the soft SUSY-breaking mass terms of left- and right-handed stops, \( \tan \beta = v_2/v_1 \) is the ratio of the vacuum expectation values of the two Higgs doublets.

From Eqs.(1)(2), we can get the expressions for \( m_{i_{1,2}}^2, \theta_t \) and \( \beta_t \):

\[ m_{i_{1,2}}^2 = \frac{1}{2} \left[ M_{i_L}^2 + M_{i_R}^2 \mp \sqrt{(M_{i_L}^2 - M_{i_R}^2)^2 + 4m_t^2|m_{LR}|^2} \right], \quad (3) \]

\[ \tan \theta_t = \frac{m_{i_L}^2 - M_{i_L}^2}{m_t (-\mu \cot \beta - |A_t| \cos \theta_{A_t}) \cos \beta_t}, \quad (4) \]

\[ \tan \beta_t = \frac{|A_t| \sin \theta_{A_t}}{\mu \cot \beta + |A_t| \cos \theta_{A_t}}, \quad (5) \]

where \( \theta_{A_t} = \text{arg}(A_t) \).

In the presence of squark mixing, the strong squark-quark-gluino interaction Lagrangian is given by

\[ L_{\tilde{g}\tilde{q}\tilde{q}} = -g_s T^a_{jk} \tilde{g}_k [(a_1 - b_1 \gamma_5) \tilde{q}_{1j} + (a_2 - b_2 \gamma_5) \tilde{q}_{2j}] \tilde{g}_a + H.C., \quad (6) \]

where \( g_s \) is the strong coupling constant, \( T^a \) are \( SU(3)_c \) generators and for top quark \( a_1, b_1, a_2, b_2 \) are given by

\[ a_1 = \frac{1}{\sqrt{2}} (\cos \theta_t - \sin \theta_t e^{i\beta_t}), \quad b_1 = -\frac{1}{\sqrt{2}} (\cos \theta_t + \sin \theta_t e^{i\beta_t}), \]

\[ a_2 = -\frac{1}{\sqrt{2}} (\cos \theta_t + \sin \theta_t e^{-i\beta_t}), \quad b_2 = -\frac{1}{\sqrt{2}} (\cos \theta_t - \sin \theta_t e^{-i\beta_t}). \quad (7) \]

The above interactions enter in the virtual corrections to the main production processes of \( t\bar{t} \) at hadron colliders: \( q\bar{q} \to t\bar{t} \) and \( gg \to t\bar{t} \). There are also weak squark-quark-neutralino and squark-quark-chargino interactions. Since their coupling constants are an order of magnitude smaller than the strong SUSY QCD squark-quark-gluino interaction, we shall not consider them here. However, they may be important when the main processes are weak interactions [16].

In Fig.1(a)–(e), the Feynman diagrams of the QCD tree level and SUSY QCD virtual corrections of the process \( q\bar{q} \to t\bar{t} \) are given. The corresponding Feynman
diagrams of $gg \rightarrow t \bar{t}$ are presented in Fig.2(a)–(m) (the u-channels of Fig.2 (b),(f)–(m) are not depicted). The dashed lines in the loop stand for scalar quarks, while the solid lines for gluinos. In Ref. [7], CP violating effects are studied using a charge energy asymmetry observable which is only sensitive to the imaginary part of the loop integrals. Therefore, there are no contributions to the charge energy asymmetry from Fig.2 (f)–(j),(m).

The one loop scattering amplitudes of $q \bar{q} \rightarrow t \bar{t}$ and $gg \rightarrow t \bar{t}$ were already presented in Refs. [17–19] for calculating the total production rates of $t \bar{t}$ pairs. To calculate the CP violating effects in the $t \bar{t}$ system, additional renormalized amplitudes are needed. In terms of the tree-level amplitude, $M^a_0$, and the next-to-leading order SUSY QCD corrections, $\delta M^a$, the renormalized amplitudes of $a \bar{a} \rightarrow t \bar{t}$ ($a = q, g$) at the one-loop level may be written as

$$M^a = M^a_0 + \delta M^a, \quad (8)$$

where $\delta M^a$ can be decomposed into two parts: $\delta M^{aS}$ which contains even combination of $\gamma_5$ and $\epsilon_{\mu\nu\rho\sigma}$, and $\delta M^{aA}$ containing odd combination of $\gamma_5$ and $\epsilon_{\mu\nu\rho\sigma}$. The symmetry breaking effects are contained in $\delta M^{aA}$ which has no contributions to the total cross sections at next-to-leading order, while $\delta M^{aS}$ will contribute to the total cross sections. We shall assume that $\delta M^{aS}$ is small enough to be within the 5% – 6% uncertainty therefore is neglected in our calculations. We also discard terms in $\delta M^{aA}$ which give only parity asymmetry [3].

Let us denote the momenta of the initial and the final state particles as $a(p_4) \bar{a}(p_3) \rightarrow t_i(p_2) \bar{t}_j(p_1)$. We may use, as a further short-hand, the notation for $a = q$ that $u_i \equiv u(p_i)$ ($v_i \equiv v(p_i)$) denotes the Dirac four-spinor corresponding to the momentum and spin of particle (anti-particle). When $a = g$ we use $\epsilon_i \equiv \epsilon(p_i)$ for the gluon polarization function. In this notation, the tree level amplitude for $a = q$ can be written as

$$M^a_0 = ig^2_s (T^c_{ji} T^e_{lm}) \bar{v}_3 \gamma^\mu u_4 \bar{u}_2 \gamma_\mu v_1 / \hat{s}, \quad (9)$$

where $\hat{s}$ is the invariant mass of the $t \bar{t}$ pair.

The tree level amplitude for $a = g$ is composed of three different production channels(s-,t-,u-channel) as following:

$$M^g_s = -ig^2_s (if_{abc} T^c)_{ji} \bar{u}(p_2) \bar{v}(p_1) / \hat{s}$$

$$= -ig^2_s (T^a T^b - T^b T^a)_{ji} M^s_0, \quad (10)$$

$$M^g_t = -ig^2_s (T^b T^a)_{ji} \bar{u}(p_2) \bar{v}(p_1) / (\hat{t} - m_t^2)$$
\[
M_0^{\tilde{s}a} = M_0^{\tilde{g}l}(p_3 \leftrightarrow p_4, \ T^a \leftrightarrow T^b, \ \hat{t} \rightarrow \hat{u})
= -ig_s^2(T^b T^a)_{ji} M_0^l,
\]
where \( q = p_2 - p_4 \), \( \Gamma^\mu \) is given in the Appendix A.

To calculate the CP violating effects induced by the SUSY QCD effects, we follow the method presented in Ref. [5] [20], in which the amplitudes were calculated numerically using the helicity amplitude method. To obtain the renormalized scattering amplitudes, we adopt the dimensional regularization scheme to regulate the ultraviolet divergences and the on-mass-shell renormalization scheme to subtract the divergences [21].

The SUSY QCD corrections to the scattering amplitude \( \delta M^{aA} \) for \( a = q \) arise from the vertex diagram, the box diagram, as well as the crossed-box diagram. The renormalized amplitudes can be written as

\[
\delta M^{qa} = \delta M^{qv} + \delta M^{DB} + \delta M^{CB},
\]
where \( \delta M^{qv} \) is the vertex corrections of Fig.1 (b)(c), and \( \delta M^{DB} \) and \( \delta M^{CB} \) are the contributions from the box diagram and crossed-box diagram of Fig.1 (d)(e), respectively.

The form factor \( D_s \) corresponding to the top quark CEDM and the form factor \( f_{DB}^{aA} \) are given in the Appendix A.

The SUSY QCD corrections to the scattering amplitude \( \delta M^{qa} \) of \( gg \rightarrow t\bar{t} \) can be written as

\[
\delta M^{qa} = -ig_s^2(T^b T^a)_{ji} M_0^{l},
\]
where \( \delta M^{qv} = \delta M^{+} - \delta M^{-} \).

The form factor \( D_s \) corresponding to the top quark CEDM and the form factor \( f_{DB}^{aA} \) are given in the Appendix A.
\[
\delta M^+ = -\frac{1}{3} \delta M^{s2} + \delta M^{self,t} + \delta M^{self,u} + \delta M^{v1,t} + \delta M^{v1,u} + \delta M^{v2,t} + \delta M^{v2,u} + \delta M^{box1,t} + \delta M^{box1,u} + \delta M^{box2,t} + \delta M^{box2,u}
\]
\[
\delta M^- = -2\delta M^{s1} + \delta M^{self,t} - \delta M^{self,u} + \delta M^{v1,u} + \delta M^{v2,u} + \delta M^{box1,t} + \delta M^{box1,u} + \delta M^{box2,t} - \delta M^{box2,u}
\]
\[
\delta M^\delta = \frac{1}{2} \delta M^{s2} + \frac{1}{6} (\delta M^{box1,t} + \delta M^{box1,u}) - \frac{3}{2} (\delta M^{box2,t} + \delta M^{box2,u})
\]
(19)

In the above expressions, the superscript \(t,u\) stand for t-, u-channel. \(\delta M^{s1}\) is the s-channel vertex corrections from Fig.2 (c)(d), \(\delta M^{s2}\) from (e), \(\delta M^{self,t}\) the self energy correction from (f), \(\delta M^{v1,t}\) from (g), \(\delta M^{v2,t}\) from (h), \(\delta M^{box1,t}\) from (i), \(\delta M^{box2,t}\) from (j), \(\delta M^{box3,t}\) from (m). In the following we only give the explicit results of the s-channel (no crossed diagram) and t-channel contributions. The u-channel results can be obtained by the following substitutions:
\[
p_3 \leftrightarrow p_4, \quad a \leftrightarrow b, \quad \hat{t} \leftrightarrow \hat{u}. \quad (20)
\]

All s-, t-channel terms in \(\delta M^+, \delta M^-, \delta M^\delta\) can be written as following according to their Lorentz structures:
\[
\delta M^X = \epsilon_4 \epsilon_3 \epsilon_2 \left[ f_1^X g_{\mu\nu} + f_2^X \gamma_\mu \gamma_\nu + f_7^X p_{1\mu} p_{1\nu} + f_8^X p_{1\mu} p_{2\nu} + f_9^X p_{2\mu} p_{1\nu} + f_{10}^X p_{2\mu} p_{2\nu} + f_{13}^X \hat{p}_{1\mu} \gamma_\nu + f_{14}^X \hat{p}_{1\mu} \gamma_\nu + f_{15}^X \hat{p}_{2\mu} \gamma_\nu + f_{16}^X \hat{p}_{2\mu} \gamma_\nu \right] \gamma_5 v_1,
\]
where \(X = s1, s2, self, v1, v2, box1, box2, box3\), respectively. The 10 form factors corresponding to each diagram are given in the Appendix A. They are not all independent when we sum over all possible channels. CP-odd property of \(\delta M^X\) requires
\[
f_{13}^X = f_{16}^X, \quad f_{14}^X = f_{15}^X, \quad f_7^X + 2 f_{14}^X = f_{10}^X. \quad (22)
\]
Those relations are verified by our explicity formula of the form factors. We find that all form factors are proportional to \(\lambda_{CP} = 2Im(a_1 b_1^*) = \sin 2\theta_t \sin \beta_t\).

The color sum of the amplitude square including the next-to-leading order correction is:
\[
\sum_{\text{color}} \{ |M^0_0| + 2 Re(M^0_0 \delta M^{A1}) \}
\]
\[
= g_s^4 \left[ \frac{7}{3} |M^{+1}_0|^2 + 3 |M^0_0|^2 \right]
\]
\[
+ \frac{14}{3} Re(M^+_0 \delta M^{+}) + 6 Re(M^-_0 \delta M^{-}) + 8 Re(M^+_0 \delta M^\delta) \} ,
\]
where
\[ M_0^+ = M_0^0 + M_0^a, \quad M_0^- = M_0^0 - M_0^a - 2M_0^s, \]
\[ (24) \]

**B. CP violation in 2HDM**

In ordinary 2HDM, there are three neutral physical Higgs bosons, namely two CP-even scalars \( H, h \), one CP-odd pseudoscalar \( A \). CP violation in the scalar potential \([22]\) induces mixing of the CP-even and CP-odd neutral Higgs bosons, thus leading to three physical mass eigenstates \( |\phi_j\rangle (j = 1, 2, 3) \) with no definite CP parity. Their Yukawa couplings to the top quark can be written as (in the notation of \([23]\))
\[
L_Y = -m_t (\sqrt{2} G_F)^{1/2} \sum_{j=1}^{3} \bar{t}(a_{jt} + i\bar{a}_{jt}\gamma_5) t \phi_j
\]
\[ (25) \]
where \( G_F \) is the Fermi constant, and
\[
a_{jt} = d_{2j}/\sin \beta, \quad \bar{a}_{jt} = -d_{3j} \cot \beta,
\]
\[ (26) \]
d_{2j}, d_{3j} are the matrix elements of a \( 3 \times 3 \) orthogonal matrix which describes the mixing of the neutral states \([23]\). We assume that the two heavier Higgs bosons \( j = 2, 3 \) may be neglected and define \( a = a_{1t}, \bar{a} = \bar{a}_{1t}, \phi = \phi_1 \). The strength of CP violation is proportional to \( 2Im[a_{1t}(-i\bar{a}_{1t})^*] = 2a\bar{a} = -2\gamma_{CP} (\gamma_{CP} \text{ is defined in } [12]) \).

The one-loop Feynman diagrams of Higgs boson contributions to the processes \( qq \to t\bar{t} \) and \( gg \to t\bar{t} \) can be represented by Fig.1(b) and Fig.2(c)(f)(g)(i)(k)(n), respectively. Now the dashed lines in the loop stand for Higgs bosons, while the solid lines for top quarks.

The one-loop amplitudes can be easily obtained from those of the MSSM by the method given in the Appendix A with no changes on \( \delta M^{re} \), \( \delta M^+ \), \( \delta M^- \) and the modification on \( \delta M^s \) which comes from Fig.2 (n):
\[
\delta M^s = \epsilon_3^3\epsilon_3^i\bar{u}_2\{a^2(f_1^{sr} g_{\mu\nu} + f_2^{sr} p_{3\mu} p_{4\nu}) - \bar{a}^2 f_3^{sr} \epsilon_{\mu\nu\rho\sigma} p_4^\rho p_3^\sigma \gamma_5
\]
\[ +a\bar{a}[i(f_1^{sr} g_{\mu\nu} + f_2^{sr} p_{3\mu} p_{4\nu})\gamma_5 - f_3^{sr} \epsilon_{\mu\nu\rho\sigma} p_4^\rho p_3^\sigma]\}v_1/(\hat{s} - m_\phi^2 + im_\phi \Gamma_\phi)
\]
\[ (27) \]
where \( \Gamma_\phi \) is the Higgs boson width and \( f_{1,2,3}^s \) are given in the Appendix A. \( \delta M^s \) is composed of two parts: \( \delta M^{se} \), which is CP-even and contains terms of \( a^2, \bar{a}^2; \delta M^{oe}, \) the CP-odd term proportional to \( a\bar{a} \). We keep the CP-even term because when \( m_\phi > 2m_t \), it may be important. The \( 8Re(\bar{M}_0^+ \delta M^s) \) term in the color sum of (23) should be replaced by
\[8\text{Re}(M_0^+\delta M^5) + 24(|\delta M^{\delta e}|^2 + |\delta M^{\delta o}|^2) + 48\text{Re}(\delta M^{\delta e}\delta M^{\delta o})\]  

(28)

We compute \(\Gamma_\phi\) by using the formula and parameters of Ref. [12].

C. CP violation from model-independent top quark CEDM

When the particles in the loops are heavy compared to the external particle momenta, it is convenient to describe the loop induced interactions by effective interaction Lagrangian. Even though when the loop masses are not too large, a model-independent study can give us knowledge about the sensitivities of given colliders. That approach is particularly useful when we do not know the underlying new physics. We assume the following additional top-quark-gluon effective interaction:

\[L_D = \frac{id_t}{2} i\sigma^{\mu\nu}\gamma_5 F^a_{\mu\nu} T^a_t,\]  

(29)

where \(\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]\), \(F^a_{\mu\nu}\) is the the gluon field strength. The coefficient \(d_t\) is the top quark chromoelectric dipole moment (CEDM) which we assume to have imaginary part as well as real part. We denote it as \(d_t^R + id_t^I\).

The CEDM \(d_t\) contributes to the CP violating amplitudes through the diagrams of Fig.3. Let us define \(\hat{d}_t = d_t/g_s = d_t^R + id_t^I\). The contribution of Fig.3 (a) to \(q\bar{q} \rightarrow t\bar{t}\) can be obtained by simply replacing \(D^s\) with \(i\hat{d}_t\) in Eq.(14) and setting \(\delta M^{\delta B} = \delta M^{\delta B} = 0\). We denote the contributions to \(gg \rightarrow t\bar{t}\) of Fig.3(b)–(e) as \(\delta M^s, \delta M^b, \delta M^v, \delta M^u\) (for the crossed diagrams), respectively. Then we have

\[\delta M^+ = \delta M^{v,t} + \delta M^{v,u},\]
\[\delta M^- = -2\delta M^s - 2\delta M^b + \delta M^{v,t} - \delta M^{v,u},\]
\[\delta M^5 = 0.\]  

(30)

\(\delta M^s, \delta M^b, \delta M^{v,t}\) can also be written as the form of Eq.(21) with the constant form factors given in the Appendix A.

III. EXTRACTING THE CP VIOLATING EFFECTS

Since \(\delta M^{aA}\) contains odd combination of \(\gamma_5\) and \(\epsilon_{\mu\nu\rho\sigma}\), its interference with the tree level amplitude will be zero if we sum up all the initial and final state spins or

\[\text{The misprint of } \Gamma_Z \text{ is corrected.}\]
polarizations. To see the CP violating effects, the spins of at least some of the particles must be identified. Because the initial state spins are difficult to be determined, the helicities of the final $t\bar{t}$ must be inferred from their decays. If we assume the SM couplings of the top quark to its decay products and set $m_b = 0$, all decay products of the top quark will have left-handed helicities. That means the spin information of the top quark can only be transferred to the momentum correlations among the decay products.

We assume the SM decay of the top quark\(^2\) and apply the narrow width approximations of the top quark and W-boson propagators:

\[
\frac{1}{|q_Y^2 - m_Y^2 + im_Y\Gamma_Y|^2} \rightarrow \frac{\pi}{m_Y\Gamma_Y} \delta(q_Y^2 - m_Y^2). \tag{31}
\]

where $Y$ stands for top quark and W-boson, $\Gamma_Y$ is the width of $Y$.

The parton level cross section for reaction $a\bar{a} \rightarrow t\bar{t} \rightarrow bl\bar{l}_1\nu\bar{\nu}_1 bl\bar{l}_2\bar{\nu}_2$ (or $b\bar{q}_1 q_2^\prime \bar{b} q_1 q_2^\prime$) can be written as:

\[
d\hat{\sigma}_{a\bar{a}} = \frac{\gamma}{(8\pi)^{10}S_a} \frac{\lambda_t |M_a|^2}{m_t^2 m_W^2 \Gamma_t^2 \Gamma_W^2} d\Omega_t d\Omega'_W + d\Omega'_W - d\Omega'_l_1 d\Omega'_l_2 \tag{32}
\]

where $\gamma = \sqrt{1 - 4m_t^2/s}$ and

\[
\lambda_t = (1 - \frac{(m_W + m_b)^2}{m_t^2})(1 - \frac{(m_W - m_b)^2}{m_t^2}) \approx (m_t^2 - m_W^2)^2/m_t^4, \tag{33}
\]

$d\Omega'_W$ is the solid angle element of $W^+$ ($W^-$) in the rest frame of the (anti) top quark, $d\Omega'_l_1$ ($d\Omega'_l_2$) denotes the solid angle element of $l_1^\pm$ ($l_2^\pm$) in the rest frame of $W^+$ ($W^-$), $|M_a|^2$ is the average amplitude square excluding the top quark and W-boson propagators after the decays of the top quarks:

\[
|M_a|^2 = 1/S_a^{-1} \sum_{\text{color,spin}} \{|M_a^s|^2 + 2Re(M_a^s \delta M^{sA})\}, \tag{34}
\]

where $1/S_a$ is the color,spin average factor: $S_q = 36$ and $S_g = 256$. In our calculations, $|M_a|^2$ is easily obtained from the amplitude of $a\bar{a} \rightarrow t\bar{t}$ by the following substitutions:

\[
\bar{u}_2 \rightarrow \frac{g^2}{8} \bar{u}_2 \gamma_\mu (1 - \gamma_5)(\not{p}_2 + m_t)\bar{u}_1 \gamma^\mu (1 - \gamma_5)v_1, \tag{35}
\]

\[
v_1 \rightarrow \frac{g^2}{8} \bar{u}_1 \gamma_\mu (1 - \gamma_5)v_2 (\not{p}_1 - m_t)\gamma^\mu (1 - \gamma_5)v_2, \tag{36}
\]

\(^2\)The top quark decay asymmetry in the MSSM is found to be small [16] [24].
where $g$ is the weak $SU(2)$ coupling constant. The above expressions are calculated numerically.

The hadronic cross section is obtained by convoluting (32) with parton distribution functions:

$$d\sigma_{pp\bar{p}} = \sum_{a=q,g} \int_0^1 dx_1 \int_0^1 dx_2 [f_a^p(x_1) f_a^{\bar{p}(p)}(x_2) + (p \leftrightarrow \bar{p}(p))] d\hat{\sigma}_{a\bar{a}}/(1 + \delta_{a,\bar{a}}),$$  \hspace{1cm} (36)

where $f_a^p(x), f_a^{\bar{p}(p)}$ are parton distribution functions (PDFs).

It has been studied extensively in the literature how to pick out the momentum correlation information contained in $2\text{Re}(M_0^a \delta M^{a'\bar{A}})$ and therefore the CP violating effects by looking at the expectation values of the CP-odd observables constructed from the momenta of the top quarks and their decay products [5]-[12] [16] [25]-[28]. In this study, we adopt the following simple observables which are constructed from observed momenta and can be easily used by experimentists:

$$A_1 = E_{l^+} - E_{l^-},$$
$$A_2 = \vec{p}_t \cdot \vec{p}_{l^+} - \vec{p}_t \cdot \vec{p}_{l^-} \equiv O_t - Q_t,$$
$$T_2 = (\vec{p}_b - \vec{p}_\bar{b}) \cdot (\vec{p}_l^+ - \vec{p}_l^-),$$
$$f_2 = \frac{\epsilon_{\mu\nu\sigma\rho} p_\mu^l p_\nu^l p_\sigma^\nu p_\rho^\mu}{(p_{l^+} \cdot p_{l^-} p_b \cdot p_\bar{b})^{1/2}},$$
$$\hat{O}_L = \frac{1}{m_t^3 |\vec{P}|^2} \vec{P} \cdot (\vec{p}_l^+ \times \vec{p}_l^-) \vec{P} \cdot (\vec{p}_l^+ - \vec{p}_l^-),$$ \hspace{1cm} (41)

where all momenta are in the laboratory frame, $E_{l^+} (E_{l^-})$ is the energy of $l^+(l^-)$ ($l = l_1 = l_2 = e, \mu$, here we do not distinguish $e, \mu$), the subscripts of the momenta denote the corresponding particles, $\vec{P}$ is the momentum of the proton in the case of $pp\bar{p}$ collision. $A_1, A_2,$ and $T_2$ are studied in Ref. [12]. $f_2$ and $\hat{O}_L$ are used in Refs. [8] [11], respectively. Because $A_1, A_2$ are $\hat{T}$-(time reflection [27] [29]) even, they are only sensitive to the absorptive parts (imaginary parts) of the loop calculations and $\hat{d}_t$. On the contrary, $T_2, f_2,$ and $\hat{O}_L$ are $\hat{T}$-odd and only sensitive to the dispersive parts(real parts). $A_1, T_2, f_2$ and $\hat{O}_L$ all require the events that two top quarks decay semileptonically. $A_2$ uses the events that one top quark decays semileptonically and the other hadronically. Among the above observables, only $f_2$ is Lorentz invariant.

While all the above naive observables use only parts of the information, the optimal observables studied in Refs. [30]-[32] use the full information in $2\text{Re}(M_0^a \delta M^{a'\bar{A}})$. Therefore they are the most effective ones.

In the case of model independent top quark CEDM, $2\text{Re}(M_0^a \delta M^{a'\bar{A}})$ contains two terms which are proportional to $\hat{d}_t^R$ and $\hat{d}_t^I$, respectively. In the MSSM, it depends on
the particle masses in the loop as well as a multiplicative constant $\lambda_{CP}$. However, since our main goal is to search for CP violation induced by this $\lambda_{CP}$, we must assume all the masses in the loop are known. In the 2HDM, from the first term in Eq.(28) which is CP-odd, one can separate a factor $\gamma_{CP} = - a \tilde{a}$. Although in the resonant region, contributions of the other three terms (belonging to higher than next-to-leading order) may be large, they are still overwhelmed by the tree level and the next-to-leading order contributions. As an approximation, we shall drop them in the definitions of the optimal observables. Therefore we can always separate a constant (denote it as $\lambda = \lambda_{CP}$, $\gamma_{CP}$, $\hat{d}_R$, $\hat{d}_I$) from $2Re(M_0^a \delta M^{A\dagger})$ in all the models considered.

Apart from some common factors in (36), the hadronic cross section can be written as

$$d\sigma_{p\bar{p}(p)} = \sum_{a=q,g} \int \sum_{\text{color,spin}} \left\{ |M_0^a|^2 + \lambda 2 Re(M_0^a \delta M^{A\dagger}) \right\} \left[ f_a^p(x_1) f_{\bar{a}}^\bar{p}(x_2) + (p \leftrightarrow \bar{p}(p)) \right] d\Phi dx_1 dx_2,$$

where $d\Phi$ denotes the phase space. In the following, $2Re(M_0^a \delta M^{A\dagger})$ is calculated by setting $\lambda_{CP} = 1$, $\gamma_{CP} = 1$, $\hat{d}_t = 1$ and $\hat{d}_t = i$ in the form factors in each model, respectively. The optimal observable is defined as

$$O_{opt} = \frac{\sum_{a=q,g} \sum_{\text{color,spin}} 2Re(M_0^a \delta M^{A\dagger}) [f_a^p(x_1) f_{\bar{a}}^\bar{p}(x_2) + (p \leftrightarrow \bar{p}(p))]}{\sum_{a=q,g} \sum_{\text{color,spin}} |M_0^a|^2 [f_a^p(x_1) f_{\bar{a}}^\bar{p}(x_2) + (p \leftrightarrow \bar{p}(p))]}. \quad (43)$$

The above observable depends on the parton distribution functions. It is inconvenient for practical use. In the $pp$ collision, it is not a CP-odd observable due to the asymmetry of quark PDFs (cf. Appendix C). Because at the 2 TeV Tevatron, the main $t\bar{t}$ production mechanism is $q\bar{q} \rightarrow t\bar{t}$, and at the 14 TeV LHC, the main process is $gg \rightarrow t\bar{t}$, we can neglect one process at each collider. We consider only $q\bar{q} \rightarrow t\bar{t}$ at the Tevatron and $gg \rightarrow t\bar{t}$ at the LHC in all the following calculations. Then we have the optimal observable

$$O_1 = \frac{\sum_{\text{color,spin}} 2Re(M_0^a \delta M^{A\dagger})}{\sum_{\text{color,spin}} |M_0^a|^2}. \quad (44)$$

Since the neglected process consists of only about 10% of the total cross section, Eq. (44) deviates from the truly optimal observable by at most 20%. $O_1$ has the property of Lorentz invariance, one can calculate it in any frame. It has the same symmetry property as $2Re(M_0^a \delta M^{A\dagger})$ which is CP-odd. In the MSSM and 2HDM, $O_1$ has no
definite $\hat{T}$ parity and depends on the loop particle masses. We note that the optimal observable defined for $\hat{d}_t^R$ is only sensitive to $\hat{d}_t^R$ independent of $\hat{d}_t^I$. The same holds true for $\hat{d}_t^I$. This is because the two terms proportional to $\hat{d}_t^R$ and $\hat{d}_t^I$ have different discrete symmetry $\hat{T}$.

To calculate $O_1$, we need to know all the momenta of the initial and final state particles. That can not always be achieved. It is still useful because it can provide us with the upper limit signal to noise ratios of any other CP-odd observables. Let us first look at the case that two top quarks decay semileptonically. The two neutrino momenta $p_{\nu_1}, p_{\nu_2}$ will be unknown. But they may be determined indirectly by the following eight equations [8]:

$$p_{\nu_1}^2 = p_{\nu_2}^2 = 0, \quad (p_{\nu_1} + p_{\nu_1^+})^2 = (p_{\nu_2} + p_{\nu_2^+})^2 = m_W^2,$$

$$ (p_{\nu_1} + p_{\nu_1^+} + p_b)^2 = m_t^2, \quad (p_{\nu_2} + p_{\nu_2^+} + p_b)^2 = m_t^2,$$

$$ (p_{\nu_2} + p_{\nu_2^+} + p_b)_{\text{transverse}} = -(p_{\nu_1} + p_{\nu_1^+} + p_b)_{\text{transverse}}.$$  

(45)

Similar situation exists in the study of $\tau\bar{\tau}$ production in $e^+e^-$ collision [33]. The above equations give rise up to fourfold solutions (see Appendix B). Considering this ambiguity, we can define a modified optimal observable:

$$O_l = \frac{\sum_{\text{color,spin,i}} 2Re(M_0^a \delta M^{aA}) \eta_i}{\sum_{\text{color,spin,i}} |M_0^a|^2 \eta_i}.$$  

(46)

where the sum $i$ is over all possible solutions of the neutrino momenta. $\eta_i = \gamma_i / \hat{s}_i [f_\nu(x_1^i) f_\nu(x_2^i) + (p \leftrightarrow p(\bar{p}))]$ comes from the $t\bar{t}$ phase space, flux factor and PDFs due to different momentum reconstruction (cf. Eqs. (32) and (36)). There may be possibility that the reconstructed initial parton energy exceeds the proton (antiproton) energy. That kind of reconstruction should be discarded in the calculations. $O_l$ also depends on PDFs. For practical use, we define a non-optimal observable as an approximation:

$$O'_l = \sum_i O_1.$$  

(47)

Again the sum $i$ is over all possible solutions of the neutrino momentum$^3$.

$^3$One can also define $O'_l$ by setting $\eta_i = 1$ in (46), the numerical difference between the two definitions is minor.
We now consider that one top quark decays semileptonically and the other hadronically. The missing neutrino momentum can be fully reconstructed [34]. But because we can not distinguish quark and antiquark jet, we still have ambiguity of twofold uncertainty. When two quarks decay all hadronically, there is fourfold uncertainty. We define therefore alternatively the optimal observables:

\begin{equation}
O_2 = \frac{\sum_{\text{color,spin},j} 2\text{Re}(M_0^a \delta M^{aA})}{\sum_{\text{color,spin},j} |M_0^a|^2},
\end{equation}

\begin{equation}
O_4 = \frac{\sum_{\text{color,spin},j'} 2\text{Re}(M_0^a \delta M^{aA})}{\sum_{\text{color,spin},j'} |M_0^a|^2},
\end{equation}

where the sum $j$ in $O_2$ is over the two possible assignments of the jet momenta, and the sum $j'$ in $O_4$ is over the four assignments.

The statistical significance of an observable $O$ can be described by the signal to noise ratio $r$ defined as

\begin{equation}
r = \frac{<O>}{\sqrt{<O^2>}},
\end{equation}

where $<O>, <O^2>$ are the expectation value of $O$, $O^2$, respectively:

\begin{equation}
<O^n> = \frac{\int O^n d\sigma_{pp(p)}}{\int d\sigma_{pp(p)}}.
\end{equation}

It is interesting to note that for the optimal observables and unit $\lambda$, we always have $<O> = <O^2>$. Care must be taken in calculating the $r$ of $A_2$. Because $A_2$ is the difference of two observables ($O_l$ and $O_t$) which are calculated using different events (i.e. different independent distribution functions), we have $<A_2^2> = <O_l^2> + <O_t^2> \approx 2 <O_l^2>$. If the experimental error comes only from statistics, the number of events $N_{\text{event}}$ needed to observe CP violating effects at 1$\sigma$ level (68% C.L.) satisfies $|r| \geq 1/\sqrt{N_{\text{event}}}$ or $N_{\text{event}} \geq 1/r^2$. To reduce the statistical errors, one can combine the measured results of the three decay modes: leptonic-leptonic, leptonic-hadronic and hadronic-hadronic modes [15]. Assuming their corresponding number of events are $N_{ll}, N_{jl}$ and $N_{jj}$, respectively, then we can define a combined signal to noise ratio:

\begin{equation}
r_c = \sqrt{((N_{ll}r_1^2 + N_{jl}r_2^2 + N_{jj}r_3^2)/N),}
\end{equation}

where $r_1$, $r_2$ and $r_3$ are the signal to noise ratios of the observables which use leptonic-leptonic, leptonic-hadronic and hadronic-hadronic events, respectively. $N = N_{ll} + N_{jl} + N_{jj}$ is the total number of events of the three modes. Note that $r_c$ depends only on the ratios $N_{ll}/N$, $N_{jl}/N$ and $N_{jj}/N$, not on $N$. The signal is detectable at 1$\sigma$ level when $|r_c| \geq 1/\sqrt{N}$.
IV. NUMERICAL RESULTS AND CONCLUSIONS

We first check our calculations with QCD gauge invariance in the process $gg \rightarrow t\bar{t}$. That can be done by replacing $\epsilon_3, \epsilon_4$ with $p_3, p_4$. We find that the correction amplitude is consistent with gauge invariance. Then we check our calculations with known results in the literature. Our parton level results are all in agreement with Refs. [7] [12]. By using $\sqrt{s} = 40$ TeV, $m_t = 160$ GeV and $10^7$ sample of leptonic events, we find the $O_1$ limit on $d_R^t$ is $2.8 \times 10^{-20}$ cm $g_s$ which is very close to $\lambda_{\text{min}}$ in Ref. [8]. That means that taking the top quark spin in its rest frame to be in the direction of lepton momentum is feasible. We can also reproduce the results of $\hat{O}_L$ in Ref. [11] with $\mu_t' = 0$ by including both $q\bar{q} \rightarrow t\bar{t}$ and $gg \rightarrow t\bar{t}$ processes in the calculations.

As mentioned previously, in the following calculations, we consider only $q\bar{q} \rightarrow t\bar{t}$ at the Tevatron and $gg \rightarrow t\bar{t}$ at the LHC. The parton distribution functions of MRS set A’ [35] with scale $Q^2 = m_t^2$ are used$^4$. $m_t$ is taken to be 176 GeV. To look at the largest possible effects, we set the CP-violating parameters $\lambda_{\text{CP}}$ and $\gamma_{\text{CP}}$ to be of order 1, namely, $\lambda_{\text{CP}} = 1$ (this needs $\theta_t = \pi/4$, cf. Appendix A), $\gamma_{\text{CP}} = 1$. In the MSSM and 2HDM, we treat the SUSY particle masses and Higgs boson mass as free parameters allowed by current experiments. We assume that all the squarks except for the light stop $\tilde{t}_1$ to be degenerate. The light stop mass is required to be above 50 GeV [37]. We choose gluino mass $m_{\tilde{g}}$ and Higgs boson mass $\geq 100$ GeV.

The signal to noise ratio results at the 2 TeV $p\bar{p}$ Tevatron and at the 14 TeV $pp$ LHC are summarized in Table I.–Table V. We do not present the results for 2HDM at the Tevatron because the effects are too small unless the Higgs boson mass is less than 100 GeV. We denote the signal to noise ratio $r$ of an observable $O$ as $r(O)$. All the tables show that $r(O')$ is slightly smaller than $r(O_1)$ and $r(O_2)$ is about $3/4$ of $r(O_1)$, $r(O_4)$ about $1/2$ of $r(O_1)$. Therefore, $O'_1$ is a good approximation for $O_1$.

Table I. shows the results at the Tevatron from five sets of SUSY parameters. We see that the naive observables have signal to noise ratios all $\leq 1%$. It is difficult to observe such small effects at the Tevatron. The optimal observables, on the other hand, have $r \geq 1\%$ as long as the gluino mass is around 200 GeV $^5$. They are about

$^4$It should be noted that all the results are insensitive to the PDF and $Q^2$ choices. CTEQ4M [36] gives similar results.

$^5$The value is close to $m_t$, so that the gluino threshold is close to the top quark one. The
3 – 10 times more effective than the naive ones. The combined signal to noise ratios of $O'_l, O_2$ and $O_4$ are all between these of $O_2$ and $O_4$. Because $N_{ll}$ is small compared with $N_{jl}$ and $N_{jj}$, the combined $r$ is just a weighted average of these of $O_2$ and $O_4$. Assuming we can obtain 30 $fb^{-1}$ integrated luminosity, then the total number of fully reconstructed $t\bar{t}$ events is about $3.5 \times 10^4$ [1]. It is reasonable to assume that we can have purely hadronic events $N_{jj} = 2 \times 10^4$, hadronic-leptonic events $N_{jl} = 1.2 \times 10^4$ and purely leptonic events $N_{ll} = 0.2 \times 10^4$. The corresponding 1 $\sigma$ level statistical errors are $r_{jj} = 0.71 \times 10^{-2}$, $r_{jl} = 0.91 \times 10^{-2}$, $r_{ll} = 2.2 \times 10^{-2}$. The combined error is 0.54%. In the Table, $O'_l$ has $r > r_{ll}$ only when $m_{\widetilde{t}_1}$ is around 50 GeV and $m_{\widetilde{g}}$ is around 200 GeV. We always have $r(O_2) > r_{jl}$, $r(O_4) > r_{jj}$ when $m_{\widetilde{g}} \sim 200$ GeV. Therefore, it is possible to detect the CP violating effects by using optimal observables $O_2$ and $O_4$ when $m_{\widetilde{g}}$ is around 200 GeV. The effects are detectable by combined measurement when $m_{\widetilde{g}}$ is in the range 100-300 GeV.

Table II. is the results at the LHC in MSSM. It shows the same features as Table I. However, at the LHC, the numbers of events are much larger than those at the Tevatron. With 150 $fb^{-1}$ integrated luminosity, we can assume $N_{jj} = 10^7$, $N_{jl} = 6 \times 10^6$ and $N_{ll} = 10^6$. We further assume the experimental systematic errors are below the statistical ones. In addition, there are also theoretical uncertainties coming from possible non-CP violating contaminations at the $pp$ LHC. Because the initial $pp$ state is not a CP eigenstate, CP conserving interactions can produce CP asymmetry effects in $t\bar{t}$ final state. We present a general analysis of the contaminations to the Lorentz invariant observables in Appendix C. We find that within the framework of parton model, there are no contaminations to these observables. $A_1$, $A_2$ and $T_2$ are discussed in Refs. [6] [12]. They are well below the signals. Therefore, the 1 $\sigma$ level errors are $r_{jj} = 0.32 \times 10^{-3}$, $r_{jl} = 0.41 \times 10^{-3}$, $r_{ll} = 1.0 \times 10^{-3}$. The combined error is $0.24 \times 10^{-3}$. The naive observable $T_2$ is on the margin to be detectable. $A_1$ and $f_2$ are better than $T_2$. They can be used to observe $\lambda_{CP}$ at few $\times 10^{-1}$ when $m_{\widetilde{g}}$ is within the range 100 – 400 GeV. The observable $O'_l$ is 2-5 times better. All $r(O_2) \geq 10 \ r_{jl}$ and $r(O_4) \geq 10 \ r_{jj}$ for $m_{\widetilde{g}} \sim 100 – 300$ GeV. We can pin down $\lambda_{CP}$ to $10^{-1}$ by using these optimal observables.

In Table III., we give the results of signal to noise ratios in 2HDM at the LHC. It is obvious that $A_1$ and $T_2$ are only detectable and can not be used to put limit on $\gamma_{CP}$. $f_2$ is 2 times better which may be used to limit $\gamma_{CP}$ to $(3 – 4) \times 10^{-1}$. All the optimal CP violating effects are large due to the threshold effects.
observables have \( r \) large than about 10 times statistical errors. Therefore, they will put a limit of order \( 10^{-1} \) on \( \gamma_{CP} \).

Table IV. and V. are the results of model-independent top quark CEDM. An overall feature of the two tables is that both at Tevatron and LHC, the accuracies of \( d_I^t \) are better than those of \( d_R^t \). With the above assumed numbers of events at the Tevatron, the best limits on \( d_R^t \) and \( d_I^t \) are \( 2.4 \times 10^{-18} cm \ g_s \) and \( 1.1 \times 10^{-18} cm \ g_s \), respectively, by using \( O_2 \). Also by using \( O_2 \) at the LHC, we can obtain the limits of \( 5.2 \times 10^{-20} cm \ g_s \) and \( 2.5 \times 10^{-20} cm \ g_s \) on \( d_R^t \) and \( d_I^t \), respectively.

To summarize, we have studied CP violating effects in top quark pair production at the future 2 TeV \( p\bar{p} \) Tevatron and 14 TeV \( pp \) LHC colliders. Three kinds of CP violating sources: the SUSY CP-odd phase of the stop trilinear soft breaking term, \( \arg(A_t) \), the CP-odd parameter in 2HDM, and the model-independent top quark CEDM are investigated. Optimal observables as well as simple observables are used. The optimal observables are usually 2 – 10 times more effective than the naive ones. It is possible to observe CP violating effects from \( \arg(A_t) \) in top quark pair production at the 2 TeV Tevatron with \( \sim 30 fb^{-1} \) integrated luminosity when \( m_{\tilde{g}} \sim 200 \) GeV. If combined measurement is applied, the range of gluino mass in which the CP violating effects are detectable is \( \sim 100 – 300 \) GeV. The LHC with \( 150 fb^{-1} \) can put a limit of order \( 10^{-1} \) on \( \lambda_{CP} \) (therefore on \( \arg(A_t) \)) in MSSM and \( \gamma_{CP} \) in 2HDM by using optimal observables provided the experimental errors are sufficient small. The CEDM of the top quark can be measured to an accuracy of \( 10^{-18} \) \( cm \ g_s \) at the Tevatron and few \( \times 10^{-20} \) \( cm \ g_s \) at the LHC. More accurate measurement on \( d_I^t \) can be obtained than on \( d_R^t \) with given number of events.
TABLES

TABLE I.
Signal to noise ratio $r$ in $p\bar{p} \rightarrow t\bar{t} + X$ at the 2 TeV Tevatron in the MSSM with $\lambda_{CP} = 1$, for five sets of SUSY parameters labeled by $(m_{\tilde{t}_1}, m_{\tilde{t}_2} = m_{\tilde{q}}, m_{\tilde{g}})$ GeV. The combined results are for $O'_l$, $O_2$ and $O_4$.

<table>
<thead>
<tr>
<th>$(m_{\phi}, m_{\tilde{f}})$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$T_2$</th>
<th>$f_2$</th>
<th>$O_1$</th>
<th>$O'_l$</th>
<th>$O_2$</th>
<th>$O_4$</th>
<th>combined</th>
</tr>
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<tr>
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<td>0.25%</td>
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<td>-0.16%</td>
<td>-0.16%</td>
<td>0.82%</td>
<td>0.79%</td>
<td>0.62%</td>
<td>0.45%</td>
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<tr>
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<td>1.82%</td>
<td>1.73%</td>
<td>1.41%</td>
<td>1.04%</td>
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</tr>
<tr>
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<td>(100,1000,200)</td>
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TABLE II.
Signal to noise ratio $r$ in $pp \rightarrow t\bar{t} + X$ at the LHC in the MSSM with $\lambda_{CP} = 1$, for six sets of SUSY parameters labeled by $(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{g}})$ GeV. The combined results are for $O'_l$, $O_2$ and $O_4$.

<table>
<thead>
<tr>
<th>$(m_{\phi}, m_{\tilde{f}})$</th>
<th>$A_1$</th>
<th>$T_2$</th>
<th>$f_2$</th>
<th>$O_1$</th>
<th>$O'_l$</th>
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<th>$O_4$</th>
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<td>1.62%</td>
<td>1.20%</td>
<td><strong>1.41%</strong></td>
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TABLE III.
Signal to noise ratio $r$ in $pp \rightarrow t\bar{t} + X$ at the LHC in the 2HDM with $\gamma_{CP} = 1$. The combined results are for $O'_l$, $O_2$ and $O_4$.

<table>
<thead>
<tr>
<th>$m_{\phi}$(GeV)</th>
<th>$A_1$</th>
<th>$T_2$</th>
<th>$f_2$</th>
<th>$O_1$</th>
<th>$O'_l$</th>
<th>$O_2$</th>
<th>$O_4$</th>
<th>combined</th>
</tr>
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<tbody>
<tr>
<td>100</td>
<td>-0.20%</td>
<td>0.10%</td>
<td>0.30%</td>
<td>1.09%</td>
<td>0.97%</td>
<td>0.82%</td>
<td>0.60%</td>
<td><strong>0.71%</strong></td>
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<tr>
<td>200</td>
<td>-0.16%</td>
<td>0.12%</td>
<td>0.30%</td>
<td>0.96%</td>
<td>0.85%</td>
<td>0.69%</td>
<td>0.49%</td>
<td><strong>0.59%</strong></td>
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<tr>
<td>300</td>
<td>-0.15%</td>
<td>0.14%</td>
<td>0.36%</td>
<td>1.12%</td>
<td>0.97%</td>
<td>0.78%</td>
<td>0.54%</td>
<td><strong>0.66%</strong></td>
</tr>
<tr>
<td>400</td>
<td>-0.13%</td>
<td>0.20%</td>
<td>0.45%</td>
<td>1.55%</td>
<td>1.52%</td>
<td>1.31%</td>
<td>0.92%</td>
<td><strong>1.11%</strong></td>
</tr>
<tr>
<td>500</td>
<td>-0.05%</td>
<td>0.15%</td>
<td>0.31%</td>
<td>0.91%</td>
<td>0.87%</td>
<td>0.76%</td>
<td>0.53%</td>
<td><strong>0.64%</strong></td>
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</tbody>
</table>
TABLE IV.
Signal to noise ratio $r$ at the Tevatron and LHC with model independent top quark CEDM $d^R_t = 1 \text{ GeV}^{-1} g_s = 1.97 \times 10^{-14} \text{ cm} \ g_s$, and the accuracies with which $d^R_t$ can be measured at the Tevatron and LHC with assumed numbers of events given in the text. $d^R_t$ is given in unit of $10^{-18} \text{ cm} \ g_s$. $O_1$ is given only for leptonic events. The combined results are for $O'_1, O_2, O_4$.

<table>
<thead>
<tr>
<th></th>
<th>$T_2$</th>
<th>$f_2$</th>
<th>$O_L$</th>
<th>$O_1$</th>
<th>$O'_1$</th>
<th>$O_2$</th>
<th>$O_4$</th>
<th>combined</th>
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<td></td>
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<tr>
<td>$r$</td>
<td>39.9</td>
<td>40.9</td>
<td>−35.2</td>
<td>124</td>
<td>88</td>
<td>75</td>
<td>47</td>
<td>61</td>
</tr>
<tr>
<td>$d^R_t$</td>
<td>10.9</td>
<td>10.6</td>
<td>12.3</td>
<td>3.5</td>
<td>4.9</td>
<td>2.4</td>
<td>3.0</td>
<td>1.7</td>
</tr>
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<td>LHC</td>
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<td></td>
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<td></td>
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<tr>
<td>$r$</td>
<td>−53.6</td>
<td>−112</td>
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<td>238</td>
<td>199</td>
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<td>0.08</td>
<td>0.10</td>
<td>0.052</td>
<td>0.064</td>
<td>0.037</td>
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</tbody>
</table>

TABLE V.
The same as Table IV., but for $d^I_t$.

<table>
<thead>
<tr>
<th></th>
<th>$A_1$</th>
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<th>$O_1$</th>
<th>$O'_1$</th>
<th>$O_2$</th>
<th>$O_4$</th>
<th>combined</th>
</tr>
</thead>
<tbody>
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<td>Tevatron</td>
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<td></td>
</tr>
<tr>
<td>$r$</td>
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<td>57.4</td>
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<td>169</td>
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<td>146</td>
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<td>$d^I_t$</td>
<td>5.4</td>
<td>4.4$^b$</td>
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<td>2.4</td>
<td>1.1</td>
<td>1.1</td>
<td>0.73</td>
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<tr>
<td>$r$</td>
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<td>−21.1</td>
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<td>332</td>
<td>332</td>
<td>248</td>
<td>282</td>
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<td>$d^I_t$</td>
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<td>5.8$^b$</td>
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<td>0.059</td>
<td>0.025</td>
<td>0.025</td>
<td>0.017</td>
</tr>
</tbody>
</table>

$^b$ Note that for $A_2$, only half of $N_{jl}$ can be used.

Acknowledgements

The author is grateful to O. Nachtmann and A. Brandenburg for helpful suggestions and discussions and to O. Nachtman for carefully reading the manuscript. The author also benefits from communications with W. Hollik. This work is financially supported by the AvH Foundation of Germany.
APPENDIX A: FORM FACTORS

We give here the non-zero form factors for the matrix elements appeared in the text. They are written in terms of the conventional one-, two-, three- and four-point scalar loop integrals defined in Ref. [38].

In the following, the form factors are given in the MSSM.

\[
D^s = \sum_{j=1,2} \left\{ F_1 C_{11}(-p_2, k, m_{\tilde{t}_j}, m_{\tilde{g}}, m_{\tilde{g}}) + F_2 (C_0 + C_{11})(-p_2, k, m_{\tilde{g}}, m_{\tilde{t}_j}, m_{\tilde{g}}) \right\} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i \lambda_{CP} \tag{A1}
\]

\[
f^{DB} = \sum_{j=1,2} \{ D_{13}(-p_2, p_4, m_{\tilde{t}_j}, m_{\tilde{g}}, m_{\tilde{g}}) \} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i \lambda_{CP} \tag{A2}
\]

\[
f^{s1}_1 = -2 D^s (p_1 \cdot p_4 - p_2 \cdot p_4) / \hat{s}, \tag{A3}
\]

\[
f^{s1}_8 = 4 D^s / \hat{s}, \tag{A4}
\]

\[
f^{s1}_9 = -4 D^s / \hat{s}. \tag{A5}
\]

\[
f^{s2}_1 = \sum_{j=1,2} \{ -B_0 \left( \hat{t}, m_{\tilde{g}}^2, m_{\tilde{t}_j}^2 \right) + B_0 \left( m_{\tilde{t}_j}^2, m_{\tilde{g}}^2, m_{\tilde{g}}^2 \right) \} \frac{(F_1 + F_2) \alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i \lambda_{CP} / (\hat{t} - m_{\tilde{t}_j}^2). \tag{A7}
\]

\[
f^{v1, t}_{16} = 2 D^v / (\hat{t} - m_{\tilde{t}}^2). \tag{A8}
\]

\[
f^{v2, t}_{2} = \sum_{j=1,2} \{ -C_0 (-p_1, -p_3, m_{\tilde{t}_j}, m_{\tilde{g}}, m_{\tilde{g}}) \} \frac{F_1 \alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i \lambda_{CP}, \tag{A9}
\]

\[
f^{v2, t}_{9} = 4 D^v / (\hat{t} - m_{\tilde{t}}^2), \tag{A10}
\]

\[
f^{v2, t}_{13} = 2 D^v / (\hat{t} - m_{\tilde{t}}^2), \tag{A11}
\]

where
\[ D' = \sum_{j=1,2} \left\{ F_1 C_{11}(-p_2, p_4, m_{i_j}, m_{\tilde{g}}, m_{\tilde{g}}) + F_2 (C_0 + C_{11})(-p_2, p_4, m_{\tilde{g}}, m_{i_j}, m_{i_j}) \right\} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} \] (A12)

\[ f_{1}^{\text{box1,t}} = \sum_{j=1,2} \left\{ -4D_{27} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_1 \right\} \] (A13)

\[ f_{2}^{\text{box1,t}} = \sum_{j=1,2} \left\{ m_{i_1}^2 (D_0 + 2D_{12} + 2D_{13} + D_{24} + D_{25}) - m_{\tilde{g}}^2 D_0 - 2p_2 \cdot p_4 (2D_{12} + D_{24} - D_{26}) - 2p_1 \cdot p_4 (D_{25} - D_{26}) + 4D_{27} \right\} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_1 \] (A14)

\[ f_{7}^{\text{box1,t}} = \sum_{j=1,2} \left\{ -4D_{26} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_1 \right\} \] (A15)

\[ f_{8}^{\text{box1,t}} = \sum_{j=1,2} \left\{ (4D_{25} - D_{26}) \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_1 \right\} \] (A16)

\[ f_{9}^{\text{box1,t}} = \sum_{j=1,2} \left\{ (4D_{12} + D_{24} - D_{26}) \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_1 \right\} \] (A17)

\[ f_{10}^{\text{box1,t}} = \sum_{j=1,2} \left\{ -4(D_{13} + D_{26}) \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_1 \right\} \] (A18)

\[ f_{13}^{\text{box1,t}} = \sum_{j=1,2} \left\{ 2D_{12} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_1 \right\} \] (A19)

\[ f_{14}^{\text{box1,t}} = \sum_{j=1,2} \left\{ -2D_{13} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_1 \right\} \] (A20)

\[ f_{15}^{\text{box1,t}} = \sum_{j=1,2} \left\{ -2D_{13} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_1 \right\} \] (A21)

\[ f_{16}^{\text{box1,t}} = \sum_{j=1,2} \left\{ 2D_{12} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_1 \right\} \] (A22)

The above \( D \) functions have the arguments \((-p_2, p_4, p_3, m_{i_j}, m_{\tilde{g}}, m_{\tilde{g}}, m_{\tilde{g}})\).

\[ f_{1}^{\text{box2,t}} = \sum_{j=1,2} \left\{ -4D_{27} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_2 \right\} \] (A23)

\[ f_{7}^{\text{box2,t}} = \sum_{j=1,2} \left\{ -4(D_{13} + D_{26}) \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_2 \right\} \] (A24)

\[ f_{8}^{\text{box2,t}} = \sum_{j=1,2} \left\{ 4(D_{25} - D_{26}) \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_2 \right\} \] (A25)

\[ f_{9}^{\text{box2,t}} = \sum_{j=1,2} \left\{ 4(D_0 + 2D_{12} + D_{24} - D_{26}) \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_2 \right\} \] (A26)

\[ f_{10}^{\text{box2,t}} = \sum_{j=1,2} \left\{ -4(D_{13} + D_{26}) \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i\lambda_{CP} F_2 \right\} \] (A27)

The above \( D \) functions have the arguments \((-p_2, p_4, p_3, m_{\tilde{g}}, m_{i_j}, m_{i_j}, m_{i_j})\).
In 2HDM, the form factors of Fig.2 (n) defined in Eq.(27) are

\[
\begin{align*}
    f_1^{box, t} & = \sum_{j=1,2} \left\{ -4D_{27} \right\} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i \lambda_{CP} F_1 F_2 \\
    f_7^{box, t} & = \sum_{j=1,2} \left\{ 4(-D_{23} + D_{26}) \right\} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i \lambda_{CP} F_1 F_2 \\
    f_8^{box, t} & = \sum_{j=1,2} \left\{ 4(-D_{25} + D_{25}) \right\} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i \lambda_{CP} F_1 F_2 \\
    f_9^{box, t} & = \sum_{j=1,2} \left\{ 4(D_{12} - D_{13} + D_{24} - D_{25}) \right\} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i \lambda_{CP} F_1 F_2 \\
    f_{10}^{box, t} & = \sum_{j=1,2} \left\{ -4(D_{11} - D_{12} + D_{21} - D_{24}) \right\} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i \lambda_{CP} F_1 F_2 \\
    f_{13}^{box, t} & = \sum_{j=1,2} \left\{ 2(D_{12} - D_{12}) \right\} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i \lambda_{CP} F_1 F_2 \\
    f_{15}^{box, t} & = \sum_{j=1,2} \left\{ -2(D_{11} - D_{12}) \right\} \frac{\alpha_s}{4\pi} m_{\tilde{g}} (-1)^{j+1} i \lambda_{CP} F_1 F_2
\end{align*}
\]

The above \( D \) functions have the arguments \((-p_2, p_4, -p_1, m_{\tilde{t}}, m_{\tilde{g}}, m_{\tilde{t}})\).

In the above, the color factors \( F_1 = \frac{3}{2}, \quad F_2 = -\frac{1}{6}, \) and \( \lambda_{CP} = 2Im(a_1 b_1^*) = \sin 2\theta_t \sin \beta_t \).

The form factors of 2HDM can be obtained from those of MSSM by setting \( j = 1, \quad F_1 = 1, \quad F_2 = 0, \quad f^{DB} = f_{a^2}^{s^2} = 0, \) and the following substitutions:

\[
\begin{align*}
    \lambda_{CP} & \rightarrow 2a\tilde{a} = -2\gamma_{CP}, \\
    m_{\tilde{g}} & \rightarrow m_t, \quad m_{\tilde{t}} \rightarrow m_{\tilde{t}} = \alpha_s \frac{\sqrt{2} m_t^2 G_F}{16\pi^2},
\end{align*}
\]

In 2HDM, the form factors of Fig.2 (n) defined in Eq.(27) are

\[
\begin{align*}
    f_1^{sr} & = \frac{\sqrt{2} m_t^2 G_F}{16\pi^2} \left\{ 4m_t [m_t^2 C_0 - p_3 \cdot p_4 (2C_{22} - 2C_{23} + C_0) + \frac{1}{2}] \right\}, \\
    f_2^{sr} & = \frac{\sqrt{2} m_t^2 G_F}{16\pi^2} \left\{ 4m_t [C_0 + 4(C_{22} - C_{23})] \right\}, \\
    f_3^{sr} & = \frac{\sqrt{2} m_t^2 G_F}{16\pi^2} \left\{ 4m_t C_0 \right\},
\end{align*}
\]

where the \( C \) function arguments are \((p_4, -k, m_t, m_t, m_t)\).

The constant model-independent form factors are:

\[
\begin{align*}
    D^a & = i\hat{d}_t, \\
    f_1^s & = -2i\hat{d}_t (p_1 \cdot p_4 - p_2 \cdot p_4) / \hat{s}, \\
    f_3^s & = 4i\hat{d}_t / \hat{s},
\end{align*}
\]
\[ f^{\ast}_0 = -4i\hat{d}_t / \hat{s} \tag{A44} \]
\[ f^{\ast\prime}_2 = 2i\hat{d}_t \tag{A45} \]
\[ f^{\ast\prime}_9 = 4i\hat{d}_t / (\hat{i} - m_i^2) \tag{A46} \]
\[ f^{\ast\prime}_{13} = 2i\hat{d}_t / (\hat{i} - m_i^2) \tag{A47} \]
\[ f^{\ast\prime}_{16} = 2i\hat{d}_t / (\hat{i} - m_i^2) \tag{A48} \]
\[ f^b_1 = -i\hat{d}_t \tag{A49} \]
\[ f^b_2 = i\hat{d}_t \tag{A50} \]

where \( \hat{d}_t \) is defined in the text.

Other definitions:
\[ k = p_1 + p_2 = p_3 + p_4, \quad \hat{s} = k^2, \quad \hat{i} = q^2 = (p_2 - p_4)^2, \quad \hat{u} = (p_2 - p_3)^2; \tag{A51} \]
\[ \Gamma^\mu = (-p_4 + p_3)\epsilon_3 \cdot \epsilon_4 + (2p_4 + p_3) \cdot \epsilon_3 \epsilon_4^\mu - (2p_3 + p_4) \cdot \epsilon_4 \epsilon_3^\mu. \]

**APPENDIX B: SOLUTION OF THE NEUTRINO MOMENTA**

We give here briefly the method of solving the neutrino momenta in Eqs.(45). Let
\[ p_{1+} = p_1, \quad p_{1'} = p_1', \quad p_6 = p'_6, \quad p_{\nu_1} = (E, X, Y, Z), \quad p_{\nu_2} = (E', X', Y', Z'), \quad p_{\nu_b} = p_t + p'_b, \quad p'_{\nu_b} = p'_t + p'_b, \quad p_0 = -(p_b + p'_b + p_t + p'_t). \]
Then we can get
\[ X = aE + bZ + \delta, \quad Y = cE + dZ + \xi, \tag{B1} \]
\[ X' = a'E' + b'Z' + \delta', \quad Y' = c'E' + d'Z' + \xi', \]
where
\[ a = \frac{E_{0b}p^y_1 - E_{1b}p^y_1}{\Delta_1}, \quad b = -\frac{p^y_1p^y_1 + p^y_1p^y_1}{\Delta_1}, \tag{B2} \]
\[ c = -\frac{E_{0b}p^y_1 + E_{1b}p^y_1}{\Delta_1}, \quad d = \frac{p^y_1p^y_1 - p^y_1p^y_1}{\Delta_1}, \]
\[ \delta = \frac{- (m_i^2 - p_{0b}^2) / 2p_{0b}^y + m_i^2 / 2p_{0b}^y}{\Delta_1}, \quad \xi = \frac{(m_i^2 - p_{0b}^2) / 2p_{0b}^y - m_i^2 / 2p_{0b}^y}{\Delta_1}, \]
\[ \Delta_1 = p^x_{0b}p^x_1 - p^x_0p^x_1, \]
and \( a', b', c', d', \delta', \xi' \) are obtained from the above equations by the substitutions of momenta without \( ' \) to those with \( ' \). We further express \( E, E' \) in terms of \( Z, Z' \):
\[ E = fZ + gZ' + h, \quad E' = f'Z + g'Z' + h', \tag{B3} \]
where
\[ f = \frac{-bc' + da'}{\Delta_2}, \quad g = \frac{-b'c' + d'a'}{\Delta_2}, \quad h = \frac{(p_0^b - \delta - \delta')c' + a'(p_0^g - \xi - \xi')}{\Delta_2}, \quad (B4) \]
\[ f' = \frac{-ad + cb}{\Delta_2}, \quad g' = \frac{-ad' + cb'}{\Delta_2}, \quad h' = \frac{(p_0^b - \xi - \xi')a + c(p_0^g - \delta - \delta')}{\Delta_2}. \]

\[ \Delta_2 = ac' - ca'. \]

Inserting the above expressions into \( E^2 = X^2 + Y^2 + Z^2, \) and \( E'^2 = X'^2 + Y'^2 + Z'^2, \) we get the following two quadratic equations for \( Z, Z': \)

\[ AZ^2 + BZZ' + CZ'^2 + D + UZ + VZ' = 0 , \quad (B5) \]
\[ A'Z'^2 + B'ZZ' + C'Z^2 + D' + U'Z' + V'Z = 0 , \quad (B6) \]

with

\[ A = f^2 - 1 - (af + b)^2 - (cf + d)^2, \quad B = 2[fg - ag(af + b) - cg(cf + d)] , \quad (B7) \]
\[ C = g^2 - a^2g^2 - c^2g^2 , \quad D = h^2 - (ah + \delta)^2 - (ch + \xi)^2 , \]
\[ U = 2[fh - (ah + \delta)(af + b) - (ch + \xi)(cf + d)] , \]
\[ V = 2[gh - ag(ah + \delta) - cg(ch + \xi)] , \]

and \( A', B', C', D', U', V' \) can be gotten from the above equations by replacing the variables without \( ' \) with the ones with \( ' \) and interchanging \( f', g' \).

Eqs.(B5)(B6) can obviously give at most four real solutions for \((Z, Z')\). They can be solved by standard methods: solving \( Z' \) in (B5) as functions of \( Z \), we get two expressions for \( Z' \), then inserting them separately into (B6), we shall get the same quartic equation of \( Z \) which has up to four real solutions. Although those solutions will be doubled when we calculate \( Z' \) with the two expressions obtained from (B5), half of them are false solutions to (B6).

**APPENDIX C: ANALYSIS ON THE CONTAMINATIONS TO CP VIOLATING EFFECTS IN \( PP \) COLLISION**

We assume the partons inside the proton have no intrinsic transverse momenta and denote the parton level reaction as \( a(x_1)\bar{a}(x_2) \rightarrow t\bar{t} \) with \( x_1 \) and \( x_2 \) being the Bjorken scale parameters. The hadronic level cross section can be written as

\[ d\sigma = 2f_p^a(x_1)f_p^{\bar{a}}(x_2)d\hat{\sigma}(a\bar{a} \rightarrow t\bar{t})dx_1dx_2/(1 + \delta_{a,\bar{a}}) \]
\[ = \{[f_p^a(x_1)f_p^{\bar{a}}(x_2) + f_p^a(x_2)f_p^{\bar{a}}(x_1)] + [f_p^{\bar{a}}(x_1)f_p^a(x_2) - f_p^a(x_2)f_p^{\bar{a}}(x_1)]\} \]
\[ \times d\hat{\sigma}(a\bar{a} \rightarrow t\bar{t})/(1 + \delta_{a,\bar{a}})dx_1dx_2 , \]

23
where \( f_p^a, f_p^{\bar{a}} \) are the parton distribution functions of \( a \) and \( \bar{a} \) in proton. The function 
\[
F^+(x_1, x_2) = f_p^a(x_1)f_p^{\bar{a}}(x_2) + f_p^{\bar{a}}(x_2)f_p^a(x_1)
\]
is CP-even (its CP transformation is just the interchanging of \( x_1, x_2 \)), so that no contamination comes from it. Now we look at the function 
\[
F^-(x_1, x_2) = f_p^a(x_1)f_p^{\bar{a}}(x_2) - f_p^{\bar{a}}(x_2)f_p^a(x_1).
\]
Contaminations should come from \( F^-(x_1, x_2) \) term since it is CP-odd. 
\[
F^-(x_1, x_2) = 0 \quad \text{for} \quad a = g.
\]
There are no contaminations from initial gluon reaction. All contaminations come from the initial quark reactions which are subdominant processes at the LHC. There are non-zero CP violating contaminations only when the observables contain asymmetry between \( x_1 \) and \( x_2 \). Since the Lorentz invariant observables can always be calculated in the center of mass frame of the parton which depending only on \( x_1x_2 \), they will receive no contaminations. Therefore, within the framework of parton model, there are no contaminations to Lorentz invariant observables. In our study, they are \( f_2, O_1, O'_1, O_2, O_4 \).
REFERENCES


Figure Captions

Fig.1 Feynman diagrams of tree level and 1-loop SUSY QCD corrections of CP violation to $q\bar{q} \to t\bar{t}$

Fig.2 Feynman diagrams of tree level and 1-loop SUSY QCD corrections of CP violation to $gg \to t\bar{t}$

Fig.3 Feynman diagrams of top quark CEDM corrections of CP violation to $q\bar{q} \to t\bar{t}$, $gg \to t\bar{t}$