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YANG-MILLS THEORY IN A NON-LINEAR GAUGE

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Abstract The Coulomb gauge is first shown to be an incomplete gauge-fixing condition for non-Abelian theories. A non-linear gauge condition, which is quadratic in A_μ and includes the Coulomb gauge is presented and its consequences explored. The effective dynamics of the new degrees of freedom strongly hint of non-perturbative effects. Finally, we show that the non-linear gauge fixing condition can no longer be generalized to include higher derivatives and higher order in A .

I. INTRODUCTION

The man we are honoring in this workshop is a friend of the Filipino physics community. In his visits to the country he has not only talked about his latest research but also about physics education and the experience of Japan in building its scientific tradition. Thus, when Chris Bernido invited me to join the workshop and present a talk, I readily agreed. I hope the ideas I present here will measure up to the prestige of our honoree, Professor Hiroshi Ezawa.

In quantizing theories with gauge symmetries, it is important that every gauge-equivalent fields are counted once and only once. It is also equally important that each orbit of gauge-equivalent fields is represented in the count for otherwise some important field configurations may be left out in the path-integral and physical effects associated with these fields brushed aside. This means that when we specify how to do the counting, i.e., when we specify the gauge condition, it better be globally unique and realizable. Uniqueness of the gauge condition implies that the orbit intersects the gauge-fixing surface only once. Realizability means that starting from a field configuration not on the gauge-fixing surface, we can follow the orbit until it reaches the surface.

Up to this time, only linear gauge conditions have been used to remove the excess degrees of freedom. For the Abelian case, linear gauge conditions are always realizable because gauge transformation is only a "translation". Also, the incomplete removal of all the gauge degrees of freedom, like in the axial and Lorentz gauges, can easily be compensated for by imposing subsidiary conditions.

The ideal gauge-fixing condition for the Abelian theory is the Coulomb gauge because it is unique and always realizable. The physical degrees of freedom of the electromagnetic field, the transverse photon, is also transparent in this gauge. Because of this it was naturally assumed that the same thing is true for the non-Abelian case, until Gribov pointed out in 1977⁽¹⁾ that gauge copies exist for certain field configurations satisfying the Coulomb gauge. This means that the gauge condition is not unique.

Since Gribov's pioneering paper, various aspects of the Coulomb gauge formulation of non-abelian gauge theories have been discussed. Peccei⁽²⁾ had shown that the existence of copies does not affect the perturbative calculations. Fujikawa,⁽³⁾ Hirschfield⁽⁴⁾ showed the possibility that the Gribov copies may not necessarily affect the functional integral. Bender, et. al.⁽⁵⁾ looked for confinement by investigating the effect of the Gribov phenomena on the continuum Hamiltonian. Cutkosky⁽⁶⁾ had a similar idea and searched for a mass gap in the Hamiltonian using the lattice approach.

On the more technical aspect of the Gribov problem, Maskawa and Nakajima⁽⁷⁾ made a geometrical formulation of the Coulomb gauge by considering the functional space of L^2 fields and showed that the Coulomb gauge follows from minimizing $|A^\Omega|^2 = \int d^3x [\Omega \bar{A} \Omega^{-1} + (\bar{\partial} \Omega) \Omega^{-1}]^2$. Zwanziger and Dell Antonio⁽⁸⁾

(i) verified the ideas of Semionov-Tian-Shansky and Franke⁽⁹⁾ that there are Gribov copies inside the horizon, (ii) claimed that all gauge orbits pass through the central Gribov region, and (iii) proved that the central Gribov region has ellipsoidal bounds and forms a convex set.

In a 1994 paper,⁽¹⁰⁾ Magpantay looked at the Coulomb gauge again and showed that there is no need to modify the Fadeev-Popov determinant $\det(\bar{\partial} \cdot \bar{D})$ even in cases where gauge copies exist. More precisely, the Fadeev-Popov determinant was shown to be exactly zero on the horizon, thus effectively showing that the horizon does not contribute to the path-integral.

In the same paper, it was also shown that there are field configurations that cannot be gauge transformed to the Coulomb gauge, thus disproving the claim of Zwanziger and Dell-Antonio that all orbits pass inside the horizon. These are the field configurations that satisfy

$$(\bar{\partial} \cdot \bar{D})(\bar{\partial} \cdot \bar{A}) = (\bar{D} \cdot \bar{\partial})(\bar{\partial} \cdot \bar{A}) = 0, \quad (1a)$$

$$\bar{\partial} \cdot \bar{A} = f(\bar{x}) \neq 0. \quad (1b)$$

An explicit example of a class of field configurations that satisfy (1;a,b) was constructed by Magpantay and Cuansing⁽¹¹⁾. These field configurations are shown to be not transformable to the Coulomb gauge if we require the square-integrability (L^2) of the vector fields.

The above results show that non-perturbative physics, like confinement, is not connected with the Gribov horizon. Furthermore, these results suggest that the Coulomb gauge has to be generalized and that the appropriate generalization is defined by equation (1a). This gauge condition has two parts – the usual Coulomb gauge $\vec{\partial} \cdot \vec{A} = 0$ and a quadratic “surface” in configuration space.

This paper presents some new results in the use of the non-linear gauge condition in SU(2). In particular, we will derive the independent degrees of freedom and their effective dynamics. We will see that the effective action of the independent degrees of freedom clearly hints of non-perturbative effects. But before we present these results, we discuss first in Section II some heuristic arguments for the non-linear gauge condition. Section III will show in a very simple way that field configurations on the Coulomb surface and those that satisfy equations (1;a,b) do not mix, i.e., they are not connected by a gauge transformation. Section IV differentiates the Hamiltonian formulations of the Coulomb gauge and the non-linear gauge. The new degrees of freedom and their effective dynamics is the subject of Section V. Why the non-linear gauge condition can no longer be extended to gauge-fixing with higher derivatives and higher order than quadratic in A_μ is the topic of Section VI. The conclusion summarizes what had been accomplished and presents a conjecture.

II. HEURISTIC ARGUMENTS

The non-linear gauge-fixing condition can be written as

$$\nabla^2(\vec{\partial} \cdot \vec{A}^a) - g \epsilon^{abc} (\vec{A}^c \cdot \vec{\partial})(\vec{\partial} \cdot \vec{A}^b) = 0 \quad (2)$$

In the Abelian limit, this gives $\nabla^2(\vec{\partial} \cdot \vec{A}) = 0$. Since ∇^2 is a positive definite operator (for simply connected spaces), we get back the Coulomb gauge.

In small distance scales, the coupling constant goes to zero and we can effectively neglect the second term again giving back the Coulomb gauge. This is the situation well within the hadrons where the quarks are loosely bound interacting with transverse vector fields.

Since the first term is proportional to p^3 (p is the momenta) and the running coupling becomes stronger at small momenta (note the second term is proportional to $p^2 g(p)$) then even before we get to the distance scales where the perturbative running coupling is no longer valid, we see that the second term becomes important. Because of this the “gluon” is no longer sufficiently described by transverse degrees of freedom. And indeed in non-Abelian theory, because of the

gluon self-interaction, the transverse degrees of freedom are not physical in large distance scales.

In this sense, the Coulomb gauge is not an appropriate gauge-fixing condition for Yang-Mills theories. The gauge-fixing condition defined by equation (2) is intuitively better because it is consistent with the asymptotic freedom and confinement phases of unbroken gauge theories.

Furthermore, as we will show in the next section, the non-Coulomb configurations of (2) cannot be gauge transformed to the Coulomb surface. And as will be shown also, these field configurations are horizons on surfaces defined by $\vec{\partial} \cdot \vec{A}^a = f^a$ (f^a defines the arbitrary surface) and that fields on the horizon belong to an orbit.

III. NO MIXING OF FIELD CONFIGURATIONS

In this section, we will supplement the results of references (10) and (11) by providing a simple proof, albeit using infinitesimal transformations, that field configurations on the Coulomb surface and those on the surface defined by (1;a,b) cannot be gauge transformed to each other.

Consider a vector field $\vec{A}$ that satisfies equations (1;a,b). Is there a gauge transformation that will take this field configuration to the Coulomb surface? Let

$$\vec{A}' = \vec{A} + \vec{D} \Lambda. \quad (3)$$

Imposing $\vec{A}'$ to be on the Coulomb surface yields

$$(\vec{\partial} \cdot \vec{D}) \Lambda = -(\vec{\partial} \cdot \vec{A}). \quad (4)$$

But since the zero mode of $(\vec{\partial} \cdot \vec{D})$ is also $(\vec{\partial} \cdot \vec{A})$, then equation (4) will not have an L^2 solution. Thus, there is no gauge transformation that will take field configurations satisfying the non-linear condition to the Coulomb surface.

Conversely, suppose $\vec{\partial} \cdot \vec{A} = 0$. Is there an $\vec{A}' = \vec{A} + \vec{D} \Lambda$ such that $\vec{A}'$ satisfies equations (1;a,b)? This means Λ must satisfy

$$(\vec{\partial} \cdot \vec{D}) \Lambda = f(\vec{x}) \neq 0, \quad (5a)$$

$$(\vec{\partial} \cdot \vec{D})(\vec{\partial} \cdot \vec{D}) \Lambda = 0. \quad (5b)$$

Equation (5b) says that $\vec{\partial} \cdot \vec{D}$ is a singular operator with $(\vec{\partial} \cdot \vec{D}) \Lambda = f(\vec{x})$ as its zero mode. To solve for Λ in equation (5a), for an L^2 solution to exist, $f(\vec{x})$ must be orthogonal to the zero mode of $(\vec{\partial} \cdot \vec{D})$. But the zero mode is also the source,

thus no solution exists. This proves that field configurations on the Coulomb surface cannot be gauge transformed to configurations that satisfy equations (1;a,b).

Let us now establish two major points about the non-linear gauge condition. First the field configurations satisfying equations (1;a,b) are actually on the horizon of the surface $\vec{\partial} \cdot \vec{A}^a = f^a(\vec{x}) \neq 0$ because given a certain $f^a(\vec{x})$, the condition $(\vec{\partial} \cdot \vec{D})f(\vec{x}) = 0$ says that the operator $(\vec{\partial} \cdot \vec{D})$ is singular. Since the Fadeev-Popov operator is linear in $\vec{A}$ then the horizon on the surface $\vec{\partial} \cdot \vec{A} = f(\vec{x})$ defined by $(\vec{\partial} \cdot \vec{D})f = 0$ is also convex just like the horizon on the Coulomb surface.

Second, since the field configurations on the surface defined by equations (1;a,b) are not gauge transformable to the Coulomb gauge, it is natural to ask where the orbits go? The answer is that the orbits of these field configurations stay on the surface and on the horizon.

Proof of this is as follows. Let $\vec{A}$ satisfy equations (1;a,b). Let $\vec{A}' = \vec{A} + \vec{D}\Lambda$. Imposing that $\vec{A}'$ is also on the same surface as $\vec{A}$, i.e. $\vec{\partial} \cdot \vec{A}' = f(\vec{x})$ then $(\vec{\partial} \cdot \vec{D})\Lambda = 0$. This means $\Lambda = c(\vec{\partial} \cdot \vec{A})$ where c is an infinitesimal constant (dimension length squared so that Λ is dimensionless). It also follows that $(\vec{\partial} \cdot \vec{D}(\vec{A}'))f = 0$, thus $\vec{A}'$ is not only on the same surface but also on the horizon. We can repeat the same procedure for $\vec{A}'$ and $\vec{A}''$ and so on with each step resulting in a gauge transformed field that is also on the surface and on the horizon. This complete the proof that the orbit of field configurations that satisfy equation (1;a,b) stay on the surface and on the horizon.

Another way to prove the second point is by explicitly verifying that any gauge transformation that attempts to get off the surface does not have an L^2 solution. Consider the field configuration (used in reference (11))

$$A_i^a = \delta_i^a k(r) + \frac{x^a x_i}{r^2} \ell(r), \quad (6a)$$

$$\partial_i A_i^a = \frac{x^a}{r} \left(\frac{dk}{dr} + \frac{d\ell}{dr} + \frac{2}{r} \ell \right) = \frac{x^a}{r} s(r) \neq 0. \quad (6b)$$

Consider the gauge transformed field $\vec{A}' = \vec{A} + \vec{D}\Lambda$, such that $\vec{\partial} \cdot \vec{A}'^a = \frac{x^a}{r} (s(r) + \epsilon(r))$ where $\epsilon(r) \ll s(r)$, i.e., the $\vec{\partial} \cdot \vec{A}'$ surface is infinitesimally close to the $\vec{\partial} \cdot \vec{A} = \frac{x^a}{r} s(r)$ surface. For a non-trivial Λ to exist, the proper treatment of the zero mode requires

$$\int_0^R dr r^2 s(r) \in(r) = 0 \quad (7)$$

We regularized the system by considering finite space of large radius R . Since $s(r) = \frac{a}{r^2} + br$ and the constants a and b have a specified R behaviour (for L^2 behaviour), then $r^2 s(r)$ is a rapidly increasing function. For equation (7) to be satisfied, $\in(r)$ must be fast decreasing and oscillatory. However, from the results of reference (11), it follows that the condition $\in(r) \ll s(r)$ cannot be satisfied for all r , thus, the only admissible $\in(r)$ is $\in(r) = 0$. This proves that the orbit cannot leave the surface and the horizon.

IV. THE HAMILTONIAN FORMULATION

The Yang-Mills Hamiltonian, whether in the Coulomb gauge or in the non-linear gauge defined by equation (1;a,b), is given by

$$H = \int d^3x \left\{ \frac{1}{2} \vec{E}^a \cdot \vec{E}^a + \frac{1}{4} F_{ij}^a F_{ij}^a + \frac{1}{2} (\partial_i \alpha^a)^2 \right\}, \quad (8)$$

where $\vec{E}^a = \partial_o \vec{A}^a$. The instantaneous Coulomb term, in both gauges, is given by

$$(\vec{D} \cdot \vec{\partial})^{ab} \alpha^b = \vec{D}^{ab} \cdot \vec{E}^b. \quad (9)$$

The differences between the two gauge-fixing conditions are summarized below:

<u>Coulomb Gauge</u>	<u>Non-Linear Gauge</u>
$\vec{\partial} \cdot \vec{A}^a = 0$	$(\vec{\partial} \cdot \vec{D})^{ab} (\vec{\partial} \cdot \vec{A}^b) = (\vec{D} \cdot \vec{\partial})^{ab} (\vec{\partial} \cdot \vec{A}^b) = 0$
$\vec{\partial} \cdot \vec{E}^a = 0$	$(\vec{\partial} \cdot \vec{D})^{ab} (\vec{\partial} \cdot \vec{E}^b) - \epsilon^{abc} \vec{\partial} \cdot [(\vec{\partial} \cdot \vec{A}^b) \vec{E}^c] = 0.$

In the Coulomb gauge, for as long as the Fadeev-Popov operator $(\vec{D} \cdot \vec{\partial})$ is non-singular, equation (9) can be solved for the instantaneous Coulomb term α^a . The solution can be expanded using the $\frac{1}{r}$ potential.

However, if $(\vec{D} \cdot \vec{\partial})$ is singular, then a non-trivial solution for α^a only exists if the zero mode of $(\vec{D} \cdot \vec{\partial})$ is orthogonal to $\epsilon^{abc} \vec{A}^c \cdot \vec{E}^b$. If this condition is

imposed, Peccei (see reference 2) showed that the instantaneous Coulomb term, when computed perturbatively, is effectively unchanged. Thus, confinement remains unproven, which agrees with the computation of Greensite.⁽¹²⁾

If we use instead the non-linear gauge, we find that we have a totally different situation. Acting the operator $(\vec{\partial} \cdot \vec{D})$ on equation (9) gives

$$(\vec{\partial} \cdot \vec{D})^{ab} (\vec{D} \cdot \vec{\partial})^{bc} \alpha^c = (\vec{\partial} \cdot \vec{D})^{ab} (\vec{D}^{bc} \cdot \vec{E}^c) \quad (10)$$

Since $(\vec{\partial} \cdot \vec{D})^{ab} (\vec{D} \cdot \vec{\partial})^{bc}$ is singular with $\vec{\partial} \cdot \vec{A}^a$ as the zero mode, for equation (10) to have a non-trivial solution, we must have

$$\int d^3x (\vec{\partial} \cdot \vec{A}^a) (\vec{\partial} \cdot \vec{D})^{ab} (\vec{D}^{bc} \cdot \vec{E}^c) = 0 \quad (11)$$

and indeed this is true because an integration by parts will yield the gauge-fixing condition. Thus unlike in the Coulomb gauge, the consistency condition is trivially satisfied in the non-linear gauge.

Equation (10) can be solved perturbatively using the Greens function for the operator $(\nabla^2)^2$ which is a linear potential. If we consider static sources given by j_o^a , equation (10) will be augmented by the term $(\vec{\partial} \cdot \vec{D})^{ab} j_o^b$ (the modification of equation (11) still holds). Perturbative expansion of the “Coulomb” term will yield the usual Coulomb term $\int d^3x d^3y j_o^a(\vec{x}) \frac{1}{|\vec{x} - \vec{y}|} j_o^a(\vec{y})$.

However, higher order effects will yield “confining” terms because of the linear potential. The lowest order “confining” potential is given by $\int d^3x d^3y \epsilon^{abc} j_o^a(\vec{x}) |\vec{x} - \vec{y}| \cdot (\vec{\partial} \cdot \vec{A}^b)_y j_o^c(\vec{y})$. Higher orders terms are even more confining.

Finally, we point out the difference between the effective potentials (static fields) in the Coulomb and non-linear gauge.

$$V_{eff} = \int d^3x \left\{ \frac{1}{2} (B_i^a)^2 + \frac{1}{2} (\partial_i \alpha_a)^2 \right\} + tr \ln \left\{ \frac{(\vec{\partial} \cdot \vec{D})}{\theta} \right\} \quad (12)$$

From equation (9), the Coulomb term is given by $(\vec{D} \cdot \vec{\partial})^{ab} \alpha^b = 0$. The first term is the classical contribution while the second term is the quantum effect. The $tr \ln(\vec{\partial} \cdot \vec{D})$ is for the Coulomb gauge while $tr \ln \theta$, where θ is the “Fadeev-Popov” given by equation (13), is for the non-linear gauge,

$$\theta^{ad} = (\vec{D} \cdot \vec{\partial})^{ab} (\vec{\partial} \cdot \vec{D})^{bd} - \epsilon^{abc} [\vec{\partial}(\vec{\partial} \cdot \vec{A}^b)] \cdot \vec{D}^{cd}. \quad (13)$$

This operator is non-singular in spite of the fact that $(\vec{\partial} \cdot \vec{D})$ is singular. The proof follows simply from first-order perturbation theory.

In the Coulomb gauge, if $(\vec{D} \cdot \vec{\partial})$ is non-singular, $\alpha^a = 0$ and the classical effective potential has minimum at $\vec{B}^a = 0$. The quantum correction the $\text{tr} \ln (\vec{\partial} \cdot \vec{D})$ leads to a vacuum with non-trivial field strength. However the imaginary part of the quantum effective potential makes the non-trivial vacuum unstable and it eventually decays.⁽¹³⁾

If $(\vec{D} \cdot \vec{\partial})$ is singular, (which is valid for fields on the horizon), the instantaneous Coulomb contribution is non-trivial. However, the $\exp[\text{tr} \ln (\vec{\partial} \cdot \vec{D})] = \det(\vec{\partial} \cdot \vec{D})$ is zero. Thus although the classical effective action may have a non-trivial minimum, the contribution of this field configurations to the path-integral is effectively zero because of the determinant.

On the other hand, for the non-linear gauge, both the “instantaneous Coulomb” and the quantum corrections are ~~both~~ non-trivial. Firstly, $\alpha^a = \ell_o (\vec{\partial} \cdot \vec{A}^a)$ where ℓ_o is a length scale. Secondly, since θ is non-singular, $\det \theta$ is non-zero and the contribution of these field configurations to the path-integral ~~do~~^{es} not vanish. Thus, the effective potential in the non-linear gauge will give a non-trivial vacuum.

V. THE NEW DEGREES OF FREEDOM AND THEIR EFFECTIVE DYNAMICS

In this section, we will shift to 4-D Euclidean space-time and consider the covariant version of equations (1;a,b). Equation (1;a,b) is solved by the following:

$$\ell_o^2 D_\mu^{ab} f^b = (A_\mu^a - \partial_\mu \frac{1}{\square^2} f^a) - t_\mu^a, \quad (14)$$

where the length scale ℓ_o is introduced for dimensional reasons. The new vector field t_μ^a is transverse and must also satisfy another condition (to have the same number of degrees of freedom).

Solving equation (14) for A_μ^a we find

$$A_\mu^a = \frac{1}{(1 + g\ell_o^2 \vec{f} \cdot \vec{f})} \left\{ \delta^{ab} + g\ell_o^2 \epsilon^{abc} f^c + g^2 \ell_o^4 f^a f^b \right\} \\ \times (\ell_o^2 \partial_\mu f^b + \partial_\mu \frac{1}{\square^2} f^b + t_\mu^b). \quad (15)$$

Rescaling the fields, $g\ell_o^2 f^a \rightarrow f^a$, so that the new field f^a is dimensionless, we have

$$A_\mu^a = \frac{1}{(1 + \vec{f} \cdot \vec{f})} \left\{ \delta^{ab} + \epsilon^{abc} f^c + f^a f^b \right\} \left(\frac{1}{g} \partial_\mu f^b + \frac{1}{g\ell_o^2} \partial_\mu \frac{1}{\square^2} f^b + t_\mu^b \right) \quad (15')$$

To guarantee that $\partial_\mu A_\mu^a = f^a$ we must impose

$$\rho^a = \left[(\vec{f} \cdot \vec{f}) \epsilon^{abc} \partial_\mu f^c + \epsilon^{adc} f^b f^c \partial_\mu f^d - \epsilon^{abc} f^c f^d \partial_\mu f^d \right. \\ \left. + f^a \epsilon^{bcd} f^c \partial_\mu f^d \right] \left(\partial_\mu \frac{1}{\square^2} f^b + t_\mu^b \right) = 0. \quad (16)$$

What we have done is to change variables from A^a which satisfies the non-linear gauge condition, to f^a and t_μ^a . The t_μ^a is not only transverse but also satisfies equation (16). Thus, the number of degrees of freedom remain^s the same.

In terms of the new degrees of freedom, the path-integral is:

$$PI = \int (df^a) (dt_\mu^a) \delta(\partial \cdot t^a) \delta(\rho^a) \det \theta \det(1 + \vec{f} \cdot \vec{f}) e^{-S_{YM}}, \quad (17)$$

where equation (15') is substituted in θ and S_{YM} . Note that the field strength can be expanded as

$$F_{\mu\nu}^a = \frac{1}{g} Z_{\mu\nu}^a(f) + L_{\mu\nu}^a(f; t) + g Q_{\mu\nu}^a(f; t) \quad (18)$$

where $Z_{\mu\nu}^a$ is zeroth order in t , $L_{\mu\nu}^a$ is linear in t and $Q_{\mu\nu}^a$ is quadratic in t . The action becomes

$$\begin{aligned}
S_{YM} \sim \int d^4x \left\{ \frac{1}{g^2} Z_{\mu\nu}^a Z_{\mu\nu}^a(f) + \frac{2}{g} Z_{\mu\nu}^a(f) L_{\mu\nu}^a(f;t) \right. \\
+ \left[2 Z_{\mu\nu}^a(f) Q_{\mu\nu}^a(f;t) + L_{\mu\nu}^a(f;t) L_{\mu\nu}^a(f;t) \right] \\
\left. + 2g L_{\mu\nu}^a(f;t) Q_{\mu\nu}^a(f;t) + g^2 Q_{\mu\nu}^a(f;t) Q_{\mu\nu}^a(f;t) \right\}. \quad (19)
\end{aligned}$$

The effective dynamics of the new degrees of freedom are read from equation (19). We notice right away the following:

- The pure f term has a typical non-perturbative behaviour.
- The f dynamics is extremely non-linear because of the $(1 + \vec{f} \cdot \vec{f})^{-1}$ term. Furthermore, the f term has four-derivatives and also non-local.
- The vector field t_μ^a is not only transverse but is also constrained to satisfy equation (16). Also, the vector field is not gauge-invariant.
- Fermions interact in a very non-linear way with the f field as given by $g \bar{\psi} \gamma_\mu T^a A_\mu^a \psi$ with A_μ^a given by equation (15').

Because of these observations, we conjecture that confinement of quarks and the transverse gluons (t_μ^a) is due to the f dynamics.

VI. GENERALIZATION TO HIGHER DERIVATIVES AND NON-LINEAR GAUGES

In section IV, we found that the expansion Greens function used in solving the “instantaneous Coulomb term” is the linear potential. This came about because the gauge-fixing yielded a “Fadeev-Popov” operator that is of order $(\nabla^2)^2$. Thus, the “hint” of confinement seems to be an artifact of gauge-fixing. Consequently, if we include more derivatives in the gauge-fixing, then we should get a more confining potential.

The above arguments are specious for two reasons. First, the arguments in references (10), (11) and Sections II and III of this paper clearly showed the Coulomb gauge is insufficient because the horizons on each of the arbitrary surfaces $\vec{\partial} \cdot \vec{A}^a = f^a(\vec{x})$ are completely ignored. Thus, physical affects associated with this ignored field configurations will be missed completely. And as we have shown in this paper, the physical effects of the missed configurations are primarily non-perturbative.

Secondly, we will now show that the non-linear gauge-fixing condition cannot be generalized to include higher derivatives and higher order non-linear terms. Note that the generalization we will be seeking must contain the Coulomb gauge. Let us consider the following gauge-fixing (note: $\vec{D} \cdot \vec{A} = \vec{\partial} \cdot \vec{A}$)

$$(\vec{\partial} \cdot \vec{D})^2 (\vec{\partial} \cdot \vec{A}) = 0 \quad (20)$$

This gauge-fixing has three regimes:

- (a) $\vec{\partial} \cdot \vec{A} = 0$; the Coulomb surface
- (b) $\vec{\partial} \cdot \vec{A} = f(\vec{x})$; $(\vec{\partial} \cdot \vec{D})f = 0$; the non-linear gauge considered in this paper.
- (c) $\vec{\partial} \cdot \vec{A} = f(\vec{x})$; $(\vec{\partial} \cdot \vec{D})f = k(\vec{x}) \neq 0$; $(\vec{\partial} \cdot \vec{D})k(\vec{x}) = 0$; the new regime.

We will now show that the regime defined by (c) is inconsistent. To express $\vec{A}$ in terms of $k(\vec{x})$ and $f(\vec{x})$ (to derive the analogue of equation (15')), we first have to solve the equation $(\vec{\partial} \cdot \vec{D})f = k(\vec{x})$. But since $k(\vec{x})$ is the zero mode of $(\vec{\partial} \cdot \vec{D})$, we cannot find an (L^2) f because the zero mode is the source. Thus, the gauge-fixing defined by equation (20) is not consistent. And in a similar way, we can argue the inconsistency of all gauge conditions of the form $(\vec{\partial} \cdot \vec{D})^n (\vec{\partial} \cdot \vec{A}) = 0$, for $n > 3$.

The other types of gauge-fixing conditions that we can consider are

$$(\nabla^2)^n (\vec{\partial} \cdot \vec{A}) = 0, \quad (21a)$$

$$(\vec{D}^+ \cdot \vec{D})^n (\vec{\partial} \cdot \vec{A}) = 0, \quad (21b)$$

and other combinations of ∇^2 , $(\vec{D}^+ \cdot \vec{D})$, and $(\vec{\partial} \cdot \vec{D})$ acting on $\vec{\partial} \cdot \vec{A}$. Equation (21a) is simply equivalent to the Coulomb gauge because the ∇^2 is a positive-definite operator. The same thing with $\vec{D}^+ \cdot \vec{D}$ because it is equal to $U^+ \nabla^2 U$ where $U = P \exp \left\{ i \int_{-\infty}^{\vec{x}} \vec{A} \cdot d\vec{z} \right\}$. Thus, generalizations which combine ∇^2 , $(\vec{D}^+ \cdot \vec{D})$ and $\vec{\partial} \cdot \vec{D}$ can either be reduced to the Coulomb gauge, the non-linear gauge or are inconsistent.

CONCLUSION

This paper had argued the importance of generalizing the Coulomb gauge to the non-linear gauge defined by equation (1;a,b). We have also shown that the gauge-fixing can no longer be generalized any further. And best of all, we had shown that non-perturbative physics is found in the non-linear regime of the non-linear gauge.

To conclude, we will speculate on an elegant proof of confinement arising from the non-linear regime. Consider the pure f term of equations (18) and (19). The relevant path-integral is

$$\int (df^a) \det(1 + \vec{f} \cdot \vec{f}) \det \tilde{\theta} \exp \left\{ -\frac{1}{g^2} \int d^4x Z_{\mu\nu}^a Z_{\mu\nu}^a \right\}, \quad (22a)$$

where

$$Z_{\mu\nu}^a = F_{\mu\nu}^a (A_\mu^a = \text{given by (15')} \text{ with } t_\mu^a = 0), \quad (22b)$$

$$\tilde{\theta} = \theta (A_\mu^a = \text{given by (15')} \text{ with } t_\mu^a = 0). \quad (22c)$$

Suppose we can transform from $f^a \rightarrow \phi^a = \phi^a(f)$, which may be a non-local transformation, such that

$$\frac{1}{4} (Z_{\mu\nu}^a)^2 = (\square^2 \phi^a - V(\phi^a))^2, \quad (23a)$$

and

$$\det \left[(\square^2 \delta^{ab} - \frac{\delta^2 V}{\delta \phi^a \delta \phi^b}) \frac{\delta \phi^b}{\delta f^c} \right] = \det(1 + \vec{f} \cdot \vec{f}) \det \tilde{\theta}. \quad (23b)$$

Then the complicated dynamics of the f^a 's is equivalent to a scalar field ϕ^a (with action $\int d^4x \left[\frac{1}{2} (\partial_\mu \phi^a)^2 - V(\phi^a) \right]$) in a random magnetic field. If this is true, then the Parisi-Sourlas mechanism⁽¹⁴⁾ is valid. Confinement is proven via dimensional reduction because of the existence of a supersymmetry. The non-perturbative aspect of QCD₄ is equivalent to a two-dimensional scalar theory.

Note that equation (23a) is plausible because $(Z_{\mu\nu}^a)^2 \approx (\partial f)^4$. Furthermore, the potential $V(\phi)$ and $\phi^a = \phi^a(f)$ are to be determined from equations, (23;a,b). From the general forms of these equations, we cannot rule out the existence of the transformation. Unfortunately, the author was not able to show, up to the present, the Parisi-Sourlas mechanism.

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