Is there a mass dependence in the spin structure of baryons?

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Abstract

We analyze the axial-vector form factors of the nucleon-hyperon system in a model with SU(3)$_\text{flavor}$ symmetry breaking due to mass dependent quark spin polarizations. This mass dependence is deduced from an analysis of magnetic moment data, and implies that the spin contributions from the quarks to a baryon decrease with the mass of the baryon. When applied to the axial-vector form factors, these mass dependent spin polarizations bring the various sum-rules from the quark model in better agreement with experimental data. As a consequence our analysis leads to a reduced value for the total spin polarization of the proton.

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I. INTRODUCTION

In the quark model the axial-vector current form factors measured in nucleon-hyperon weak decays are related to the spin polarizations of quarks. In principle there are three quark spin polarization parameters, $\Delta u$, $\Delta d$ and $\Delta s$ that can be determined from the experimental data. Unfortunately the four axial-vector form factors can all be expressed in term of two parameters, $a_3 = \Delta u - \Delta d$ and $a_8 = \Delta u + \Delta d - 2\Delta s$. To determine the quark spin polarizations we therefore also need the total spin polarization $\Delta \Sigma = \Delta u + \Delta d + \Delta s$ of the nucleon.

The relation between the axial-vector currents and the nucleon spin has been discussed in the recent literature.

McCarthy et al [1] have performed an analysis of the recent deep inelastic scattering measurements to obtain the quark spin content and concluded that in view of current limited understanding of the nature and magnitude of SU(3)$_{\text{flavor}}$ symmetry breaking, the use of SU(3) underestimates the uncertainties of the flavor-specific quark helicity contributions to the nucleon spin. In another paper [2] Song et al have discussed small SU(3) breaking contributions to the analysis.

Also Lipkin [3] and Lichtenstadt and Lipkin [4] have analyzed possible SU(3) symmetry breaking in a model to estimate its influence on the value of the axial vector form factors and its effect on the spin polarization analysis. Their conclusion is that SU(3) symmetry breaking does not have a large influence on the nucleon spin evaluation.

Ehrnsperger and Schäfer [5] have also performed an analysis of hyperon beta decay to extract the $F/D$ ratio and study its implication on the spin polarization sum rules. They argue that there is no evidence for a strong spin polarization of the strange quark sea.

Ratcliffe [6] has analyzed hyperon semi-leptonic decays and come to the conclusion that SU(3) symmetry with some small corrections for its violation gives a very satisfactory description of the experimental data.

In all these analyses, both with and without SU(3) symmetry breaking, the authors have used the Review of Particle Properties (RPP) data set [7] for the hyperon decays, which are obtained by assuming that the SU(3) symmetry breaking induced form factors $g_2$ can be neglected.

In our opinion an analysis with SU(3) breaking to evaluate the axial-vector form factors should use the best experimental results available in the literature, including the symmetry breaking form factor $g_2$. This is the attitude adopted in the present work and elaborated upon later. Using the RPP data set already presumes that the symmetry breaking effects are small. This is clearly undesirable in an analysis of such effects.

In two earlier papers [8,9] we have discussed the magnetic moments of baryons in an extension of the quark model, which allows for general flavor symmetry breaking and where the quark magnetic moments are allowed to vary with the isomultiplet $B$. The magnetic moments of the baryons in this model can be written as a linear sum of contributions from the various flavors

$$\mu(B^i) = \mu_u^B \Delta u^{B^i} + \mu_d^B \Delta d^{B^i} + \mu_s^B \Delta s^{B^i},$$

(1)

where $\mu_f^B$ is the effective magnetic moment of the quark of flavor $f$ in the isomultiplet $B$ and $\Delta f^{B^i}$ is the corresponding spin polarization for baryon $B^i$, $i$ being the baryon charge state.
By symmetry arguments the $\Delta f^{B_i}$'s in the octet baryons can be expressed as constant linear combinations of the three $\Delta f$'s for the proton, which are the only spin polarizations needed to describe the octet:

$$\Delta f^{B_i} = \sum_{f'} M(B_i)_{f f'} \Delta f', \quad (2)$$

where $f, f'$ runs over $u, d, s$, and the $M(B_i)$'s are matrices with constant elements. In particular for the six mirror symmetric baryons of type $B(xyy)$, where $x$ and $y$ are different flavors, we have $\Delta y^{B_i} = \Delta u$, $\Delta x^{B_i} = \Delta d$ and $\Delta z^{B_i} = \Delta s$ where the flavor $z$ is the non-valence quark flavor.

In the non-relativistic quark model (NQM) the values of the spin polarizations are $\Delta u = \frac{4}{3}$, $\Delta d = -\frac{1}{3}$ and $\Delta s = 0$.

Due to the homogeneity of the right hand side of (1), it is a question of definition if the dependence on the baryon multiplet is considered to be associated with the quark magnetic moment rather than with the spin polarization. In Refs. [8] and [9] we have chosen to analyze the data by keeping the spin polarizations fixed throughout.

Here we will analyze the opposite situation, where the spin polarization is instead assumed to vary with the baryon multiplet and the quark magnetic moments are the same for all multiplets.

This scheme has the advantage of making the properties of the quarks static and environment independent. Since the effective magnetic moment of a quark in the NQM has the form

$$\mu_f = \frac{e_f}{2m_f}, \quad (3)$$

$e_f$ being the quark charge, this means that there is no dependence of the effective quark mass $m_f$ on $B$. This is in accordance with the fact that the same constituent quark masses can be used successfully to predict the baryon octet and decuplet masses with only a hyperfine splitting interaction in the Hamiltonian. The disadvantage is that the spin structure varies from multiplet to multiplet.

This variation of spin polarization with the isospin multiplet introduces an explicit SU(3)$_{\text{flavor}}$ symmetry breaking in the analysis, which is different from earlier symmetry breaking schemes and is evaluated from the measured baryon magnetic moments.

A further merit of this interpretation, and the one that we are going to analyze here, is that the sum-rules governing the axial-vector form factors are better fulfilled in this scheme, although the errors are still somewhat large to definitely decide between either of the two ways of attributing the mass dependence effect.

Since we are going to allow for SU(3) symmetry breaking in our analysis, we will use the experimental data including the symmetry breaking form factors $g_2$ whenever available.

**II. CALCULATING THE MODEL PARAMETERS**

The spin structure parameters in the expressions for the magnetic moments and in the deep inelastic scattering experiments and axial-vector form factors are not a priori the same.
In many models they are nevertheless proportional \([10]\), and can be normalized to be the same. We normalize them to the axial-vector form factor \(g_{np}^A = 1.2573\), as is generally done.

We write the baryon magnetic moments as

\[
\mu(B^i) = \sum_{f,f'} \mu_f \alpha(B)f f' \Delta f',
\]

where \(\mu_f\) is the magnetic moment of the quark of flavor \(f\) and \(\Delta f\) is the corresponding spin polarization. The factor \(\alpha(B)\) is an overall factor, the same for all flavors, depending only on the isomultiplet \(B\). The flavor symmetry breaking within the isomultiplets is then accounted for by the quark magnetic moments that are free parameters. We stress that the value of \(\alpha(B)\) is independent of the isospin symmetry breaking parameter \(T = \mu_u^B / \mu_d^B\) discussed in Ref. \([8]\). Thus we can, if we like, assume that we have isospin symmetry.

In our previous analysis we associated the factor \(\alpha(B)\) with the quark magnetic moments and defined \(\mu_B^f = \alpha(B)\mu_f\) as in equation (1). We can choose to normalize \(\alpha(B)\) to \(\alpha(N) = 1\), in which case \(\mu_N^f = \mu_f\). The other values of \(\alpha\) can then be obtained from the previously extracted values of the \(\mu_B^f\)'s as \([8,9]\]

\[
\alpha(\Lambda) = \mu_d^\Lambda / \mu_d^N = 0.88 \pm 0.04, \\
\alpha(\Sigma) = \mu_d^\Sigma / \mu_d^N = 0.91 \pm 0.01, \\
\alpha(\Xi) = \mu_d^\Xi / \mu_d^N = 0.85 \pm 0.03.
\]

We will now instead associate \(\alpha(B)\) with the spin polarizations. Equation (2) is then rewritten as

\[
\Delta f^B = \sum_{f'} M(B^i)_{f f'} \alpha(B)\Delta f'.
\]

Thus, \(e.g.\) in the mirror symmetric baryons \(B(xy y)\), we instead have \(\Delta y^B = \alpha(B)\Delta u\), \(\Delta x^B = \alpha(B)\Delta d\) and \(\Delta z^B = \alpha(B)\Delta s\).

The values of \(\alpha(B)\) can be well fitted to a linear function of the mean mass of \(B\) as shown in Fig. 1. The linear relation is

\[
\alpha(m) = 1 - 0.376 (m - 0.939),
\]

when \(m\) is expressed in GeV. We will continue to use \(\alpha(B) \equiv \alpha(m_B)\) in the following.

We will test this linear relation in what follows by using the interpolated \(\alpha\)'s from the equation above. These values are

\[
\alpha(\Lambda) = 0.93 \pm 0.02, \\
\alpha(\Sigma) = 0.90 \pm 0.02, \\
\alpha(\Xi) = 0.86 \pm 0.02.
\]

To illustrate why this \(B\) dependent factor is needed we regard the sum-rule \([11]\]

\[
\mu(p) + \mu(\Xi^0) + \mu(\Sigma^-) - \mu(n) - \mu(\Xi^-) - \mu(\Sigma^+) = 0,
\]
which follows when the quark magnetic moments and spin polarizations both are independent of $B$. It is badly broken by the experimental data by ten standard deviations so that the left hand side is instead $0.49 \pm 0.05$. In our more general parameterization this sum-rule becomes

$$
\mu(p) + \frac{\mu(\Xi^0)}{\alpha(\Xi)} + \frac{\mu(\Sigma^-)}{\alpha(\Sigma)} - \mu(n) - \frac{\mu(\Xi^-)}{\alpha(\Xi)} - \frac{\mu(\Sigma^+)}{\alpha(\Sigma)} = 0.
$$

(14)

Due to the construction of the $\alpha$’s this sum-rule is satisfied.

There is no doubt that the correction factors are needed. However, if a totally different structure is envisaged with more degrees of freedom, like pion-clouds etc. there is of course room for other possibilities. Our aim is to investigate the symmetry-breaking in the more restricted framework of magnetic moments, spin polarizations, and axial-vector form factors, since these are extensively used to investigate baryon structure.

III. THE AXIAL-VECTOR FORM FACTORS

Whether we associate the $\alpha(B)$ factors to the quark magnetic moments or the spin polarizations, obviously does not affect the analysis of the magnetic moments. However, the analysis of the axial-vector form factors will be modified when we let the spin polarizations be given by (8).

The axial-vector form factors can in this parameterization now be written

$$
g_{A}^{np} = \Delta u - \Delta d,
$$

(15)

$$
g_{A}^{\Lambda p} = \frac{1}{3}(2\Delta u - \Delta d - \Delta s)\alpha(\Lambda),
$$

(16)

$$
g_{A}^{\Xi \Lambda} = \frac{1}{3}(\Delta u + \Delta d - 2\Delta s)\alpha(\Xi),
$$

(17)

$$
g_{A}^{\Sigma n} = (\Delta d - \Delta s)\alpha(\Sigma).
$$

(18)

This can be used to derive the two sum-rules

$$
\frac{g_{A}^{\Xi \Lambda}}{\alpha(\Xi)} + \frac{g_{A}^{\Lambda p}}{\alpha(\Lambda)} = \frac{g_{A}^{\Sigma n}}{\alpha(\Sigma)} + g_{A}^{np},
$$

(19)

$$
\frac{g_{A}^{\Xi \Lambda}}{\alpha(\Xi)} + g_{A}^{np} = 2\frac{g_{A}^{\Lambda p}}{\alpha(\Lambda)},
$$

(20)

As mentioned in the introduction, since $\alpha(m)$ is an SU(3) breaking parameter, it would in our opinion be inconsistent to use $g_{A}$’s extracted from experimental data in the SU(3) symmetric limit. In the RPP data table [7] the values are effective values of the $g_{A}$’s in the SU(3) symmetric limit where the induced axial vector form factors $g_{2}$ are taken to be zero. We want here to explore the consequence of taking the symmetry breaking into account through the $g_{2}$’s. The values of the $g_{A}$’s are in general sensitive to the values of the $g_{2}$’s [12].

For $g_{A}^{\Sigma n}$ we use the value $g_{A}^{\Sigma n} = -0.20 \pm 0.08$ from Hsueh et al. [13] and for $g_{A}^{\Lambda p}$ the value $g_{A}^{\Lambda p} = 0.731 \pm 0.016$ from Dworkin et al. [14].
The value $g^\Sigma_A = -0.340 \pm 0.017$ in RPP assumes that the induced form factor $g_2 = 0$ (SU(3) limit). The value measured by Hsueh et al. is $g_2 = 0.56 \pm 0.37$ which is indeed far from zero.

Similarly, we use the value for $g^\Lambda_p$ from Dworkin et al. because they have not assumed that the weak magnetism coupling $g_W = 0.97$. However, it would not make any significant difference to use the value $g^\Lambda_p = 0.718 \pm 0.015$ given in the RPP data table.

The two sum-rules are barely satisfied without the $\alpha$'s. The relations are satisfied as follows

\begin{align}
(0.98 \pm 0.07) & \quad 1.07 \pm 0.06 = 1.04 \pm 0.09 \quad (1.06 \pm 0.08), \quad (21) \\
(1.51 \pm 0.05) & \quad 1.55 \pm 0.06 = 1.57 \pm 0.05 \quad (1.46 \pm 0.03), \quad (22)
\end{align}

corresponding to the two equations above. The numbers in parentheses are the values without the $\alpha$'s (i.e. $\alpha \equiv 1$).

Although the improvement relative to the case without the $\alpha$'s might not be dramatic, both sum-rules are definitely better satisfied with the $\alpha$'s.

As a further test we calculate the constant $R = \frac{\Delta u - \Delta d}{\Delta u - 2\Delta s}$ defined in Ref. [8]. This constant has the value $R = 1.18 \pm 0.01$ from the magnetic moment data. Our expression for this constant, expressed in terms of axial-vector form factors, is now

$$R = \frac{2g^{np}_A}{g^{LP}_A/\alpha(\Lambda) + g^{\Xi\Lambda}_A/\alpha(\Xi) + g^\Sigma n_A/\alpha(\Sigma) + g^{np}_A} = 1.19 \pm 0.06. \quad (23)$$

This is again an improvement over the value $R = 1.23 \pm 0.06$ found in Ref. [8].

The four axial-vector form factors can be parameterized by two variables, which we choose as $\Delta u - \Delta d = g^{np}_A$ and $a_8 = \Delta u + \Delta d - 2\Delta s$.

Since $g^{np}_A = 1.2573 \pm 0.0028$ is by far the best measured parameter we will use this as a fix parameter and express the three other axial-vector form factors in terms of $g^{np}_A$ and $a_8$. This gives

\begin{align}
\frac{g^{\Lambda}_A}{\alpha(\Lambda)} &= \frac{1}{6} a_8 + \frac{1}{2} g^{np}_A, \quad (24) \\
\frac{g^{\Sigma n}_A}{\alpha(\Sigma)} &= \frac{1}{2} a_8 - \frac{1}{2} g^{np}_A, \quad (25) \\
\frac{g^{\Xi \Lambda}_A}{\alpha(\Xi)} &= \frac{1}{3} a_8. \quad (26)
\end{align}

We have performed two least square fits of $a_8$ using these formulas and the experimental numbers quoted above, one with the $\alpha$'s and one without. With the $\alpha$'s we get $a_8 = 0.89 \pm 0.08$ with $\chi^2 = 0.31$ and without the $\alpha$'s we get $a_8 = 0.70 \pm 0.08$ with $\chi^2 = 1.9$. We see that there is a considerable improvement when the $\alpha$'s are included. This is further illustrated in Fig.2. The left picture illustrates the fit to Eqs. (24)-(26), without the $\alpha$ factors. The gray bands are the experimental values of the $g_A$'s including the errors. The
linear functions are the corresponding right hand sides of (24)-(26). The thick central vertical line is the value for $a_8$ obtained in the fit, and the thinner vertical lines are the error bounds of the fit. The right picture similarly illustrates the fit when the $\alpha$’s are included. We can see that all $g_A$’s are better accounted for when using the $\alpha$ factors.

The value $a_8 = 0.89 \pm 0.08$ differs rather much from the value $a_8 = 0.601 \pm 0.038$ used by Ellis and Karliner [15] in their proton spin polarization analysis.

The difference has two sources: the $\alpha$ factors and the different choice of experimental data for the $g_A$’s. This shows, in our opinion, that there is a rather large uncertainty in the determination of the quark spin polarization of the nucleon, which involves the constant $a_8$. The conventional wisdom is to use the experimental values with the assumption that $g_2 = 0$ for all $g_A$’s. Using these values in Eqs. (24)-(26) with $\alpha \equiv 1$ leads to a value of $a_8 = 0.58 \pm 0.06$. However, such an analysis ignores the experimental fact that $g_2 \neq 0$ for $g_A^\Sigma$, making the determination uncertain. On the other hand, in our analysis we have only set $g_2 \neq 0$ for one $g_A$ as the others are experimentally unknown. This obviously also leads to an uncertainty in the determination.

There are basically three different ways that the data can be analyzed at present:

a) We ignore the non-zero value of $g_2$ for $g_A^\Sigma$ and hope that there is no effect in the other form factors. In this case $\alpha \equiv 1$.

b) We include the correction $g_2$ for $g_A^\Sigma$ and hope that this correction is small for the other $g_A$’s. ($\alpha \equiv 1$.)

c) We include the correction for $g_2$ that is measured, and hope that they are not too large for the unmeasured ones. We also include the phenomenological SU(3) correction factors from the magnetic moment analysis. ($\alpha \neq 1$.)

The option c) is in our view essentially on the same level of rigor as the option a), which tries to ignore the problem entirely. We cannot find that it is more consistent to try to ignore a problem that has been experimentally shown to exist, rather than to treat it, even if the treatment is incomplete, due to lack of data.

Our calculation has shown that the sum-rules for the $g_A$’s seem to be better accounted for using the approach c). This indicates that the corrections are important.

We can also see that our approach is more consistent with the sum-rules if we calculate $a_8$ separately from Eqs. (24)-(25) and (26). Option a) gives $a_8 = 0.57 \pm 0.03$ from (24)-(25) and $a_8 = 0.75 \pm 0.15$ from (26). Option c) gives the values $a_8 = 0.90 \pm 0.06$ and $a_8 = 0.87 \pm 0.13$ respectively. These two values are certainly much closer to each other.

IV. IMPLICATIONS FOR THE PROTON SPIN POLARIZATION ANALYSIS

The values of $g_A^{np}$ and $a_8$ are commonly used in analyses to determine the proton spin polarization. A change in the value of $a_8$ therefore has a non-negligible influence on such a spin polarization analysis. We will illustrate this using the formulas from the analysis of Ellis and Karliner [15]. Their evaluation of $\Delta\Sigma = 0.31 \pm 0.07$ can be expressed as

$$\Delta\Sigma(Q^2) = 9 \Gamma_1^p(Q^2) - \left(\frac{g_A^{np}}{12} + \frac{a_8}{36}\right) f(\alpha_s) \frac{h(\alpha_s)}{h(\alpha_s)},$$

(27)
where \( f(\alpha_s) \) and \( h(\alpha_s) \) are as in [15]. The constant \( g_{A}^{np} \) has the usual value \( g_{A}^{np} = 1.2573 \), but the constant \( a_8 \) has in Ref. [15] the value \( a_8 = 0.601 \pm 0.038 \).

The value of \( \Delta \Sigma \) will change with the value of \( a_8 \). Let the change in \( a_8 \) be denoted \( \delta a_8 \), and the new value of \( \Delta \Sigma \) be denoted \( \Delta \Sigma' \). We then have

\[
\Delta \Sigma' = \Delta \Sigma - \frac{\delta a_8}{4} \left( 1 - \mathcal{O}(2) \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 \right) \approx \Delta \Sigma - \frac{\delta a_8}{4}.
\]

The value of \( a_8 = 0.89 \pm 0.08 \) found above will thus lead to a different estimate of the total spin polarization of the proton. \( \Delta \Sigma \) will change to

\[
\Delta \Sigma' = 0.31 - \frac{0.29}{4} = 0.24 \pm 0.09.
\]

In our previous analysis of isospin symmetry breaking in the baryon magnetic moments [8] this value favors a slightly smaller isospin symmetry breaking than the value \( \Delta \Sigma = 0.31 \). It also changes slightly the quark spin content of the proton to the values

\[
\begin{align*}
\Delta u &= 0.86 \pm 0.04, \\
\Delta d &= -0.40 \pm 0.04, \\
\Delta s &= -0.22 \pm 0.05.
\end{align*}
\]

calculated by means of \( \Delta \Sigma' \), \( a_8 \) and \( g_{A}^{np} \). The main effect is to allow \( \Delta s \) to be larger.

V. DISCUSSION AND CONCLUSIONS

As we have seen above there is support from the axial-vector form factor data, when \( g_2 \) is taken into account, that the spin polarizations of the quarks are diminishing with the increase of mass of the host particle. This mass dependence is born out in the sum-rules that can be written and are well satisfied by the experimental data. These mass dependent spin polarizations also account for the breaking of the sum-rule for the baryon magnetic moments.

Our analysis leads to a change in the evaluation of the axial-vector coupling constant \( a_8 \), that will affect the proton spin polarization analysis. Our value for the constant \( a_8 \) favors a lower value of the proton quark spin sum.

The data for the axial-vector form factors used in the sum-rules are subject to statistical and systematic errors. There are essentially three form factor values used by us in these sum-rules, that are still uncertain. These are \( g_{A}^{\Xi \Lambda}, g_{A}^{\Sigma n}, \) and \( g_{A}^{\Lambda p} \). For the first one, which is a low statistics experiment (1992 events), the quoted error is the combined error. The two last ones are high statistics experiments. For the \( g_{A}^{\Sigma n} \) form factor the quoted error is probably dominated by the error in the determination of the induced form factor \( g_2 \). For the \( g_{A}^{\Lambda p} \) form factor measured by Dworkin et al, there might be a further systematic error \( \pm 0.012 \) to be added to the quoted error. If this is done the difference between the two fits in Fig 2. becomes somewhat less pronounced, but the main features remain.

The final result for \( a_8 \) should of course therefore be taken with some caution and await further measurements of the other \( g_2 \) factors to be complete. However, it does, in our opinion,
merit consideration and shows the uncertainty at present in the determination of $a_8$, which has a non-negligible influence on the determination of the nucleon quark spin polarizations.

The studies performed by Leinweber et al. [16] supports indirectly the findings here. Their lattice gauge calculations have been done in quenched QCD, and to define the effective quark magnetic moments, the spin polarizations were set to their NQM values once the results were extracted from the lattice. The quark magnetic moments then show a decrease in value with increasing mass of the host particle, in much the same way as we found in Ref. [9], where we also chose the keep the spin polarizations mass independent.

One possible interpretation of this effect could be that the quarks simply gradually lose their orientation as the excitation energy increases.

As the total angular momentum of the proton is fixed to $1/2$, this means that there must be a contribution from some other electrically neutral component that increases its angular momentum with baryon mass to compensate for the decrease in the contribution coming from the quarks.

One possibility is to attribute such a contribution to the presence of gluonic components in the baryons. This is perfectly consistent with the findings from deep inelastic scattering experiments, that only about half of the proton momentum is carried by the quarks. Also a collective mode of the Skyrmmion type, with a rather small contribution to the magnetic moment, could be envisaged to manifest in this way.

The new feature found here is that this contribution seems to vary linearly with the mass of the baryon multiplet.

This emphasizes the importance of trying to measure the magnetic moments of high mass baryon states and also to try to calculate them with lattice gauge techniques.

Also lattice gauge calculations of the axial-vector form factors for the heavier states, would possibly shed light on the behavior found here. Such calculations have already been performed for the nucleon system [17].

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REFERENCES

FIGURES

FIG. 1. The ratio $\alpha(B) = \mu_d^B / \mu_d^N$ as a function of the baryon mass. The points are the data for the nucleon, $\Lambda$, $\Sigma$ and $\Xi$ as given by equations (5)-(7). The straight line represents the linear fit according to equation (9).

FIG. 2. The left picture illustrates the fit to Eqs. (24)-(26), without the $\alpha$ factors. The gray bands are the experimental values of the $g_A$'s including the errors. The linear functions are the corresponding right hand sides of (24)-(26). The thick central vertical line is the value for $a_8$ obtained in the fit, and the thinner vertical lines are the error bounds of the fit. The right picture similarly illustrates the fit when the $\alpha$'s are included. We can see that all $g_A$'s are better accounted for when using the $\alpha$ factors.