Outbursts of Irradiated Accretion Discs

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ABSTRACT

I solve analytically the viscous evolution of a self-irradiated accretion disc, as seen during outbursts of soft X-ray transients. The solutions predict steep power-law X-ray decays \( L_X \sim (1 + t/t_{\text{visc}})^{-4} \), changing to \( L_X \sim (1 + t/t_{\text{visc}})^{4} \) at late times, where \( t_{\text{visc}} \) and \( t_{\text{c}} = t_{\text{visc}} \) are viscous timescales. These forms closely resemble the approximate exponential and linear decays inferred by King and Ritter (1997) in these two regimes. The decays are much steeper than for unirradiated discs because the viscosity is a function of the central accretion rate rather than of local conditions in the disc.

Key Words: accretion, accretion discs - instabilities - stars: X-rays: stars.

1 INTRODUCTION

Soft X-ray transients (SXTs) are close binary systems with black-hole or neutron-star primaries; they undergo large outbursts, reaching X-ray luminosities of order the Eddington limit. The outbursts have some resemblance to those of dwarf novae, in which the primary is a white dwarf, with the important difference that the timescales are far longer. SXT outbursts last of order a year, and recur at intervals of several years or greater, whereas these timescales are typically days and weeks for dwarf novae. It is by now generally accepted that dwarf nova outbursts result from a thermal-viscous disc instability (see e.g. Cannizzo 1993 for a review). While this picture naturally explains observed dwarf nova timescales it requires essentially ad hoc modification to explain SXT light curves. However, a crucial observed difference between dwarf nova disc and those in SXTs is that the latter are heavily irradiated by the central X-ray source (van Paradijs & McClintock, 1994; Shahbaz & Kuulkers, 1997). A recent paper (King & Ritter, 1997, hereafter KR) has pointed out that long decays were a feature of the irradiated-disc model of KR; rather than exponentials, the irradiated-disc model of KR would give a poor representation of observed SXT light curves. I consider this question further in this paper.

In the unirradiated disc models considered by Pringle (1974) and Cannizzo, Lee & Goodman, (1990) the viscosity controlling the time evolution of the disc is a function of purely local variables, either through assumed power-law dependences (Pringle) or the alpha-prescription

\[ \nu = \alpha c_S H \]

with the sound speed \( c_S \propto T_m^{1/2} \) and scaleheight

\[ H = c_S \left( \frac{R^3}{GM} \right)^{1/2} \]

fixed by the disc midplane temperature \( T_m \), resulting from local viscous dissipation (here \( R \) is the radial disc coordinate and \( M \) the central mass). The irradiated-disc model of KR is fundamentally different; the temperature and viscosity are no longer fixed locally, but instead by \( M_r \), and have explicit time dependences. I shall find analytic solutions of the disc diffusion equation for this case. These give much steeper power-law behaviour for \( \dot{M}_r \), which is in practice indistinguishable from the exponential decay predicted by KR.

2 IRRADIATED DISCS

The surface temperature of a disc irradiated by a central source is given by

\[ T_{\text{irr}}(R)^4 = \frac{\eta \dot{M}_r c_S^2(1 - \beta)}{4\pi\sigma R^2} \left( \frac{H}{R} \right)^n \left[ \frac{d\ln H}{d\ln R} - 1 \right], \]

where \( \eta \) is the efficiency of rest-mass energy conversion into X-ray heating, \( \dot{M}_r \) the central accretion rate, \( H \) the disc scaleheight at disc radius \( R \), \( \beta \) the albedo of the disc faces.
and the factor in square brackets lies between 1/8 and 2/7. The index $n = 1, 2$ for irradiation by a central point source or the inner disc respectively (van Paradijs, 1996; Fukue, 1992; King, Kolb & Susszkiezwi 1997). Neutron–star accretion always has $n = 1$, while $n = 2$ is possible in the black hole case because of the lack of a hard stellar surface; this effect probably allows black–hole binaries to be transient at mass transfer rates which would otherwise give a persistent system (King, Kolb & Susszkiezwi 1997). However during outbursts, observations of a strong power–law continuum strongly suggest that the disc develops an extended X–ray corona, thereby reverting to $n = 1$. The distinction between these two possibilities will in any case be unimportant for the purposes of this paper (I shall assume $n = 1$ below): the main effect comes from the radial dependence of $T_{\text{eff}}$, which is the same regardless of $n$. The ratio $H/R$ is roughly constant in a disc, so $T_{\text{eff}}$ falls off as $R^{-1/2}$. Thus for a disc with a large enough ratio of outer radius $R_0$ to inner radius $R_*$, $T_{\text{eff}}$ dominates the disc’s own effective temperature $T_{\text{disc}}$, given by

$$T_{\text{eff}}^4 = \frac{3GM\dot{M}}{8\pi R^3}$$

(e.g. Frank et al., 1992) which goes as $R^{-3/4}$ (here $\dot{M}$ is the accretion rate at radius $R$). Thus

$$T_{\text{eff}}^4 \frac{T_{\text{eff}}^4}{T_{\text{disc}}^4} = \frac{4}{3} \frac{\dot{M}_*}{M_*} \eta(1 - \beta) \frac{H}{R} \left[ \frac{\ln H}{\ln R} - 1 \right],$$

where $R_* = 2GM/c^2$ is the Schwarzschild radius of the central object. In X–ray binaries one typically finds $H/R \approx 0.2$, $\beta = 0.9$ (de Jong, van Paradijs & Augusteijn, 1996), while the last factor on the rhs lies between 2/7 and 1/8. Further, the inner disc radius $R_0$ is close to $R_*$ in all X–ray binaries, so the efficiency $\eta \approx 0.1$. With a typical value $\eta = 0.2$ we find

$$\frac{T_{\text{disc}}^4}{T_{\text{eff}}^4} \approx 6.7 \times 10^{-4} \frac{R}{R_*}$$

for a quasi–steady disc ($\dot{M} \approx \dot{M}_*$). Since we typically have $R_* \lesssim 2 \times 10^6 \mathrm{cm}$ and a disc radius $R_0 \gtrsim 10^{13} \mathrm{cm}$, we find $T_{\text{disc}}^4 / T_{\text{eff}}^4 \gtrsim 34$. This result shows that irradiation dominates the output of the disc over most of its surface area where the central luminosity is significant. In fact we can go further and show that irradiation completely alters the disc structure over most of its area. For $T_{\text{disc}} \times c_S$ we replace $T_{\text{eff}}$ by $T_{\text{disc}} \ll M_*/R_*^{1/2}$ so that (1, 2) give

$$\nu = \nu_* \left( \frac{M_*}{M_*} \right)^{1/4} \left( \frac{R}{R_*} \right),$$

where $\nu_*$, $M_*$ are the viscosity and accretion rate at the inner disc radius $R_*$ at time $t = 0$, measured from the beginning of the decay (the X–ray peak). Then the diffusion equation determining the disc surface density $\Sigma(R, t)$

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left( R^{1/2} \Sigma \frac{\partial R^{1/2}}{\partial R} \right)$$

(e.g. Frank et al., 1992) can be rewritten as

$$\frac{\partial}{\partial t} (R^{3/2}\Sigma) = 3\nu\frac{M_*}{R_*} \left( \frac{M_*}{R_*} \right)^{1/4} \left( R^{1/2} \frac{\partial}{\partial R} \right)^2 (R^{3/2}\Sigma).$$

Now defining $p = 2R^{3/2}$ and $S = R^{3/2}\Sigma$ we get finally

$$\frac{\partial S}{\partial t} = 3\nu\frac{M_*}{R_*} \left( \frac{M_*}{R_*} \right)^{1/4} \frac{\partial^2 S}{\partial p^2}.$$  

Thus we obtain a diffusion equation for $S(R, t)$, which is however nonlinear because of the factor $M_*^{1/4}$ on the rhs: this itself depends on $S$ and thus $t$ through the relation

$$M_\epsilon = 3\pi\nu S(R, t)(R_\epsilon S(R, t))$$

(e.g. Frank et al., 1992). Using (8) and the definition of $S$ we have

$$M_\epsilon = 3\pi\nu\left( \frac{M_*}{R_*} \right)^{1/4} R_*^{-3/2} S(R_*, t)$$

and thus

$$\left( \frac{M_*}{M_*} \right)^{1/4} = \left[ \frac{3\pi\nu \epsilon S(R_*, t)}{R_*^{3/2} M_*} \right]^{1/3}.$$  

It is clear that (11) has a steady solution of the form $S = a + b/R$, or

$$\Sigma = \frac{a}{R} + \frac{b}{R^{3/2}}.$$  

Since from (8) $\nu \propto R$, this is simply the usual steady solution $\nu\Sigma = A + B R^{-1/2}$ (e.g. Frank et al., 1992), in which the first term represents a steady inflow with accretion rate given by $A$, and the second term gives zero net mass inflow.

I now seek separated time–dependent solutions of (11) of the form

$$S(R, t) = R^{3/2} \epsilon(t) P(p)$$

with $\epsilon(t)$ dimensionless and $P$ with the dimensions of $\Sigma$. Eq. (14) gives

$$\left( \frac{M_*}{M_*} \right)^{1/4} = \left[ \frac{3\pi\nu \epsilon P(p_\epsilon)}{M_*} \right]^{1/3}$$

with $p_\epsilon = 2R_*^{1/2}$. Choosing $\epsilon(0) = 1, P(p_\epsilon) = \Sigma(R_\epsilon, 0)$, the definition of $M_\epsilon$ implies that the square bracket = 1, so that

$$\left( \frac{M_*}{M_*} \right)^{1/4} = \epsilon^{1/3}(t).$$

Then inserting (16) into (11) and using (18) gives

$$\frac{\dot{\epsilon}}{3\theta^{3/2}} = \frac{\nu\epsilon}{R_*^2 P} \equiv - \frac{1}{T}$$

where $T$ is a separation constant with dimensions of time. Since $\epsilon(0) = 1$ we find

$$\epsilon = \left( 1 + \frac{t}{T} \right)^{-3}.$$
and also
\[ P = A \cos \left( \frac{R_c}{\nu_{se} x} \right)^{1/2} p + B \sin \left( \frac{R_c}{\nu_{se} x} \right)^{1/2} p. \]  
\hspace{1cm} (21)

Replacing \( p \) by \( 2R^{1/2} \) and assuming that the disc is irradiated out to a fixed outer edge \( R_0 \), i.e.
\[ \Sigma(R_0, t) = 0, \]
we get the eigenvalues
\[ T_n = \frac{4R_0 R_0}{\nu_s (n - 1/2)^2 \pi^2}, \quad n = 1, 2, 3, ... \]  
\hspace{1cm} (22)

and
\[ T'_n = \frac{4R_0 R_0}{\nu_s n \pi^2}, \quad n = 1, 2, 3, ... \]  
\hspace{1cm} (23)

for the cosine and sine terms respectively. We can simplify these by noting from (8) that
\[ \nu_s \equiv \nu(R_0, 0) = \nu_s R_0 / R, \]
so that
\[ T_n = \frac{t_{v_{\text{vis}}}}{(2n - 1)^2}, \]
\hspace{1cm} (24)

\[ T'_n = \frac{t_{v_{\text{vis}}}}{(2n)^2} \]
\hspace{1cm} (25)

for the two cases, with \( t_{v_{\text{vis}}} = 16R_0^2 / \pi^2 \nu_1 \) a measure of the viscous time at \( R_0 \). Then in terms of the variable
\[ x = \pi \left( \frac{R}{R_0} \right)^{1/2}, \]
we finally get the general solution of the diffusion equation for a fully irradiated disc obeying (22) as:
\[ \Sigma(R, t) = \frac{1}{\pi} \sum_{n=1}^{\infty} \left\{ \frac{A_n \cos(2n - 1)x}{1 + (2n - 1)^2 t/v_{\text{vis}}} + \frac{B_n \sin 2nx}{[1 + (2n)^2 t/v_{\text{vis}}]} \right\}. \]  
\hspace{1cm} (26)

Here the coefficients \( A_n, B_n \) are to be found by expanding the initial surface density distribution \( \Sigma(R, 0) \) as a Fourier series in \( \cos(2n - 1)x \) and \( \sin 2nx \), viz
\[ A_n = \frac{4}{\pi} \int_0^{\pi/2} x^2 \Sigma(R, 0) \cos(2n - 1)x \, dx \]  
\hspace{1cm} (27)

\[ B_n = \frac{4}{\pi} \int_0^{\pi/2} x^2 \Sigma(R, 0) \sin 2nx \, dx. \]  
\hspace{1cm} (28)

(If we had not used the boundary condition (22) we would have found a Fourier transform rather than a series.) For an initial density distribution with no discontinuities we therefore have \( A_n, B_n \sim 1/n^2 \). Note that the steady solution (15) does not obey the boundary condition (22), as mass has to be fed in from \( R > R_0 \) to maintain the steady state. Accordingly if we start with initial conditions \( \Sigma(R, 0) = ax^{-2} + bx^{-3} \) in (29) we do not obtain the steady solution, since the expansion assumes that (15) holds. It therefore demands a discontinuity in \( \Sigma(R, 0) \) at \( R = R_0 \) (i.e. \( A_n, B_n \sim 1/n \)) which subsequently diffuses away.

Given the solution (29), the central accretion rate \( \dot{M}_c(t) \) can be found from (17). (I do not consider here the possible glitch in the light curve caused by irradiation of the outer disc (cf KR).) The disc mass within a radius given by \( x \) at any time can be expressed as
\[ M(R, t) = \frac{64R_0^2}{\pi^3} \int_0^{x} x^3 \Sigma(x, t) \, dx. \]  
\hspace{1cm} (29)

Using (29) with \( x = \pi/2 \), the total mass in the nth eigenmode is
\[ M_n = \frac{64R_0^2}{\pi^3} \left( -1 \right)^{n+1} A_n \]
\[ + \frac{64R_0^2}{\pi^3} \left( -1 \right)^{n+1} B_n \]  
\hspace{1cm} (30)

\[ M_n = \frac{64R_0^2}{\pi^3} \left( -1 \right)^{n+1} A_n \]  
\hspace{1cm} (31)

The structure of the expansion (29) is simple to understand. The viscous time of the nth eigenmode varies as \( t_n \sim t_{v_{\text{vis}}}/n^2 \), reflecting the fact that steeper spatial gradients cause more rapid diffusion. While the full spectrum of eigenmodes is in general present in the surface density distribution at the beginning of the outburst, the higher eigenmodes will decay more rapidly (as \( (1 + t/t_m)^{-1} \)) as the outburst proceeds, causing \( \Sigma \) to approach the first term of the expansion \( (\cos x) \). The disc mass is dominated by this mode in any case, as from (33) we have \( M_n \sim n^{-3} (1 + t/t_m)^{-3} \). Thus
\[ \Sigma(R, t) \sim A_1 \left[ 1 + \frac{t}{t_{v_{\text{vis}}} - 3} \right] ^{-3} \frac{R_0}{R} \cos \frac{\pi}{2} \left( \frac{R}{R_0} \right)^{1/2}. \]  
\hspace{1cm} (32)

For this term, (17) gives simply
\[ \dot{M}_c = \dot{M}_c \left[ 1 + \frac{t}{t_{v_{\text{vis}}} - 3} \right] ^{-1}. \]  
\hspace{1cm} (33)

Accordingly the X-ray luminosity must decay at least as steeply as this. From (35), the typical ‘exponential’ decay phase of SXT light curves, during which the flux drops by a factor \( \sim 10 \), corresponds to \( t \) running from 0 to 1.26 \( R_0^2 / v_1 \). The paper by KR derived the simple law \( \dot{M}_c \propto e^{-t/\tau} \) for the typical ‘exponential’ decay phase of SXT light curves, with \( \tau = R_0^2 / v_1 \). The best-fitting function of the form (35) to this exponential, over a dynamic range of 10, has normalization 1.05 and \( t_{v_{\text{vis}}} = 3.117 \). Thus the treatment of KR agrees surprisingly well with the present detailed calculation. As can be seen from Fig. 1, the power-law decay is almost indistinguishable from the exponential over this range. It is much steeper than the \( \dot{M}_c \sim t^{-(1+\epsilon)} \) decay of an unirradiated disc (see Section 4).

I note finally in this Section that the fundamental disc outburst mode (34) has the mass concentrated quite close to the centre: from (32) we find
\[ M_1(R, t) = \frac{64R_0^2 A_1}{\pi^3} \sin \frac{\pi}{2} \left( \frac{R}{R_0} \right)^{1/2}. \]  
\hspace{1cm} (34)

Thus one-half of the mass of this mode (i.e. effectively one-half of the total disc mass) lies within \( R = 0.11R_0 \). This contrasts with outbursts of unirradiated discs, where the usual \( \Sigma \sim R^{-3/4} \) distribution leads to \( M(R, t) \sim R^{5/4} \), so that much of the mass is still in the outer parts of the disc. The reason for the difference is the larger viscosity in the outer parts of an irradiated disc (see Section 4 below), which drives the mass inwards more rapidly.

This difference in mass distribution holds only in outburst. In quiescence irradiation is of course completely neg-
ligible. The disc will evolve in the usual way to a state in which most of the mass is near its outer edge (i.e. $\Sigma \sim R$) and begin the next outburst in this state (cf Cannizzo, 1993).

3 THE LATE DECLINE

The solutions derived above assume that the whole disc is dominated by irradiation. However as explained by KR, at late times this cannot be true, and the region where irradiation is important will retreat inwards as $R \propto M_{\bullet}^{1/2}$. Indeed sufficiently large discs will already be in this regime at the start of the outburst, as even the Eddington limit luminosity will be unable to ionize hydrogen out to the edge of such a disc. KR showed that the decline of $\dot{M}$ changes from approximately exponential to linear in this regime.

The variables $(R, t)$ or $(p, t)$ used in the diffusion equation up to now are not useful in this regime, as one wants to set boundary conditions where the disc structure changes from irradiated to effectively unirradiated, i.e. at a time–dependent radius $R \propto M_{\bullet}^{1/2}$. Accordingly we change the variables from $(p, t)$ to $(u, t)$, with

$$u = \frac{p}{M_{\bullet}^{1/4}(t)} \quad (37)$$

After some manipulation, the diffusion equation (11) can then be cast in the form

$$\frac{\partial S}{\partial t} - \frac{uM_{\bullet}}{4M_{\bullet}^{1/4} \partial u} S = \frac{3\nu_*}{R_{\bullet}M_{\bullet}} \left( \frac{1}{uM_{\bullet}^{1/4} \partial u} + \frac{1}{M_{\bullet}^{1/4} \partial u^2} \right) \quad (38)$$

One can again look for separated solutions, this time of the form

$$S = U(u) \phi(t), \quad (39)$$

which implies

$$M_{\bullet}^{1/4} \frac{\partial \phi}{\partial \phi} = \frac{\dot{M}_{\bullet}}{4M_{\bullet}^{1/4} U} U' + \frac{3\nu_*}{R_{\bullet}M_{\bullet}^{1/4}} \left[ \frac{1}{u U} + \frac{U''}{U} \right] \quad (40)$$

Evidently the only hope of finding separated solutions is to require

$$\frac{\dot{M}_{\bullet}}{4M_{\bullet}^{3/4}} = A \quad (41)$$

with $A$ a constant. Then

$$\dot{M}_{\bullet} = (At + B)^4 \quad (42)$$

with $B$ also constant. Substituting in (40) gives

$$\left(At + B\right)^{3/4} \frac{\partial \phi}{\partial \phi} = A U' + \frac{3\nu_*}{R_{\bullet}M_{\bullet}^{1/4}} \left[ \frac{1}{u U} + \frac{U''}{U} \right], \quad (43)$$

which requires each side to equal a separation constant, say $\beta$. Thus we find

$$\phi = \phi_0 (At + B)^{3/A}. \quad (44)$$

Now (14) and (42) require $\phi \propto M_{\bullet}^{3/4} \propto (At + B)^3$, which is compatible with (44) only if $\beta = 3A$. The $u$ equation can be rearranged as

$$U'' + \frac{U'}{u} + C'U' - 3CU = 0, \quad (45)$$

where $C = AR_{\bullet}M_{\bullet}^{1/4} / 3\nu_*$. This has a regular singular point at $u = 0$ and an irregular singular point at $u = \infty$, suggesting a relation to the confluent hypergeometric equation. A change of independent variable shows that

$$U = M \left[ -\frac{3}{2}, 1; -\frac{Cu^2}{2} \right], \quad (46)$$

where $M(a, b : u)$ is the confluent hypergeometric (Kummer) function. The full solution is

$$\Sigma(u, t) = \int f(C)(Ct + D)^3 M \left[ -\frac{3}{2}, 1; -\frac{Cu^2}{2} \right] dC, \quad (47)$$

recalling that $C$ is proportional to $A$. We can use the freedom in $f(C)$ to satisfy the boundary condition specifying $\Sigma(u_0, t)$ on the irradiation boundary $u = u_0$, where $T_{\text{irr}} = T_H$.

Although (47) appears complex, its meaning is again straightforward, and similar to that of the expansion (29). The time development $\dot{M}_{\bullet} \propto (At + B)^4$ shows that $A^{-1}$ is a viscous timescale. Since $\dot{M}_{\bullet}$ is decaying, we have approximately

$$\dot{M}_{\bullet} \propto \left[ 1 - \frac{t}{t_{\text{visc}}} \right]^4, \quad (48)$$

where $t_{\text{visc}}$ is the longest viscous time of the partially–irradiated disc. This expression predicts that the outburst of the irradiated disc shuts off completely at $t = t_{\text{visc}}$. In reality the unirradiated disc outside the irradiation boundary $u = u_0$ may produce further outburst activity. Note also that the innermost part of the disc may not be irradiation–dominated, as for small $R$ the effective temperature $T_{\text{eff}}$ is larger than $T_H$ for similar local accretion rates.

Equation (48) thus gives an approximate description of the regime in which the central emission is unable to keep the disc ionized all the way to its outer edge. KR found a linear decay of the central X–rays in this case, which is approximately what is observed (Giles et al., 1996) in the decline of GRO J1744–28, the transient with the longest known orbital period. As can be seen (Fig. 2), eq. (48) gives a rather similar light curve over the typical observed dynamic range. Note in particular that the linear decay of GRO J1744–28 extends only over a dynamic range of about 4 in flux. After this the observed flux is persistently higher than the linear trend, just as in Fig. 2.

4 DISCUSSION

I have shown that analytic solutions for the viscous evolution of an irradiated disc predict a steep power–law decay in the central accretion rate $\dot{M}_{\bullet}$. In practice this is quite close to the exponential decay predicted by the dimensional arguments of KR; this is a good representation of typical observed X–ray light curves. It is important to ask why these solutions are so different from the very shallow $\dot{M}_{\bullet} \propto t^{-(1+\nu)}$ decay of an unirradiated disc.

As pointed out by Cannizzo (1997), the latter is easy to understand. The equation for vertical radiative energy transport within a disc

$$\frac{4\sigma T_m^4}{3k\Sigma} = \frac{9}{8} \frac{GM}{R^3}, \quad (49)$$
where $T_m$ is the midplane temperature can be combined with the alpha–prescription (1) to give
\[ \nu \propto \kappa^{1/3} \Sigma^{2/3} R, \]
where $\kappa$ is the opacity. The factor $\kappa^{1/3}$ generally varies only slowly with $\Sigma$ and $T_m$, so the diffusion equation (9) predicts
\[ \frac{\partial \Sigma}{\partial t} \propto \Sigma^{5/3}, \]
and thus $\Sigma \sim t^{-3/2}$ for a separated solution for an unirradiated disc.

The fundamental difference for an irradiated disc is that, from (8), the viscosity $\nu$ is independent of the local value of $\Sigma$. Thus (51) is replaced by
\[ \frac{\partial \Sigma}{\partial t} \propto \dot{M}_{L}^{5/4}. \]

Were it not for the weak $\dot{M}_{L}^{5/4}$ dependence we would get an exactly exponential decay for $\Sigma(t)$ and thus for $L_x$: this is in any case clearly a good approximation until $\dot{M}_{L}$ has decreased by large factors. Note that this dependence of $\nu$ on disc properties is global, i.e. on the value of $\dot{M}$ or $\Sigma$ at the inner edge of the disc, rather than a local dependence on $\Sigma$ at each radius. The effect is to convert the exponentials into steep power laws since the relation $\dot{M}_{L} \propto \Sigma(R_c)^{1/3}$ implies $\dot{M}_{L} \propto \dot{M}_{L}^{5/4}$ and thus $\dot{M}_{L} \sim t^{-4}$ for separated solutions.

The different viscosity behaviour in an irradiated disc is also responsible for the greater central concentration of the disc mass noted at the end of Section 2. The unirradiated viscosity (50) drops as $\Sigma$ decreases, tending to slow the depletion of the lower surface density of the outer disc regions. By contrast the irradiated viscosity (8) is independent of the local value of $\Sigma$ and increases with $R$, thus driving mass inwards from large radii. The depleted density in the outer regions implies a smaller local accretion rate there: from (8) and the local equivalent of (12) we find
\[ \dot{M}(R,t) = \dot{M}_{e}(t) \left( \frac{R_e}{R} \right)^{1/2} \frac{\cos \frac{\pi}{2} \left( R/R_0 \right)^{1/2}}{\cos \frac{\pi}{2} \left( R_e/R_0 \right)^{1/2}}, \]
so that the local effective temperature goes as $\sim R^{-7/8}$ instead of the usual $\sim R^{-3/2}$.

5 CONCLUSIONS

The presence of a central irradiating point source completely changes the viscous evolution of an accretion disc. Because the viscosity is controlled by conditions in the centre, the disc mass in outburst is more centrally condensed than for unirradiated discs. Most importantly, the decay of the central accretion rate is much steeper, being approximately exponential. I conclude that irradiated discs give a good representation of soft X–ray transient outbursts.

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FIGURE CAPTIONS

Figure 1. Comparison of the exponential light curve $L_x \propto e^{-t/\tau}$ (solid curve) derived by KR, and the best–fitting curve of the form (35) of this paper (dashed curve): this has $L_x \propto 1.05(1 + t/(3.11)^{-4}).$

Figure 2. Comparison of the linear light curve $L_x \propto 1 - t/\tau'$ derived by KR (solid curve), and the best–fitting curve of the form (48) of this paper (dashed curve): this has $L_x \propto 1.073(1 - 2\times 647 \times 1.1)^{4}$. 

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