Quark Mixing and CP Violation

Ahmed Ali
Deutsches Elektronen-Synchrotron DESY
Notkestraße 85, D-22603 Hamburg, FRG

Boris Kayser
Division of Physics, National Science Foundation,
4201 Wilson Boulevard, Arlington, VA 22230, USA

Abstract

We review the historical developments leading to the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Present status of the CKM matrix from direct measurements is summarized, giving also the present profile of the unitarity triangle. CP Violation in the $K^0 - K^0$ complex and in $B$-meson decays are discussed in the context of the CKM matrix.

1 Quark Flavour Mixing

Elementary particles carry many additive attributes (quantum numbers) which are conserved in the strong and electromagnetic interactions. These quantum numbers are called flavours and are used to characterize hadrons (strongly interacting particles). If electromagnetism and strong nuclear forces were the only interactions in nature, there would have been no flavour changing reactions seen in laboratory experiments. However, it has been known since the early days of weak interactions that the neutron is unstable and it decays into a proton by emitting an electron and its associated antineutrino $n \rightarrow p e^- \bar{\nu}_e$ with a mean life of about 15 minutes. On the other hand, to date not a single proton decay has been observed. Laboratory experiments have put the proton lifetime in excess of $10^{32}$ years, which is some 22 orders of magnitude larger than the age of our universe!

In quark language, the two lightest quarks, called up (or $u$), having the fractional electric charge $+2/3$, and down (or $d$), having the fractional electric charge $-1/3$ are at the base of the neutron beta (electron-emitting) decay and the stability of the proton. One can imagine that the $u$ and $d$ quarks form a doublet and the charged weak interaction causes a transition from the heavier $d$ to the lighter $u$ component. Then, a neutron, which consists of two $d$ and one $u$ quarks ($n = ddu$) turns into a proton ($p = uud$), which consists of two $u$ and one $d$ quarks with the charged weak interaction causing the transition $d \rightarrow ue^- \bar{\nu}_e$. The lightest of the quarks, the $u$ quark, is then stable, as ordinary charged weak interactions do not allow the transition of a quark into a lepton. The consequence of this is that a proton, being the lightest known baryon (a hadron with spin 1/2), remains stable. This example shows that charged weak interactions allow transitions between hadrons (or quarks) with different flavour quantum numbers. Here the quark flavours are up and down.
Back in 1933, Enrico Fermi wrote an effective (i.e., low energy) theory of charged weak interactions by introducing an effective coupling constant $G_F$, the Fermi coupling constant. In the example given above, the mean lifetime of the neutron determines the strength of $G_F$ and present day experiments have measured it very precisely: $G_F = 1.166392(2) \times 10^{-5}$ GeV$^{-2}$ in units used by particle physicists in which the reduced Planck constant $\hbar/2\pi$ and the velocity of light are both set to unity. There are other known reactions in which charged weak interactions are at work. An example is the decay of a muon into an electron, a neutrino and an antineutrino, $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$. The decay rate, hence the lifetime of the muon, is also determined by the Fermi coupling constant. For a long time, until experimental precision improved, it was generally accepted that the Fermi coupling constants in neutron beta decay, $G_n$, and in the muon decay, $G_\mu$, were one and the same. However, as the experimental precision improved and theoretical calculations became more sophisticated, by including quantum corrections as well as nucleus-dependent effects in nuclear beta decays, from where most of the information on neutron beta decay comes, it was established that indeed $G_n \neq G_\mu$, though the difference is small. Today, thanks to very precise experiments, this difference is known very precisely: $G_n/G_\mu = 0.9740 \pm 0.001$. So, it seemed that experiments on nuclear beta decay and muon decay required not one but two different Fermi coupling constants.

As the particle zoo enlarged, in particular with the discoveries of hadrons which carry a new quantum number called strangeness, it became clear that the decay rates of these newly discovered weakly decaying particles were different. A successful description of the decay widths (a measure of transition rate) of kaons and hyperons (nucleon-like particles with a non-zero strangeness quantum number) required introduction of effective coupling constants which were very different than either $G_n$ or $G_\mu$. A good example is
the decay of a charged kaon, $K^-$, which was found to decay into a neutral pion $\pi^0$, an electron and an electron-anti neutrino, $K^- \rightarrow \pi^0 e^- \bar{\nu}_e$. In this case, the effective Fermi coupling constant was found to have an empirical value of $G_K/G_\mu \simeq 0.22$. In quark language, this transition is induced by the mutation $s \rightarrow ue^- \bar{\nu}_e$, as the charged kaon has the quark content $K^- = s\bar{u}$ and a neutral pion is built up from the linear combination of the up and down quarks and their antiquarks $\pi^0 = 1/\sqrt{2}(u\bar{u} - d\bar{d})$, reflecting its isospin properties. So, experiments seemed to have implied the existence of at least three different Fermi coupling constants, $G_n$, $G_\mu$ and $G_K$. The question in the theory of weak interactions being asked in the early sixties was: Should one give up the concept of a universal charged weak interaction, as opposed to electromagnetism and the strong nuclear force?

The answer came in the hypothesis of flavour mixing, implying that the quantum eigenstates of the charged weak interactions are rotated in quark flavour space with respect to the mass eigenstates. In other words, the states which have simple charged weak interactions are not the states of definite mass, but linear combinations of them. The concept of ‘rotated’ charged weak currents (involving the $W$ bosons) in the flavour space was introduced by Nicola Cabibbo in 1963, following an earlier suggestion by Murray Gell-Mann and Maurice Levy. It solved the two outstanding problems in weak interactions, explaining the strongly suppressed weak decays of the kaons and hyperons compared to the weak decays of the non-strange light hadrons (containing the $u$ and $d$ quarks), and the difference in the strength of the nuclear $\beta$-decays compared to $\mu$-decay. Calling the Fermi coupling constant in $\mu$-decay $G_F$, the coupling constants in neutron $\beta$-decay and the strange hadron decays in the Cabibbo theory are given by $G_F \cos \theta_C$ and $G_F \sin \theta_C$, respectively. Here, the Cabibbo angle $\theta_C$ is the angle between the weak eigenstate and the mass eigenstate of the quarks. A value $\theta_C \simeq 13$ degrees
describes all data involving weak decays of light hadrons with the same Fermi coupling constant, preserving the universality of weak interactions.

The rates for numerous weak transitions involving light hadrons or leptons are adequately accounted for in the Cabibbo theory in terms of two quantities, $G_F$ and $\theta_C$. This was a great triumph. However, Cabibbo rotation with three light quarks turned out to cause havoc for so-called flavour-changing-neutral-current (FCNC) processes, which in this theory were not in line with their effective measured strengths.

What are these FCNC processes? One example from the Cabibbo epoch characterizes the processes in question. Consider particle-antiparticle mixing involving the neutral kaon ($K^0 - \bar{K^0}$) complex, in which, through a virtual transition, a $K^0(= \pi d)$ meson turns into its charge conjugate antiparticle $\bar{K^0}(= s\bar{d})$. Now, the $K^0$ meson has the strangeness quantum number $S$ equal to +1. The strangeness quantum number of its conjugate antiparticle $\bar{K^0}$ is then $S = -1$. So, in the virtual $K^0 - \bar{K^0}$ transition, the electric charge $Q$ does not change, i.e., in this process $\Delta Q = 0$, but $S$ has changed by two units, i.e., $\Delta S = 2$. Such transitions, and we shall see several counterparts in heavy meson systems, are FCNC processes.

Since the quantum number $S$ is conserved in strong and electromagnetic interactions, the $K^0 - \bar{K^0}$ states can not be mixed by these forces. The charged weak force is the only known force which changes flavours, so it must be at work in inducing the $K^0 - \bar{K^0}$ mixing. Now, it is known that mixing of two degenerate levels must result in level splitting, introducing mass differences between the mass eigenstates, named $K_S$ and $K_L$, being the short-lived and longer-lived of the two mesons. The mass difference $\Delta M_K \equiv M(K_L) - M(K_S)$ in the Cabibbo theory turned out to be several orders of magnitude larger than the observed mass difference, whose present day value is $\Delta M_K \simeq 3.49 \times 10^{-6}$ eV. (As a fraction of $M_K$, the average of
the $K_L$ and $K_S$ masses, this mass difference is only $7.0 \times 10^{-15}$).

This great disparity in the effective strength of the $K^0 - \bar{K}^0$ transition in the Cabibbo theory and experiment remained for a long time a stumbling block in developing a consistent theory of neutral weak currents involving hadrons. For example, it was not at all obvious if the same weak force which causes the decays of the muon, neutron, and the charged kaon discussed above was at work in $K^0 - \bar{K}^0$ transition, or whether a new effective force had to be invented to explain $\Delta M_K$. During this epoch came the seminal papers by Steven Weinberg and Abdus Salam in 1967/68, proposing renormalizable weak interaction models for leptons unifying weak and electromagnetic interactions (see Chapter Rubbia), in which the outstanding problem of FCNC hadronic weak interactions was pushed to one side. It took several years after the advent of this electroweak theory before the FCNC problem was solved elegantly through the ‘GIM’ mechanism, invented by Sheldon Glashow, John Iliopoulos and Luciano Maiani in 1970, using the hypothesis of the fourth (charm or simply c) quark. According to the GIM proposal the charge-changing ($W$-emitting) weak current involving quarks has the form

$$J = \bar{u}d' + \bar{c}s', \quad (1)$$

where $d'$ and $s'$ are rotated (orthogonal) combinations of the $d$ and $s$ quarks which can be described in terms of the Cabibbo angle $\theta_C$ as:

$$d' = d \cos \theta_C + s \sin \theta_C$$

$$s' = -d \sin \theta_C + s \cos \theta_C. \quad (2)$$

The GIM construction of the charge-changing weak current, involving four quark flavours ($u, d, s, c$), removed the leading contribution to the $K_L - K_S$ mass difference. Quantum (loop) effects, such as the ones shown in the box diagram of Fig. 1, with the contribution of the $u$ and $c$ quarks in the intermediate states, give nonzero contributions to the $K^0 - \bar{K}^0$ mass difference.
The result of the box diagrams can be written as (here $m_\mu$ is the mass of the muon)

$$\Delta M_K \simeq \frac{4(m_c^2 - m_u^2) \cos^2 \theta_C}{3\pi m_\mu^2} \Gamma(K^+ \to \mu^+\nu_\mu).$$

(3)

Figure 1: The box diagram contributing to the mass difference $\Delta M_K$ in the GIM theory. In the six quark theory, also the top quark contributes whose contribution is small and hence not shown.

With the other quantities known, $\Delta M_K$ could be predicted in terms of the mass difference of the charm and up quark. This led Benjamin Lee and Mary Gaillard in 1972 to estimate a mass of 1 - 2 GeV for the charm quark in the Cabibbo-GIM four-quark theory. The GIM proposal remained a curiosity until the charm quark was discovered in 1974 by the experimental groups led by Samuel Ting, at the Brookhaven National Laboratory, and Burt Richter, at the Stanford Linear Accelerator SLAC, through the $c\bar{c}$ bound state $J/\psi$ (see Chapter Schwitters), with the charm quark mass compatible with theoretical estimates. Subsequent discoveries of the charmed hadrons $D^0(=c\bar{u}), D^+(=c\bar{d}), D_s(=c\bar{s})$ at SLAC, DESY and elsewhere and their weak decays have confirmed the Cabibbo-GIM current, with again all the decays governed essentially by the parameters $G_F$ and $\theta_C$, thus restoring the universality of charged weak interactions.
We now know that there are not four but six quarks. The charged weak currents then would involve linear combinations of these quarks. We leave the discussion of flavour mixing in the six quark theory to a subsequent section and discuss first another development in kaon decays which had a profound effect on the theoretical developments, namely CP violation.

2 CP Violation in the $K^0 - \bar{K}^0$ Complex

For every elementary particle, there is a corresponding antiparticle. However, a particle and its antiparticle do not always behave in the same way. For example, in the process $\pi^+ \rightarrow \mu^+ + \nu$, in which a positively charged pion decays into a positively charged muon and a neutrino, the muon emerges with its spin vector antiparallel to its momentum. By contrast, in the process $\pi^- \rightarrow \mu^- + \bar{\nu}$, in which every particle in the previous decay has been replaced by its antiparticle, the muon emerges with its spin parallel to its momentum. This difference between the two processes shows that the world is not invariant under charge conjugation $C$, which replaces every particle by its antiparticle. However, one may wonder whether the world is nevertheless unaltered by matter-antimatter interchange in the sense that it is invariant under charge conjugation combined with a parity (space) reflection $P$. Under $P$, particle spins do not change, but their momenta are reversed. Thus, the CP-mirror image of the process $\pi^+ \rightarrow \mu^+ + \nu$ (spin antiparallel to momentum) $+ \nu$ is $\pi^- \rightarrow \mu^- + \bar{\nu}$ (spin parallel to momentum) $+ \bar{\nu}$. Experimentally, the rates for these two processes are equal. Thus, CP invariance holds for $\pi \rightarrow \mu\nu$.

However, it has been discovered that CP does not hold everywhere. There are, as already noted, two neutral $K$ mesons, the short-lived $K_S$ (decaying mainly into two pions) and the longer-lived $K_L$ (decaying mainly into $\pi\nu, \pi\mu\nu$, or three pions). If CP invariance held, $K_S$ and $K_L$ would each be its
own CP-mirror image. Thus, the CP-mirror image of the decay $K_L \to \pi^- e^+ \nu$ would be the decay $K_L \to \pi^+ e^- \bar{\nu}$, with all the momenta in the first decay reversed. If we ask about the rates for these two decays integrated over all possible directions of the outgoing particles, the momentum reversals become irrelevant, and CP invariance would require that the two rates be equal. However, these rates differ by 0.3%. Thus, the world is noninvariant, not only under $C$, but under CP as well. Noninvariance under the symmetry operation CP is accompanied by nonconservation of the associated CP quantum number, and the first observation that either of these phenomena occurred was the discovery in 1964 that the CP quantum number is not conserved in the decays of neutral $K$ mesons to pairs of pions. The CP quantum number of a system, referred to as its CP parity, can be either $+1$ or $-1$. If CP were conserved, $K_S$ and $K_L$ would be CP eigenstates with opposite CP parity. The CP parity of the pion pair $\pi^+ \pi^-$ (the dominant decay of the $K_s$) is even. However, in 1964 it was discovered by J. Christenson, James Cronin, Val Fitch, and René Turlay that the process $K_L \to \pi^+ \pi^-$ also occurs. That is, both $K_S$ and $K_L$, one of which would have CP = $-1$ if CP were conserved, decay to $\pi^+ \pi^-$, which has CP = +1. Thus CP is not conserved in neutral $K$ meson decays, although the observed nonconservation is small: the ratio of the CP-violating amplitude to the CP-conserving one, $|\text{Amp}(K_L \to \pi^+ \pi^-)/\text{Amp}(K_S \to \pi^+ \pi^-)|$, is only $2.3 \times 10^{-3}$. However, much larger effects may be revealed in the future, as we shall see.

Now, CP violation has so far been seen only in the decays of neutral $K$ mesons. Thus, this violation could perhaps be a feature of $K^0 - \bar{K}^0$ mixing, rather than of particle decay amplitudes. Then there would be no CP violation in the decays of charged $K$ mesons. (The charged $K$ mesons, $K^+$ and $K^-$, certainly do not mix, because the conversion of one of them into the other would violate charge conservation.) Several very challenging experi-
mental efforts are being made to see whether decay amplitudes do violate CP. So far, the results are inconclusive. One experiment finds that the quantity \( \epsilon'_{K}/\epsilon_{K} \), whose nonvanishing would signal that the decay amplitudes violate CP, is \((23.0 \pm 6.2) \times 10^{-4}\), but another finds that it is \((7.4 \pm 5.9) \times 10^{-4}\), consistent with zero. The experiments continue.

Regardless of the value of \( \epsilon'_{K}/\epsilon_{K} \), the fact that nature violates CP invariance and CP conservation has been established. What is the origin of this CP violation? In addressing this question, we note that, as remarked earlier, CP-violating effects have thus far been observed only in the decays of neutral \( K \) mesons. These decays are known to be due to the weak interaction. Thus, it is natural to ask whether CP violation is also due to the weak interaction, rather than to some so-far unknown, mysterious force.

Among the discrete symmetries \( C, CP, T, \) and \( CPT \), the CPT symmetry is considered to be exact as it follows from fundamental principles underlying all field theories, namely positivity of the norm and locality (a particle is represented by a local field). Lately, the invariance of natural laws under CPT has been put in question in the context of the superstring theories of particle physics, in which the particles are described by extended objects in space-time, such as a string, lifting the assumption of locality (point-like nature) ascribed to the particles in field theories (see Chapter Ross). However, even if CPT invariance should prove to be broken in superstring theory, the effects on the flavour physics being discussed in this Chapter would very likely be negligible. Therefore, in what follows, we shall assume that CPT holds exactly. The CPT-invariance principle has a number of implications, such as the equality of the masses and of the lifetimes of a particle and its antiparticle. The best limit on CPT violation stems from the upper limit of the ratio of mass difference to the mass, \( m(K^{0}) - m(\bar{K}^{0})/m(K^{0}) \leq 10^{-18} \).

If CP violation is indeed produced by the known weak interaction de-
scribed by the Standard Model, then it is caused by complex phase factors in the quark mixing matrix. How these complex phases produce physical CP-violating effects will be explained shortly.

3 The Cabibbo-Kobayashi-Maskawa Matrix

We now know that there are six quarks in nature. The fifth and the sixth quarks are called the beauty (or simply $b$) and top (or simply $t$) quarks. The $b$ quark was discovered in the form of its bound state $\Upsilon = (b\bar{b})$ and excited states $\Upsilon', \ldots$ by the group of Leon Lederman working at Fermilab in 1977. The discovery of the top quark had to wait until 1994 when two large experimental groups (D0 and CDF) working again at Fermilab finally discovered the top quark in the process $p\bar{p} \rightarrow t\bar{t}X$ and the subsequent decays of the top quarks $t \rightarrow bW^+$ (see Chapter Shochet). However, indirect evidence of a top quark with a rather large mass, $m_t = O(100)$ GeV, was found earlier from $B^0 - \bar{B}^0$ mixing by the UA1 experiment at CERN’s proton-antiproton collider (see Chapter Rubbia), the ARGUS experiment at DESY’s DORIS electron-positron collider, and the CLEO experiment at the Cornell electron-positron collider CESR. The expectation that $m_t$ is large was further strengthened by electroweak precision measurements at CERN’s LEP. No top meson has so far been constructed from its decay product, but there exists an impressive amount of data on the properties of the beauty hadrons from experiments at DORIS (DESY), CESR (Cornell), LEP (CERN), SLAC (Stanford) and Fermilab.

Given that there are six quarks, arranged in terms of three “weak isospin” doublets $(u, d; s, c; t, b)$, the obvious questions are: How are the weak interaction eigenstates involving six quarks related to the quark mass eigenstates? And what does this rotation imply for the decays of the light (containing only
$u, d, s$ quarks) and heavy hadrons (containing $c, b, t$ quarks)?

The answers follow if the two-dimensional rotation of Eq. (2) is now replaced by a three-dimensional one, where the $W$-emitting weak current takes the form

$$J = \bar{u}d' + \bar{c}s' + \bar{t}b' = (\bar{u}, \bar{c}, \bar{t}) V \begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$  \hspace{1cm} (4)

Here $V$ is a $3 \times 3$ matrix in the quark flavour space and can be symbolically written as:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$ \hspace{1cm} (5)

Thus, in this case the weak (interaction) eigenstate $d'$ is: $d' \equiv V_{ud}d + V_{us}s + V_{ub}b$. (Likewise, the other rotated states $s'$ and $b'$ follow from eq. (5).) Every weak process involving the $W$ boson is proportional to some product of the elements of $V$. Now comes a crucial observation: if some of the elements of the $3 \times 3$ matrix $V$ are not real but complex (so that $V$ is not, strictly speaking, a rotation matrix, but a “unitary” one), then the hadronic weak interactions can violate CP. However, if there were only four quarks, so that we did not have a $3 \times 3$ quark mixing matrix but only the two-dimensional rotation of Eq. (2) of the Cabibbo-GIM theory, then making the coefficients in that rotation complex would not lead to any physical effects, as in this case these complex phases in the $(2 \times 2)$ rotation matrix can be eliminated by a redefinition of the quark fields. Not so, if there are six or more quarks.

These facts were first pointed out by M. Kobayashi and T. Maskawa in 1972, long before the $c, b,$ and $t$ quarks were actually discovered. In fact, the GIM mechanism, put forward to describe FCNC transitions in the $K^0 - \bar{K}^0$ complex, was immediately followed by the KM hypothesis to accommodate
CP violation in the same \( K^0 - \bar{K^0} \) system and which predicted the existence of all the three heavy quarks.

Now that all these quarks have been discovered, the six quark theory of Kobayashi and Maskawa stands on firm experimental ground - as far as its quark content is concerned. The crucial question now is whether the complex phases in the matrix \( V \) are the only source of CP violation in flavour physics. These phases predict CP-violating phenomena in the decay amplitudes (also called direct CP violation) of many hadrons. They also predict indirect CP violation, which resides in the mass matrix of a neutral meson complex, \( M^0 - \bar{M^0} \), in particular \( K^0 - \bar{K^0} \), and can manifest itself only when such mixings are involved. It is widely appreciated that \( B \) physics has the potential of providing crucial tests of the KM paradigm. The \( 3 \times 3 \) flavour mixing matrix, which is now aptly called the Cabibbo-Kobayashi-Maskawa (CKM) matrix, plays a central role in quantifying CP-violating asymmetries.

### 3.1 Present status of the CKM Matrix

The magnitudes of all nine elements of the CKM matrix have now been measured in weak decays of the relevant quarks, and in some cases in deep inelastic neutrino nucleon scattering. The precision on these matrix elements varies for each entry, reflecting both the present experimental limitations but often also theoretical uncertainties associated with the imprecise knowledge of the hadronic quantities required for the analysis of data. In most cases, the decaying particle is a hadron and not a quark and one has to develop a prescription for transcribing the simple quark language to that involving hadrons. Here, the theory of strong interactions, Quantum Chromodynamics (QCD), comes to the rescue. Powerful calculational techniques of QCD, in particular renormalization group methods, lattice-QCD, QCD sum rules and effective heavy quark theory, have been used to estimate, and in some
cases even determine, the hadronic quantities of interest, thereby reducing theoretical errors on the CKM matrix elements.

This theoretical development is very impressive though the QCD technology has not quite reached its goal of achieving an accuracy of a few percent in the determination of the hadronic matrix elements. Nevertheless, it has been crucial in quantifying the CKM matrix elements. Fascinating as these calculational aspects are, their discussion here would take us far from our mainstream interest and we refer to the suggested literature for further reading.

Present knowledge of $V$ comes from a variety of different sources and the present status can be summarized as follows:

$$
|V| = \begin{pmatrix}
0.9730 - 0.9750 & 0.2173 - 0.2219 & 0.0023 - 0.0040 \\
0.208 - 0.24 & 1.20 - 0.88 & 0.038 - 0.041 \\
0.0065 - 0.0102 & 0.026 - 0.040 & 1.14 - 0.84 \\
\end{pmatrix}. \quad (6)
$$

The following comments about the entries are in order:

1. $|V_{ud}|$: This is based on comparing nuclear beta decays $(A,Z) \to (A,Z+1) + e^- + \bar{\nu}_e$ that proceed through a conserved vector current to muon decay $\mu^- \to \nu_\mu e^- \bar{\nu}_e$. In the three-quark Cabibbo theory, this matrix element was identified with $\cos \theta_C$.

2. $|V_{us}|$: This is based on the analyses of the decays $K^+ \to \pi^0 \ell^+ \nu_\ell$ and $K^0 \to \pi^- \ell^+ \nu_\ell$ and beta decays of the hyperons. In the Cabibbo theory, this matrix element was identified with $\sin \theta_C$.

3. $|V_{cd}|$: This is derived from the neutrino and antineutrino production of charm quarks from $d$ quarks in a nucleon in deep inelastic neutrino nucleon scattering experiments, $\nu_\ell + d \to \ell^- + c$. In the GIM-Cabibbo current, this matrix element is $\sin \theta_C$.

4. $|V_{cs}|$: This comes from the semileptonic decays of the charmed hadrons $D^\pm$ and $D^0$, involving for example the decay $D^\pm \to K^0 \ell^\pm \nu_\ell$. Again, in the
GIM-Cabibbo current, this matrix element is identified with $\cos \theta_C$.

(5) $|V_{cb}|$: From the semileptonic decays of $B$ hadrons, such as $\overline{B}^0 \rightarrow D^{*+} + \ell^- + \nu_\ell$, or the inclusive decay of the $b$ quark $b \rightarrow c + \ell^- + \nu_\ell$.

(6) $|V_{ub}|$: Obtained from the semileptonic decays of $B$ hadrons into non-charmed hadrons, such as $\overline{B}^0 \rightarrow \pi^+ + \ell^- + \nu_\ell$, or the inclusive semileptonic decays of a $b$ quark into a non-charm quark $b \rightarrow u + \ell^- + \nu_\ell$.

(7) $|V_{td}|$: From the measured mass difference between the mass eigenstates in the $B^0 - \overline{B}^0$ meson complex. Being an example of a FCNC process, this transition is a quantum effect and in the Standard Model takes place through a box diagram very similar to the one shown in Fig. 1 for the $K^0 - \overline{K}^0$ system, except that in this case the transition amplitude is dominated by the top quark due to its very large mass ($m_t \simeq 175$ GeV).

(8) $|V_{ts}|$: From the measured branching ratio of the electromagnetic process $b \rightarrow s + \gamma$, measured by the CLEO experiment at CESR (Cornell) and recently also by the ALEPH collaboration at CERN. Again, an example of a FCNC process, this is also a quantum effect and again in the Standard Model the transition rate is dominated by the top quark.

(9) $|V_{tb}|$: From the production and decay of the top quark in the process $p\overline{p} \rightarrow t\overline{t} + X$ followed by the decay $t \rightarrow b + W^+$.

One sees that present knowledge of the matrix elements in the third row of the CKM matrix involving the top quark in eq. (6), but also of the matrix elements $V_{cs}$, $V_{cd}$ and $V_{ub}$ is still rather imprecise. A check of the unitarity of the CKM matrix from the entries in eq. (6) makes this quantitatively clear. Unitarity requires, among other things, that the absolute squares of the elements in any row of the CKM matrix add up to unity. We have at present

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.997 \pm 0.002,$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.18 \pm 0.33,$$
This shows that except for the first row, the information on the unitarity of the CKM matrix is very imprecise. However, all data, within errors, are consistent with the CKM matrix being unitary.

3.2 Unitarity Triangles

The unitarity of the CKM matrix also requires that any pair of rows or any pair of columns of this matrix be orthogonal. This leads to six orthogonality conditions. These can be depicted as triangles in the complex plane of the CKM parameter space. The constraint stemming from the orthogonality condition on the first and third column of $V$, 

$$V_{ud}^*V_{ub} + V_{cd}^*V_{cb} + V_{td}^*V_{tb} = 0$$

is at the centre of contemporary theoretical and experimental attention. Since present measurements are consistent with $V_{ud} \simeq 1$, $V_{tb} \simeq 1$ and $V_{cd} \simeq -\lambda$, where $\lambda = \sin \theta_C$, the unitarity relation (8) simplifies to:

$$V_{ub}^* + V_{td}^* \simeq -V_{cd}^*V_{cb}^* \simeq +\lambda V_{cb}^*$$

which can be conveniently depicted as a triangle relation in the complex plane, as shown in Fig. 2. In drawing this triangle, we have used a representation of the CKM matrix due to Wolfenstein, characterized by four constants $A, \lambda, \rho$ and $\eta$. We have also rescaled the sides of the triangle by $\lambda V_{cb}$, which makes the base of the triangle real and of unit length and the apex of the triangle given by the point $(\rho, \eta)$ in the complex plane. This is usually called the unitarity triangle (UT). Knowing the sides of the UT, the three angles of this triangle $\alpha, \beta$ and $\gamma$ are determined. But these angles can also, in principle, be measured directly through observation of CP violation.
Figure 2: The unitarity triangle. The angles $\alpha$, $\beta$ and $\gamma$ can be measured via CP violation in the $B$ system, and the sides from the rates for various CC- and FCNC-induced $B$ decays.

in various $B$ decays. By measuring both the sides and the angles, the UT will be overconstrained, which is one of the principal goals of the current and forthcoming experiments in flavour physics.

In the Wolfenstein representation,

$$\tan(\alpha) = \frac{\eta}{\sqrt{\rho^2 - \rho (1 - \rho)}} , \quad \tan(\beta) = \frac{\eta}{1 - \rho} , \quad \tan(\gamma) = \frac{\eta}{\rho} . \quad (10)$$

A profile of the UT based on our present knowledge of the CKM matrix is now given from which the CP-violating asymmetries which will be measured in forthcoming experiments in B Physics can be estimated. For this, the present experimental input can be summarized as follows:

$$\sqrt{\rho^2 + \eta^2} = 0.363 \pm 0.073 ,$$

$$(f_{B_d}\sqrt{\hat{B}_{B_d}/1 \text{ GeV}})\sqrt{(1 - \rho)^2 + \eta^2} = 0.202 \pm 0.017 ,$$

$$\hat{B}_K\eta[0.93 + (2.08 \pm 0.34)(1 - \rho)] = 0.79 \pm 0.11 , \quad (11)$$
which come from the measurements of the CKM matrix element ratio $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$, the mass difference induced by the $B^0 - \bar{B}^0$ mixing, which is measured very accurately, $\Delta M_d = (3.12 \pm 0.20) \times 10^{-4}$ eV, and the CP-violating parameter in the $K^0 - \bar{K}^0$ system, $\epsilon_K = (2.28 \pm 0.013) \times 10^{-3}$, which is likewise known very precisely. The quantities $f_{B_d}$, $\hat{B}_{B_d}$ and $\hat{B}_K$ are various hadronic quantities whose knowledge is needed to analyze data. Present estimates, based mostly on lattice QCD calculations, put them in the range $f_{B_d}\sqrt{\hat{B}_{B_d}} = 200 \pm 40$ MeV and $\hat{B}_K = 0.75 \pm 0.10$. The resulting allowed regions in the $(\rho, \eta)$ parameter space from each of these constraints individually and the resulting overlap region from all the constraints put together are shown in Fig. 3. The triangle drawn is to guide the eye and represents the presently preferred solution. Two messages are clear: First, current theoretical uncertainties in hadronic quantities translate into rather large uncertainties in the profile of the unitarity triangle. Second, and despite this, a good part of the allowed parameter space is now ruled out by data and the CKM matrix provides a consistent solution only over a limited parameter space.

3.3 CP Violation in $B$ Decays

The paramount interest in $B$ physics lies in that it will test the CKM paradigm of CP violation in flavour-changing weak interactions. The $B$ mesons can decay in many different ways, and a large number of their decay modes are potentially interesting from the point of view of observable CP-violating effects. In some of the decay modes, these effects can yield clean information, free of theoretical uncertainties, on the angles in the unitarity triangle of Fig. 2. Since these angles are just the relative phases of various combinations of CKM elements, the clean information on the angles will stringently test the hypothesis that CKM phases cause CP violation.
Figure 3: Allowed region in $\rho$-$\eta$ space obtained by overlaying the individual constraints following from $|V_{ub}/V_{cb}|$ (dashed curves), $\epsilon_K$ (solid curves), and $\Delta M_d$ (dotted curves), by letting the hadronic quantities vary in the range shown above. The 95% C.L. contour resulting from a simultaneous fit of the data is also shown ("Haggis"-type curve). The triangle shows the best fit.
The decay modes which can provide clean information on the angles include
the decays of neutral $B$ mesons to final states which are CP eigenstates or
at least can come from both a pure $B^0$ and a pure $\bar{B}^0$, and certain decays
of charged $B$ mesons. The decays of neutral $B$ mesons to CP eigenstates
provide a particularly pretty example of how CP violation comes about, and
how the phases of CKM matrix elements can be determined. In any decay,
for CP violation to be non-zero, there must be interfering amplitudes with
clashing phases. Now, in neutral $B$ decay to a CP eigenstate $f_{CP}$, there
are two routes to the final state. If the parent $B$ was born as a $B^0$, it may
(1) decay directly to $f_{CP}$, or else it may (2) turn via weak mixing into a
$\bar{B}^0$, and then this $\bar{B}^0$ decays to $f_{CP}$. The amplitudes for these two routes
must be added coherently, and will interfere. If the parent $B$ was born as
a $\bar{B}^0$, decay to $f_{CP}$ can again proceed through two routes: $\bar{B}^0 \rightarrow f_{CP}$, and
$\bar{B}^0 \rightarrow B^0 \rightarrow f_{CP}$. As before, the amplitudes for these two routes will inter-
fere. If the CKM matrix elements have complex phases, then these (weak)
amplitudes will have different phases than when the $B$ was born as a $B^0$. As
a result, the interferences encountered in $(B$ born as $B^0) \rightarrow f_{CP}$ and $(B$ born
as $\bar{B}^0) \rightarrow f_{CP}$ will differ, and consequently the rates for these two decays
will differ as well. Since the two decays are CP-mirror-image processes, the
difference between their rates is a violation of CP.

Since the rates $\Gamma[B$ born as $\bar{B}^0 \rightarrow f_{CP}$ after time $t]$ $\equiv (\bar{\Gamma}(t)$ depend
nontrivially on the time $t$ that the $B$ lives before decaying, experiments will
study the time-dependent CP-violating asymmetry

$$a_{f_{CP}}(t) \equiv \frac{\Gamma(t) - \bar{\Gamma}(t)}{\Gamma(t) + \bar{\Gamma}(t)} . \quad (12)$$

When the unmixed $B$ decays, $B^0 \rightarrow f_{CP}$ and $\bar{B}^0 \rightarrow f_{CP}$, are each dominated
by one diagram, $a_{f_{CP}}(t)$ is given by the simple expression

$$a_{f_{CP}}(t) = \eta_{f_{CP}} \sin(\phi_{f_{CP}}) \sin(\Delta M t) . \quad (13)$$

19
Here, $\eta_{fCP} = \pm 1$ is the CP parity of the final state, $\Delta M$ is the mass difference between the two mass eigenstates of the $B^0 - \bar{B}^0$ system, and $\phi_{fCP}$ is the phase of a certain product of CKM elements. Namely, $\phi_{fCP}$ is the relative phase of the product of CKM elements to which the amplitude for $B^0 \rightarrow f_{CP}$ is proportional, and the product to which the amplitude for the alternate decay route, $B^0 \rightarrow B^0 \rightarrow f_{CP}$, is proportional. Of course, the identity of $\phi_{fCP}$ depends on the choice of $f_{CP}$.

There are two neutral $B$ systems: $B^0_d(\bar{b}d)$ and its antiparticle, and $B^0_s(\bar{b}s)$ and its antiparticle. For the $B^0_d - \bar{B}^0_d$ system, the mass splitting $\Delta M_d$ between the mass eigenstates is already known, as previously mentioned. For the $B^0_s - \bar{B}^0_s$ system, the analogous splitting $\Delta M_s$ will no doubt eventually be determined as well. Thus, the $\Delta M$ in Eq. (13) for the CP asymmetry may be assumed known. For any chosen final state, the CP parity $\eta_{fCP}$ is also known. Thus, we see from Eq. (13) that once the asymmetry $a_{fCP}(t)$ is measured, $\sin(\phi_{fCP})$ is cleanly determined, with no theoretical uncertainties. This makes it possible to cleanly test whether complex phases of CKM matrix elements are indeed the origin of CP violation.

As an example, suppose $f_{CP}$ is $J/\psi K_S$. In the decays ($B$ born as $\bar{B}^0_d$) $\rightarrow J/\psi K_S$, each of the unmixed $B$ decays, $B^0_d \rightarrow J/\psi K_S$ and $\bar{B}^0_d \rightarrow J/\psi K_S$, is expected to be dominated by one diagram. Thus, Eq. (13) for $a_{fCP}(t)$ should hold. The dominating diagrams are such that for this final state, $\phi_{fCP}$ is simply $2\beta$, where $\beta$ is one of the angles in the unitarity triangle of Fig. 2. Thus the decays ($B$ born as $\bar{B}^0_d$) $\rightarrow J/\psi K_S$ can give us clean information on $\beta$. It appears that obtaining information on the other angles in the UT will be more difficult, but should still be possible. A major experimental effort will be made to determine all the angles of the UT.

How large are the CP-violating asymmetries in $B$ decays? They depend in part on the mass-mixing related quantities $x_d \equiv \Delta M_d \cdot \tau(B_d)$ for the $B^0_d - \bar{B}^0_d$
system, which is well measured with \( x_d \simeq 0.74 \), and on \( x_s \equiv \Delta M_s \cdot \tau(B_s) \) for the \( B^0_s - \bar{B}^0_s \) system, for which experiments at LEP (CERN) have provided only lower limits \( x_s \geq 16 \). Here, \( \tau(B_d)(\tau(B_s)) \) is the lifetime of the \( B^0_d(B^0_s) \) meson. But, the CP asymmetries depend crucially on the angles of the UT, which can be estimated from the unitarity fits. With the help of the relations given in eqs. (10), the CP-violating asymmetries in \( B \) decays can be expressed straightforwardly in terms of the CKM parameters \( \rho \) and \( \eta \). The constraints on \( \rho \) and \( \eta \) discussed above can then be used to predict the correlated ranges of the angles \( \alpha, \beta \) and \( \gamma \) in the Standard Model. Representative of the current theoretical expectations are the following ranges for the CP-violating rate asymmetries parametrized by \( \sin 2\alpha, \sin 2\beta \) and \( \sin^2 \gamma \), which are estimated by Ali & London in the context of the Standard Model at the end of 1997:

\[
-1.0 \leq \sin 2\alpha \leq 1.0, \\
0.30 \leq \sin 2\beta \leq 0.88, \\
0.27 \leq \sin^2 \gamma \leq 1.0,
\]

with all ranges corresponding to 95% C.L. (i.e., \( \pm 2\sigma \)). The currently preferred solutions of the unitarity fits yield: \( \rho \simeq 0.12 \) and \( \eta \simeq 0.34 \), which then translate into \( \alpha \simeq 88^\circ \), \( \beta \simeq 21^\circ \) and \( \gamma \simeq 72^\circ \). The central values of the parameters which determine the asymmetries are then: \( \sin 2\alpha \simeq 0.07 \), \( \sin 2\beta \simeq 0.67 \) and \( \sin^2 \gamma \simeq 0.89 \). These parameters will be measured in decays such as \( (B \text{ born as } \bar{B}^0_d) \to J/\psi K_S \), where the CP-violating asymmetry is proportional to \( \sin 2\beta \), \( (B \text{ born as } \bar{B}^0_d) \to \pi^+ \pi^- \), which can determine \( \sin 2\alpha \), and \( (B \text{ born as } \bar{B}^0_s) \to D^\pm_s K^\mp \) or \( B^\pm \to DK^\pm \), which can yield \( \sin^2 \gamma \). The actual asymmetries in the partial rates are expected to be quite large in some of these decays, which will make them easier to measure in the next round of \( B \) physics experiments.

Additional decay modes which appear to be promising places to study CP
violation include $B^\pm \rightarrow \pi^\pm K$, ($B$ born as $B_d^0 \rightarrow \pi^\pm K^\mp$, $B^\pm \rightarrow \pi^\pm \eta'$, ($B$ born as $B_d^0 \rightarrow K_S \eta'$, ($B$ born as $B_d^0 \rightarrow D(\ast)^{\pm} \pi^\mp$, ($B$ born as $B_d^0 \rightarrow K^0 \bar{K}^0$, and many others. Moreover, one expects measurable CP violation in the inclusive radiative decays such as $B \rightarrow X_d + \gamma$, where $X_d$ is a system of light, non-strange, hadrons, and in exclusive radiative decays such as $B \rightarrow \rho + \gamma$, which are governed by the FCNC process $b \rightarrow d + \gamma$. These processes are similar to the observed decays $B \rightarrow K^* + \gamma$ and $B \rightarrow X_s + \gamma$, but are suppressed by about a factor 20. Measurements of CP asymmetries in these processes do not directly determine the angles of the unitarity triangle. However, they all depend on the parameters $\rho$ and $\eta$ and hence their measurement will contribute to determine the UT more precisely, and to the understanding of CP violation. However, most of these measurements will require sufficiently many $B$ hadrons that they will probably have to await the second round of experiments in $B$ factories at SLAC (Stanford), KEK (Japan) and CESR (Cornell).

Apart from the CP violation measurements discussed above, some of the anticipated landmark measurements in $B$ physics include: (1) Determination of the mass splitting $\Delta M_s$ in the $B_s^0 - \bar{B}_s^0$ complex, (2) Rare $B$ decays, such as $b \rightarrow d + \gamma$, $B \rightarrow \rho^0(\omega) + \gamma$, $b \rightarrow s\ell^+\ell^-$, $b \rightarrow d\ell^+\ell^-$ - all examples of FCNC processes, which have been the driving force behind theoretical developments in flavour physics.

Likewise, several planned and ongoing experiments in $K$ physics will measure rare decays such as $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^\pm \rightarrow \pi^\pm \nu \bar{\nu}$, and the CP-violating ratio $\epsilon_K^/'/\epsilon_K$. These important $K$-system measurements will complement the $B$-system experiments, and will help us to determine the properties of the unitarity triangle and to explore the origin of CP violation.
4 Concluding remarks

The elegant synthesis of seemingly diverse, and in their effective strengths widely differing, empirical observations involving weak interactions in terms of a universal constant $G_F$ and a $3 \times 3$ unitary matrix is one of the great simplifications in elementary particle physics. All data on weak interactions can at present be analyzed and understood in terms of a few universal constants, and the consistency of the picture is indeed remarkable. With improved theoretical and experimental precision, this consistency will provide in the future one of the most promising search strategies for finding physics beyond the six quark Standard Model of particle physics. A good candidate in that context is supersymmetry which may contribute to many of the FCNC processes discussed here but whose anticipated effects are quite subtle and their detection would require high precision data (see Chapter Ross).

Despite this success, there are many discomforting features which deserve attention. It must be stressed that the parameter $\epsilon_K$, which describes CP violation in $K^0 - \bar{K}^0$ mixing, and whose first measurement dates back some 35 years, still remains the only source of information on CP violation in laboratory experiments. This state of affairs is deeply disturbing, in particular as CP violation has a direct bearing on a fundamental phenomenon in nature, namely the observed large-scale preponderance of matter over antimatter in the universe. The next round of experiments in $B$ (and $K$) physics will certainly help fill in some of the numerous blanks. At a deeper level, however, the connection between complex phases in the CKM matrix and the observed matter-antimatter asymmetry in the universe remains very much a matter of speculation. It is conceivable that fundamental progress here may come from completely different quarters, such as observation of CP violation in the lepton sector and the understanding of baryo-genesis at the grand unification scale – all aspects not directly related to the flavour physics of quarks
We started this article with the discussion of the Fermi theory postulated some sixty five years ago. The physics behind the effective Fermi coupling constant, $G_F$, has come to be understood in terms of a fundamental gauge interaction. The question is: Are the elements of the CKM unitary matrix also some kind of effective parameters, which some day one would be able to derive in terms of more fundamental quantities? Some ideas along these lines are being pursued enthusiastically in grand theoretical schemes where the CKM matrix elements are derived in terms of quark masses. As theoretical and experimental precision on the CKM matrix improves, many of these relations will come under sharp experimental scrutiny. The emerging pattern will help us to discard misleading theories, and perhaps single out a definitive and unique theoretical perspective. The flavour problem - understanding the physics behind the parameters of the CKM matrix which seem to describe all flavour interactions at present energies consistently - remains one of the most challenging problems of particle physics.

**Bibliography:**


