Jeans instability of interstellar gas clouds in the background of weakly interacting massive particles

David Tsiklauri

Physics Department, University of Cape Town, Rondebosch 7701, South Africa

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1email: tsiklauri@physci.uct.ac.za, http://pc021.phy.uct.ac.za/tsiklauri/
ABSTRACT

Criterion of the Jeans instability of interstellar gas clouds which are gravitationally coupled with weakly interacting massive particles is revisited. It is established that presence of the dark matter always reduces the Jeans length, and in turn, Jeans mass of the interstellar gas clouds. Astrophysical implications of this effect are discussed.

Subject headings: dark matter — gravitation — instabilities — ISM: clouds — ISM: kinematics and dynamics — stars: formation

Investigation of physical processes in clouds of interstellar gas is important for a number of reasons. There is a strong evidence that the formation of stars occurs through gravitational collapse of the interstellar gas clouds. An important mechanism in triggering the formation of stars and/or stellar clusters is believed to be a high-velocity (supersonic) cloud-cloud collisions in which a dense gaseous slab is formed with two plane-parallel shock fronts propagating away from the interface (Usami et al. 1995). Then the slab grows in mass becoming unstable against gravitational instability which causes its fragmentation. The fragments or the gaseous clumps, in turn, collapse further and evolve into stars and/or stellar clusters. Yet another mechanism for triggering of star formation is Rayleigh-Parker instability which leads to an increase in the curvature of the magnetic field lines that initially were directed parallel to the galactic plane. The interstellar gas clouds move along the field lines (due to the “frozen in” condition) and fall into the “wells” of the field. This causes further stretching of the magnetic field lines, making “wells” deeper, thus large amounts of gas can fall into the “wells” creating large concentrations of gas which then becomes gravitationally unstable (Gorbatskii, 1977).

In this letter we consider stability of the interstellar gas clouds against Jeans instability in the presence of background weakly interacting massive particles (WIMP). We find that presence of WIMP matter yields an unavoidable reduction of Jeans length, Jeans mass and Jeans time (time-scale of the collapse via Jeans instability).

Existence of WIMP matter, one of the possible form of dark matter, the latter itself a dominant mass component of the universe, is strongly motivated both by standard models of particle physics and cosmology. Generally speaking, mass density associated with the luminous matter (stars, hydrogen clouds, x-ray gas in clusters, etc.) cannot account for the observed dynamics on galactic scales and above, (Trimble,
1987), thus, revealing the existence of large amounts of dark matter or otherwise pointing to a breakdown of Newtonian dynamics (Milgrom, 1994, 1995) or the conventional law of gravity (Mannheim, 1995). The role of dark matter could be played by anything from novel weakly interacting elementary particles to normal matter in some invisible form (brown dwarfs, primordial black holes, cold molecular gas, etc.). A global, homogeneously distributed dark matter component can be provided by the vacuum in the form of a cosmological constant (see Raffelt, 1995 for a review). Therefore, a revision of classical theory of Jeans instability in the presence of WIMP matter, or generally speaking any type of microscopic dark matter which is coupled with an interstellar gas cloud only via gravitational interaction, seems to be of a considerable importance, especially in the context of star and/or stellar cluster formation.

The low-surface brightness dwarf spiral galaxy DDO 154 has one of the most extended and best studied dark matter halo rotation curves (Carignan & Freeman, 1988; Carignan & Beaulieu, 1989; Burkert & Silk, 1997), with a precisely known contributions from stars, gas and dark matter. DDO 154 is one of most gas-rich systems known, but what is more important, the shape of its rotation curve is completely dominated by the dark matter even in the innermost regions. Thus, the galaxies like DDO 154 where gas dynamics is governed mostly by the dark matter gravitational potential are the best examples where novel effects revealed in this letter would be pronounced at most. However, because of their relative proximity, detailed observations of interstellar gas clouds are possible only for our galaxy and one has to extrapolate this knowledge on other galaxies, because there is no reason to believe that our own galaxy is exceptional in any respect. Moreover, our own galaxy is known to have a dark matter halo, which makes results presented here relevant for the description of physical processes in the interstellar gas clouds in the Milky Way (see discussion below).

Equations that govern dynamics of two self-gravitating fluids (an interstellar gas cloud and WIMP matter) inter-coupled only via gravitational interaction can be written in the following way:

$$\frac{\partial \rho_i}{\partial t} + \nabla (\rho_i \vec{V}_i) = 0,$$

$$\frac{\partial \vec{V}_i}{\partial t} + (\vec{V}_i \cdot \nabla) \vec{V}_i = -\nabla \phi_i - \frac{\nabla P_i}{\rho_i},$$

$$P_i = K_i \rho_i^{\gamma_i},$$

$$\Delta \phi_i = 4\pi G \sum \rho_i,$$

where notation is standard: $\vec{V}$, $P$, $\rho$ and $\phi$ denote velocity, pressure, mass density and gravitational potential of the fluids. The subscript $i = g, w$ denotes two components, interstellar gas and WIMP.
matter respectively. Now, writing every physical quantity, for brevity commonly denoted by \( \vec{f} \), in a form of \( \vec{f} = \vec{f}^0 + \vec{f}' \exp[i(\vec{k} \cdot \vec{r} - \omega t)] \) and doing usual linearization of the Eqs.(1)–(4) we can obtain following dispersion relation for the perturbations

\[
\omega^2 = \frac{1}{2} \left[ (\omega_g^J)^2 + (\omega_w^J)^2 \pm \sqrt{[(\omega_g^J)^2 - (\omega_w^J)^2]^2 + 4\delta^2} \right],
\]

where \((\omega_g^J)^2 \equiv 4\pi G \rho_g^0 - c_g^2 k^2\), \((\omega_w^J)^2 \equiv 4\pi G \rho_w^0 - c_w^2 k^2\). \(c_g \equiv \sqrt{\gamma_g \pi^g / \rho_g^0}\) and \(c_w \equiv \sqrt{\gamma_w \pi^w / \rho_w^0}\) denote speeds of sound of the gas and dark matter respectively and finally, \(\delta \equiv 4\pi G \sqrt{\rho_g^0 \rho_w^0}\), which is the term providing the gravitational coupling between the two fluids.

From Eq.(5) a criterion for the onset of instability can be easily obtained

\[(\omega_g^J)^2(\omega_w^J)^2 < \delta^2.\]  

(6)

Now, introducing Jeans length for the gas and dark matter in an usual way

\[
\lambda_g^J = c_g \sqrt{\pi G \rho_g^0}, \quad \lambda_w^J = c_w \sqrt{\pi G \rho_w^0},
\]

we can rewrite the instability criterion, Eq.(6), as

\[\lambda > \lambda_g^J \sqrt{1 + (\lambda_g^J / \lambda_w^J)^2} \equiv \lambda^J\]

(8)

Note that in the case of a gas cloud alone, i.e. without the dark matter background, the criterion for the onset of Jeans instability, Eq.(8), is simply \(\lambda > \lambda_g^J\). That is presence of the gravitationally coupled dark matter component results in an additional factor of \(1/\sqrt{1 + (\lambda_g^J / \lambda_w^J)^2}\). Thus, it is clear that in the case of presence of the dark matter, Jeans length for the gas cloud is always reduced to a value smaller than without dark matter. The latter, of course, is in accordance with general physical grounds, since the dark matter induces additional gravitational pull upon the gas inside the cloud. However, to the best of our knowledge this effect has not been studied quantitatively so far.

It is instructive to give a more specific estimate for \(\lambda^J\). For this purpose we assume, further, that a proposed dark matter component is in a form of a neutrino ball, one of the possible candidates of WIMP dark matter which recently has been studied by some authors (Bilić & Viollier 1997; Bilić et al. 1997; Tsiklauri & Viollier, 1996, 1998a,b,c; Viollier et al., 1992,1993; Viollier, 1994). That is a gas cloud is immersed in a neutrino ball which is composed of massive, self-gravitating, degenerate neutrinos, an object in which self-gravity of the neutrino matter is compensated by its degeneracy pressure, likewise in an ordinary polytropic star except for thermal pressure is replaced by the degeneracy pressure due to Pauli
exclusion principle. We start calculation with an estimate of the squared Jeans length for WIMP matter, $(\lambda_w^J)^2$. Using, Eq.(7) and polytropic equation of state for the neutrino matter (Viollier, 1994)

$$P_\nu = K\rho_\nu^{5/3}, \quad K = \left(\frac{6}{g}\right)^{2/3} \frac{\pi^{1/3} h^2}{5 m_\nu^{8/3}}$$

(9)

where, $g$ is spin degeneracy factor (in these notations $g = 1$ corresponds to the case when there are only neutrinos in the ball, whereas $g = 2$ corresponds to the case when there are both neutrinos and antineutrinos present), $m_\nu$ is the neutrino mass, we obtain

$$P_\nu = \frac{\pi G}{3} K\rho_c^{-1/3}.$$  

(10)

Eqs.(1-8) have been derived under assumption that the both gas and dark matter components are homogeneous. In the neutrino ball, however, the physical quantities vary with the radius, like in an ordinary polytropic star. A gravitational potential of the neutrino ball may well be approximated by a constant density distribution, with an average density given by $\bar{\rho} = \frac{3\left(-\xi^2 \theta'\right)_1}{\xi^1_1}\rho_c$. Here, $\rho_c$ denotes the central density in the neutrino ball and $\xi_1 = 3.65375$ and $\left(-\xi^2 \theta'\right)_1 = 2.71406$ are usual notations from polytropic theory of stars (Cox & Giuli, 1968). Thus, putting $\left(3\left(-\xi^2 \theta'\right)_1/\xi^1_1\right)\rho_c \rightarrow \rho$ in the Eq.(10) and using standard definition of the Lane-Emden unit of length (Cox & Giuli, 1968) (except that we have $8\pi$ instead of $4\pi$ since we have contribution both from neutrinos and antineutrinos)

$$\rho_n \equiv \frac{1}{2} \frac{1}{8\pi G} K\rho_c^{-1/3},$$

(11)

we finally obtain

$$\left(\lambda_w^J\right)^2 = \frac{4\pi}{3^{2/3}(-\xi^2 \theta')_1^{1/6} \xi^1_1^{1/2}} R_\nu = 2.6760 R_\nu.$$  

(12)

Here, $R_\nu$ is the radius of a neutrino ball defined in an usual way by $R_\nu = r_n \xi_1$.

Now, to estimate the reduction factor in the Jeans length, $1/\sqrt{1 + (\lambda_w^J/\lambda_g^J)^2}$, we use a typical physical parameters for an interstellar gas cloud, namely, $\lambda_g^J \approx 30\text{pc}$, its size $\approx 2\text{ pc}$ (Gorbatskii, 1986, 1977) and assume that radius of a neutrino ball is $1\text{ pc}$, i.e. the gas cloud is fully immersed in the neutrino ball. Using these values we obtain $1/\sqrt{1 + (\lambda_g^J/\lambda_w^J)^2} = 8.8848 \times 10^{-2}$. This estimate clearly demonstrates that presence of background dark matter which is gravitationally coupled to the gas cloud significantly reduces its Jeans length, $\lambda_g^J$, and in turn, its Jeans mass, which is proportional to $(\lambda_g^J)^3$. Therefore we conclude that the star formation rate which is obviously related to the Jeans length of a star-forming cloud can, in principle, serve as a test for the amount of dark matter in the Galaxy.

As it was mentioned above, typical physical parameters of an interstellar gas cloud in our galaxy are a characteristic size 2-3 pc, density $\approx 10^{-22}\text{ g/cm}^{-3}$, and Jeans length $\lambda_g^J \approx 30\text{ pc}$, i.e. a typical cloud
is stable against gravitational instability (its size is an order of magnitude less than corresponding Jeans length). However, as it is known from the observations, in our galaxy there are some gigantic cold gas clouds whose age is larger than their typical Jeans time. The lifetimes of these molecular clouds have been inferred to be few times $10^7$ years from the total fraction of gas mass in the Galaxy in the form of molecular gas, and from the lifetimes of young stars associated with them (Blitz & Shu, 1980). It has been argued that the shock formation due to the observed hypersonic velocities would dissipate the energy stored in clouds quickly enough so that entire cloud would collapse and form stars in a time span not much longer than its free-fall time, that is, $t_{\text{ff}} = 1.4 \times 10^6/\sqrt{2n(H_2)/10^4 \text{cm}^{-3}}$ yr, where $n(H_2)$ is the number density of molecular hydrogen (Goldreich & Kwan, 1974; Field, 1978). Three approaches have been put forward to explain this discrepancy (Mac Low, 1997). Arons & Max (1975) suggested that strong enough magnetic fields can prevent creation of shocks which will increase the dissipation time. Scalo & Pumphrey (1982) have argued that hydrodynamic turbulence, which is thought to be capable of preventing the collapse, will dissipate more slowly than expected. However, as shown by Mac Low et al. (1997), via performed numerical studies of compressible, decaying turbulence, with and without magnetic fields, the observed long lifetimes and supersonic motions in molecular clouds must be due to some kind of external driving (e.g. stellar outflows (Silk & Norman 1980), photoionization (McKee 1989, Bertoldi & McKee 1996), galactic shear (Fleck 1981) of combination of these thereof) as undriven turbulence decays far too fast to account for observations. On the other hand, existence of the dark matter halo around our galaxy has been thoroughly established through gravitational microlensing events of stars in the Large Magellanic Cloud (Alcock et al., 1996, 1997). It has been established that the dark matter within 50 kpc (14 disk scale length) is order of $2.5 \times 10^{11} M_\odot$, which is 4 to 5 times the mass of the galactic disk ($\approx 6 \times 10^{10} M_\odot$). Therefore, in the light of the results of this letter, problem of existence of the long-lived star-forming interstellar clouds of molecular gas, pointed out above, becomes even more enigmatic. At the same time, one has to admit that obviously, simple minded, classical theory of Jeans instability cannot be used for drawing of detailed picture of star formation process in the molecular clouds. It is likely that mechanisms that trigger and govern star formation are related to those, yet not clearly identified, processes which prevent the gigantic molecular clouds from gravitational collapse. However, it is probably also not a mere coincidence that, for typical molecular clouds having mass in the range $10^3$–$10^6 M_\odot$, number densities of the order of $10^6 \text{ cm}^{-3}$ and temperature of the order of 10K, classical Jeans theory sets correct mass scale, i.e. Jeans mass is of the order of $\sim M_\odot$ — a mass of a typical star.

Finally, we would like to remark that as an illustrative example we have considered a neutrino ball
as a possible form of dark matter. However, the results of this work are equally valid for any type of microscopic dark matter, microscopic in a sense that it can be described as a continuous medium, i.e. by the hydrodynamic equations.

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