Heavy neutrino ball as a possible solution to the ”blackness problem” of the galactic center

David Tsiklauri and Raoul D. Viollier
Physics Department, University of Cape Town, Rondebosch 7701, South Africa

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ABSTRACT

It has been recently shown (Tsiklauri & Viollier, 1998a) that the matter concentration inferred from observed stellar motion at the galactic center (Eckart & Genzel, 1997, MNRAS, 284, 576 and Genzel et al., 1996, ApJ, 472, 153) is consistent with a supermassive object of $2.5 \times 10^6$ solar masses, composed of self-gravitating, degenerate heavy neutrinos. It has been furthermore suggested (Tsiklauri & Viollier, 1998a) that the neutrino ball scenario may have the distinct advantage that it could naturally explain the so-called "blackness problem" of the galactic center. Here, we present a quantitative proof of this statement, by calculating the emitted spectrum of Sgr A* in the framework of standard accretion disk theory.

Subject headings: accretion, accretion disks — dark matter — Galaxy: center — radiation mechanisms: thermal
1. Introduction

The enigmatic radio source Sgr A* at the galactic center has been a longstanding puzzle. Observations of stellar motions at the galactic center (Eckart & Genzel, 1997; Genzel et al., 1996) and low proper motion (≤ 20 km sec\(^{-1}\); Backer, 1996) of Sgr A* indicate that, on the one hand it is a massive \((2.5 ± 0.4) \times 10^6 M_\odot\) object dominating the gravitational potential in the inner \(< 0.5\) pc region of the galaxy. On the other hand, observations of stellar winds and other gas flows in the vicinity of Sgr A* suggest that the mass accretion rate \(\dot{M}\) is about \(6 \times 10^{-6} M_\odot\) yr\(^{-1}\) (Genzel et al., 1994). This implies that the luminosity of the object should be more than \(10^{40}\) erg sec\(^{-1}\), provided the radiative efficiency is the customary 10%. However, observations indicate that the bolometric luminosity is actually less than \(10^{37}\) erg sec\(^{-1}\). This discrepancy has been a source of exhaustive debate in the recent past. The broad-band emission spectrum of Sgr A* can be reproduced either in the quasi-spherical accretion model (Melia, 1992, 1994) with \(\dot{M} \approx 2 \times 10^{-4} M_\odot\) yr\(^{-1}\) or by a combination of disk plus radio-jet model (Falcke et al., 1993a, 1993b). As pointed out by Falcke and Melia (1997), quasi-spherical accretion seems unavoidable at large radii, but the low actual luminosity of Sgr A* points toward a much lower accretion rate in a starving disk and therefore, Sgr A* can be described by a model of a fossil disk fed by quasi-spherical accretion. Another successful model which is consistent with the observed emission spectrum of Sgr A* has been developed by Narayan et al., 1995, 1997 and independently by Manmoto et al., 1997. This model is based on the concept of advection dominated accretion flow, in which most of the energy released by viscosity in the disk is carried along with the gas and lost into the black hole, while only a small fraction is actually radiated off.

Recently, Tsiklauri & Viollier (1998a) have proposed an alternative model for the mass distribution at the galactic center in which the customary supermassive black hole is replaced by a ball composed of self-gravitating, degenerate neutrinos. They showed that a neutrino ball with a mass \(2.5 \times 10^6 M_\odot\), composed of neutrinos and antineutrinos with masses \(m_\nu \geq 12.0\) keV/c\(^2\) for \(g = 2\) or \(m_\nu \geq 14.3\) keV/c\(^2\) for \(g = 1\), where \(g\) is the spin degeneracy factor, is consistent with the current observational data. The purpose of this paper is to present calculations of the spectrum emitted by Sgr A* in the framework of standard accretion disk theory, assuming that Sgr A* is a neutrino ball with the abovementioned physical properties, and to show that this could resolve the ”blackness problem”.

In the recent past, Viollier et al. have proposed that massive, self-gravitating, degenerate neutrinos arranged in balls, where the degeneracy pressure balances self-gravity, can form long-lived configurations that could mimic the properties of dark matter at the centers of galaxies (Viollier, 1994; Viollier et al., 1993;
Viollier et al., 1992). Tsiklauri & Viollier (1996) have shown that a neutrino ball could play a similar role as a stellar cluster in the 3C 273 quasar, revealing its presence through the infrared bump in the emitted spectrum. Tsiklauri & Viollier (1998b) further investigated the formation and time evolution of neutrino balls via two competing processes: annihilation of the particle-antiparticle pairs via weak interaction and spherical (Bondi) accretion of these particles. Bilić & Viollier (1997a) showed how the neutrino balls could form via a first-order phase transition of a system of self-gravitating neutrinos in the presence of a large radiation density background, based on the Thomas-Fermi model at finite temperature. They find that, by cooling a non-degenerate gas of massive neutrinos below a certain critical temperature, a condensed phase emerges, consisting of quasi-degenerate supermassive neutrino balls. General relativistic effects in the study of the gravitational phase transition in the framework of the Thomas-Fermi model at finite temperature were taken into account in Bilić & Viollier (1997b). A theorem was proven by Bilić & Viollier (1997c) which in brief states that the extremization of the free energy functional of a system of self-gravitating fermions, described by the general relativistic Thomas-Fermi model, is equivalent to solving Einstein’s field equations.

2. The model

The basic equations which govern the structure of cold neutrino balls have been derived in the series of papers (Viollier, 1994; Viollier et al., 1993; Viollier et al., 1992 and Tsiklauri & Viollier, 1996); here we adopt the notation of Tsiklauri and Viollier (1996). In this notation the enclosed mass of the neutrinos and antineutrinos within a radius \( r = r_n \xi \) of a neutrino ball is given by

\[
M_\zeta = 8\pi \rho_c r_n^3 \left( -\xi^2 \frac{d\theta(\xi)}{d\xi} \right) \equiv 8\pi \rho_c r_n^3 \left( -\xi^2 \theta' \right),
\]

where \( \theta(\xi) \) is the standard solution of the Lane-Emden equation with polytropic index \( 3/2 \), \( r_n \) is the Lane-Emden unit of length and \( \rho_c \) is the central density of the neutrino ball.

In the standard theory of steady and geometrically thin accretion disks, the power liberated in the disk per unit area is given by (Perry & Williams, 1993)

\[
D(r) = -\frac{\dot{M} \Omega' r}{4\pi} \left[ 1 - \left( \frac{R_i}{r} \right)^2 \left( \frac{\Omega_i}{\Omega} \right) \right].
\]

Here \( \Omega \) is the angular velocity of the accreting matter, \( R_i \) is the inner edge of the disk and \( \Omega_i \) denotes the angular velocity at the radius where its derivative with respect to \( r \) vanishes due to the deviation from the Keplerian law of rotation. Finally, the prime denotes the derivative with respect to \( r \). Since the motion of
accreting matter in the bulk of the disk is Keplerian, we assume that the angular velocity is given by
\[ \Omega(r) = \sqrt{\frac{GM(<r)}{r^3}}. \] (3)

In the case of a back hole \( M(<r) = \text{const} = M_{bh} \), whereas in our case \( M(<r) \) is determined by Eq.(1).
Throughout this paper we take the outer radius of the disk as \( 10^5 \) Schwarzschild radii, since for larger radii the disk is unstable against self-gravity (e.g. Narayan et al., 1997). The radius of a neutrino ball with a mass \( 2.5 \times 10^6 M_\odot \), composed of neutrinos and antineutrinos with masses \( m_\nu = 12.0 \text{ keV}/c^2 \) for \( g = 2 \) or \( m_\nu = 14.3 \text{ keV}/c^2 \) for \( g = 1 \), is equal to \( 1.06 \times 10^5 \) Schwarzschild radii of a black hole with the same mass, thus the accretion disk is fully immersed in the neutrino ball. Moreover, as in our case there is no last stable orbit, accretion may in principle continue as \( r \) tends to zero, where \( \Omega(r) \) and \( \Omega'(r) \) assume the values
\[ \Omega(0) = \sqrt{\frac{8\pi G\rho_c}{3}}, \quad \Omega'(0) = \frac{1.5\pi G\rho_c}{r_n\Omega(0)}. \] (4)

Of course the latter result is of rather academic interest, because in reality, the accreting matter will be diverted at the origin in the form of an outflow which will inevitably stream away perpendicular to the disk plane. The excess matter which has spiraled down to the very center will be pushed out of the plane due to the gas pressure of the accreting matter in the disk. It is important to note that this outflow will differ considerably from a jet shooting out of an accretion disk around a black hole. In the latter case, the jets manifest themselves as strong emitters mostly in the radio band due to the synchrotron radiation produced by the electrons moving at highly relativistic velocities, whereas the outflow from the accretion disk immersed in a neutrino ball will be practically unobservable, since the outflowing matter will be cold as it radiated off its energy while spiraling down in the disk (see further Fig.2). Moreover, the particles will be moving at non-relativistic velocities because of the shallowness of the gravitational potential of the neutrino ball that is much more spatially extended than a black hole. Also, it is worthwhile to note that, even at a constant accretion rate of \( 6 \times 10^{-6} M_\odot \text{ yr}^{-1} \), the baryonic mass acquired by the neutrino ball within the age of the universe of 10 Gyr would be of the order of \( 6 \times 10^4 M_\odot \) which is small compared to the mass of the neutrino ball.

Numerical analysis shows that initially, as the matter spirals towards the center, \( \Omega'(r) \) is negative. From Eq.(4) we gather that the central value for \( \Omega' \) is finite and positive, thus there exits a point at which \( \Omega' \) crosses zero. This is precisely the point where the angular velocity attains its maximal value. Numerically, this happens at \( \xi_i = R_i/r_n = 8.25 \times 10^{-4} \). Note that this position is quite close to the center of the ball since its radius in dimensionless units is \( \xi_1 = 3.65375 \) (Cox & Giuli, 1968). Such a behavior of \( \Omega(r) \) is quite interesting since, in the neutrino ball scenario, there is neither last stable orbit (as in the case
of a black hole) nor a stiff stellar surface (as in the case of accretion onto a neutron star). Basically, this is a consequence of the non-trivial mass distribution determined by the Lane-Emden equation.

We now assume that the gravitational binding energy released is immediately radiated away locally according to the Stefan-Boltzmann’s law

\[ D(r) = \sigma T_{\text{eff}}^4(r), \]

with \( \sigma \) denoting Stefan-Boltzmann constant. The effective temperature can be derived using Eqs.(1-3) and (5) yielding

\[ T_{\text{eff}}(\xi) = \left[ \frac{-M\Omega' r_n}{4\pi \sigma} \left( \frac{\xi \theta^{1.5} + 3\theta'}{\xi} \right) \left( 1 - \xi^2 \Omega_1 \sqrt{-1/\theta' \xi^3} \right) \right]^{1/4}. \]

Here we have introduced the quantities

\[ \Omega = \sqrt{\frac{2.5 \times 10^6 M_\odot G}{r_n^3 (-\xi^2 \theta')_1}}, \quad \Omega' = \frac{2.5 \times 10^6 M_\odot G}{2M r_n^4 (-\xi^2 \theta')_1}, \]

\((-\xi^2 \theta')_1 = 2.71406\) (Cox & Giuli, 1968) and \( \bar{\Omega}_i = \Omega(\xi_i)/\bar{\Omega}. \)

Once the temperature distribution in the accretion disk is specified, we may calculate its luminosity using

\[ L_\nu = \frac{16\pi^2 \hbar r_n^2 \cos i \nu^3}{c^2} \int_{\xi_i}^{\xi_f} \frac{\xi d\xi}{\exp[h\nu/k_b T(\xi)] - 1}, \]

where \( h \) is Plank constant, \( k_b \) denotes Boltzmann’s constant and \( i \) is the disk inclination angle which we assume to be 60° as in Narayan et al. (1997). Following the same paper, we parameterize the accretion rate in terms of the Eddington limit accretion rate, i.e. \( \dot{M} = \dot{m} \dot{M}_{\text{Edd}} M_\odot \text{yr}^{-1} \), where \( \dot{M}_{\text{Edd}} = 10L_{\text{Edd}}/c^2 = 1.39 \times 10^{18}(M/M_\odot) \text{g sec}^{-1} = 2.21 \times 10^{-8}(M/M_\odot)M_\odot \text{yr}^{-1} \). Melia (1992) has estimated \( \dot{M} \) as \( \approx 2 \times 10^{-4} M_\odot \text{yr}^{-1} \) using 600 km sec\(^{-1}\) for the wind velocity, whereas Genzel et al. (1994) obtained \( \dot{M} \approx 6 \times 10^{-6} M_\odot \text{yr}^{-1} \) using 1000 km sec\(^{-1}\) for the wind velocity. These values translate into \( 10^{-4} < \dot{m} < 4 \times 10^{-3} \) in terms of the Eddington units. Following again Narayan et al., 1997 we use these two values as the lower and upper limits for this quantity.

3. Discussion

Results of our numerical calculations are presented in Fig.1, where we plot the quantity \( \nu L_\nu \), calculated using Eq.(7). Data points are taken from Table 1 in Narayan et al., 1997. The thick solid line corresponds to the case of a neutrino ball with \( \dot{m} = 4 \times 10^{-3} \), whereas the thin solid line corresponds to \( \dot{m} = 10^{-4} \). The short-dashed line represents the calculation with a \( 2.5 \times 10^6 M_\odot \) black hole with \( \dot{m} = 10^{-4} \) and an accretion
disk extending from 3 to $10^5$ Schwarzschild radii. The long-dashed line corresponds to the case when $\dot{m}$ is artificially brought down to $10^{-9}$. As we see from Fig.1, and as also was pointed out by Narayan et al., 1997, the latter two curves provide a poor fit to the observational data. Actually, this is the major reason why the standard accretion disk theory was abandoned as a possible candidate for the description of the emitted spectrum from Sgr A$^*$. However, as originally was pointed out in Tsiklauri & Viollier (1998a) in the neutrino ball scenario, the accreting matter experiences a much shallower gravitational potential than in the case of the black hole with the same mass, and therefore less viscous torque will be exerted. The radius of a neutrino ball of total mass $2.5 \times 10^6 M_\odot$, which is composed of self-gravitating, degenerate neutrinos and antineutrinos of mass $m_\nu = 12.0$ keV/c$^2$ for $g = 2$ or $m_\nu = 14.3$ keV/c$^2$ for $g = 1$, is $1.06 \times 10^5$ larger than the Schwarzschild radius of a black hole of the same mass. In this context it is important to note that the accretion radius $R_A = 2GM/v_w^2$ for the neutrino ball, where $v_w \approx 700$ km/sec is the velocity of the wind from the IRS 16 stars, is approximately 0.02pc (Coker & Melia, 1997), which is slightly less than the radius of the neutrino ball, i.e. 0.02545 pc (for $m_\nu = 12.0$ keV/c$^2$ for $g = 2$ or $m_\nu = 14.3$ keV/c$^2$ for $g = 1$). Therefore, in the neutrino ball scenario, the captured accreting matter will always experience a gravitational pull from a mass less than the total mass of the ball. One can see from Fig. 1, that for this very reason the theoretical spectrum in the case of the neutrino ball with $\dot{m} = 4 \times 10^{-3}$ gives a much better fit than in the case of a black hole for any (even unrealistically lowered) values of $\dot{m}$. Discrepancies between the theoretical and observed spectra appear in the case of the neutrino ball for frequencies $< 40$ GHz and $\gtrsim 10^{14}$ Hz. However, as it has been pointed out before, electron scattering in the interstellar medium between Sgr A$^*$ and the observer leads to source broadening. The apparent size of the source is larger than the intrinsic one, thus the observed flux may have been contaminated by e.g. winds or jets which exist on scales smaller than the scattering size (Narayan et al., 1997). Therefore, in the $< 40$ GHz region of the spectrum, the data points should be regarded as upper bounds. At the higher end ($\gtrsim 10^{14}$ Hz), the discrepancy is due to the fact that our model does not incorporate effects of Compton-scattered synchrotron radiation (which causes the second peak on the left in Fig.1 of Narayan et al., 1997). Our model is based on the simple-minded assumption of a steady, geometrically thin accretion disk which radiates off the gravitational binding energy locally, according to the black-body radiation law. However, even in this simplified framework, our model gives an acceptable fit in the radio to near infrared part of the spectrum. Besides, it is important to note that, as it has been shown by Falcke & Melia (1997), the evolution of an accretion disk can be considerably influenced by the deposition of mass and angular momentum by an infalling Bondi-Hoyle wind. The major result of their paper is that the modification of the standard accretion disk model, by taking into account the contribution from the Bondi-Hoyle wind and considering the physical picture of accretion process in
dynamics, yields significant changes in the emitted spectrum. In fact, it produces an infrared bump, in addition to the Big Blue Bump, due to the deposition of energy in the outer part of the fossil accretion disk. Our paper is based on the standard accretion disk model i.e. without modifications arising from taking into account effects from the wind. In our case the gravitational potential is shallower than in the case of a supermassive black hole with the same mass. Therefore, taking into account effects from the Bondi-Hoyle wind and considering the non-steady problem (as in the case of Falcke & Melia’s paper), both bumps will be shifted into the lower frequency domain. Thus the incorporation of Falcke & Melia’s model of the accreting flow into our scenario of the dark matter distribution at the galactic center would presumably produce a better fit in the $\leq 40$ GHz part of the spectrum.

It is important to address the issue of consistency of our model with intrinsic source size versus frequency data. For the test we take the data of emission wavelength $\lambda = 7$ mm (Bower & Backer, 1998) and 3.5 mm (Rogers et al., 1994; Krichbaum et al., 1994). The upper limits on the intrinsic source size are $< 4.1$ AU (Bower & Backer, 1998) for 7 mm and $< 1.1$ AU (Rogers et al., 1994) and $2.8 \pm 1.2$ AU (Krichbaum et al., 1994) for 3.5 mm assuming a distance to the galactic center of 8.5 kpc. Now, we have to estimate the radial location of the circles of the accretion disk in our model, emitting at these two wavelengths. For this purpose we assume that the corresponding temperature of a circle can be determined by the Wien displacement law $\nu_m \approx 3k_bT/h \approx 6 \times 10^{10}T$ Hz (Lang, 1974), i.e. we assume that the maximal frequency (wavelength) in the brightness distribution given the black body law determines the temperature of the emitting region. This assumption seems reasonable recalling the sharpness of the maxima in the brightness distribution (see Fig. 1 in Lang, 1974). Therefore we obtain $T_{7\text{mm}} = 0.71K$ and $T_{3.5\text{mm}} = 1.43K$. To find out to which values of the radial distance in the accretion disk these two values correspond, we have to use Eq.(6), graphically depicted in the Fig.2. The values are $\xi_{7\text{mm}} = 8.71 \times 10^{-4}$ and $\xi_{3.5\text{mm}} = 1.145 \times 10^{-3}$. The final predictions of our model would be twice these values (diameter of the emitting circle) which in dimensional units are 2.50 AU and 3.29 AU for 7 mm and 3.5 mm, respectively. Thus we conclude that for 7mm our model is consistent with the observations by Bower & Backer (1998); the same applies to the data by Krichbaum et al. (1994) for 3.5 mm. However, currently some of the VLBI observations at millimeter wavelength stand in conflict with each other (Bower & Backer, 1998). Therefore, discrepancy of our estimates with Rogers et al., (1994) data can not be claimed as an established fact. Another important requirement which our model does satisfy is the lower limit on the size derived from the scintillation experiments (Gwinn et al., 1991). These experiments imply that the source diameter should be $> 0.1$ AU for 0.8 mm wavelength. Our estimates show that at this wavelength the source diameter is 34.11 AU which
corroborates the validity of our model.

Finally, we would like to emphasize that the idea that Sgr A* may be an extended object rather than a supermassive black hole is not new (see e.g. Haller et al., 1996; Sanders, 1992). To our knowledge all previous such models assume that the extended object is of a baryonic nature, e.g. a very compact stellar cluster. However, it is commonly accepted that these models face problems with stability and it has been questioned whether such clusters are long-lived enough, based on evaporation and collision time-scales stability criteria (for different point of view see Moffat, 1997). Our model of Sgr A* is surprisingly simple while it satisfies all current observational constraints: First, a neutrino ball is a stable object quite alike an ordinary baryonic star though much more massive, with the difference that its self-gravity is compensated by the degeneracy pressure of the neutrinos rather than thermal pressure as in the case of a baryonic star. Second, a neutrino ball with the abovementioned physical parameters is compact enough as to be virtually indistinguishable from the $2.5 \times 10^6 M_\odot$ black hole with current observational resolution ($\approx 10^5$ Schwarzschild radii) of the observations of proper stellar motions (Eckart & Genzel, 1997; Genzel et al., 1996). Third, a neutrino ball of this mass can explain its low proper motion ($\lesssim 20$ km sec$^{-1}$; Backer, 1996). Fourth, as a bonus of our model, the neutrino ball is extended enough to provide a much shallower gravitational potential than a $2.5 \times 10^6 M_\odot$ black hole for the accreting matter, thus producing reasonable emission flux.

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Figure captions

Fig. 1. Comparison of theoretical and observed spectra of Sgr A*. The thick solid line corresponds to the case of a neutrino ball of total mass $2.5 \times 10^6 M_\odot$ with $\dot{m} = 4 \times 10^{-3}$, while the thin solid line represents $\dot{m} = 10^{-4}$. The short-dashed line describes the calculation with a $2.5 \times 10^6 M_\odot$ black hole, with $\dot{m} = 10^{-4}$ and an accretion disk extending from 3 to $10^5$ Schwarzschild radii. The long-dashed line corresponds to the case when $\dot{m}$ is artificially reduced to $10^{-9}$. Data points in the $< 40$ GHz region are upper bounds. Note, that the thick solid line fits the most reliable data points with the error bars.

Fig. 2. Temperature of the accretion disk as a function of radial distance from the center. The thick line corresponds to the case of a neutrino ball with $2.5 \times 10^6 M_\odot$ and $\dot{m} = 4 \times 10^{-3}$, whereas, the thin line corresponds to a black hole with the same mass and accretion rate. The unit of length is $r_n = 2.14934 \times 10^{16}$ cm. Note that for values larger than 3.65375, which corresponds to the radius of the neutrino ball, the back hole and neutrino ball lines do overlap because the potentials are equal.