The osp(1,2)–covariant Lagrangian quantization of reducible massive gauge theories

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Abstract

The osp(1,2)–covariant Lagrangian quantization of irreducible gauge theories [1] is generalized to L–stage reducible theories. The dependence of the generating functional of Green’s functions on the choice of gauge in the massive case is discussed and Ward identities related to osp(1,2) symmetry are given. Massive first–stage theories with closed gauge algebra are studied in detail. The generalization of the Chapline–Manton model and topological Yang–Mills theory to the case of massive fields is considered as examples.

I. INTRODUCTION

In a previous paper [1], a generalization of the Sp(2)–covariant Lagrangian quantization for irreducible (or zero–stage) general gauge theories [2, 3, 4] has been proposed which is based on the orthosymplectic algebra osp(1,2). Within this approach it is possible to consider massive fields thus avoiding infrared divergencies otherwise occurring within the renormalization procedure. Moreover, this approach ensures symplectic invariance to all orders of perturbation theory. This is due to the fact that for nonvanishing mass m the quantum action $S_m$ (and the related gauge fixed action $S_{m,\text{ext}}$) is required to satisfy the generating equations of Sp(2)–symmetry in addition to the $m$–extended quantum master equations generating the extended BRST symmetry.

The aim of the present paper is to extend this formalism to L–stage reducible gauge theories, i.e. to theories having a redundant set of linearly dependent gauge generators. In principle, every such theory permits to single out a basis of linearly independent generators but then, in general, either locality or manifest relativistic covariance will be lost.
The paper is organized as follows. In Section II we shortly review the basic definitions concerning the reducibility properties of the theory. The extended configuration space of $L$–stage reducible gauge theories is introduced and the $osp(1,2)$–covariant quantization procedure for these theories is formulated. To be able to express this $osp(1,2)$–algebra through operator identities and to have nontrivial solutions of the generating equations it is necessary to introduce additional sources not present in the $Sp(2)$–covariant formulation. Furthermore, the explicit construction of generating differential operators fulfilling this algebra is outlined. As in the case of irreducible theories mass terms destroy gauge independence; however, this gauge dependence disappears in the limit $m = 0$. In Section III we consider first–stage reducible massive theories with closed gauge algebra, thereby extending the solution given in [1]. The problem of how to find the full set of necessarily required (anti)ghost and auxiliary fields has also been tackled in Ref. [5] for the massless case by introducing additional structure constants and postulating some new structure relations. But we were neither able to confirm one of these relations (Eq. (15) in Ref. [5]) nor to prove the nilpotency of the corresponding extended BRST transformations. The same inaccuracy was adopted in Ref. [6]. Re–analysing that problem we proved that the above mentioned relation had to be generalized (see Eq. (32) below) in order to ensure nilpotency. As a consequence, also quartic (anti)ghost terms enter into the extended BRST transformations and do not disappear as has been claimed in Ref. [5]. In Section IV as an application we consider the Chapline–Manton model [7] as well as topological Yang–Mills theory [8] and generalize the corresponding (anti)BRST transformations for the massive case.

Throughout this paper we have used the condensed notation introduced by DeWitt [9] and conventions adopted in Ref. [1]; if not otherwise specified, derivatives with respect to the antifields are the (usual) left ones and that with respect to the fields are right ones. Left derivatives with respect to the fields are labeled by the subscript $L$, for example, $\delta_L/\delta \phi^A$ denotes the left derivative with respect to the fields $\phi^A$.

II. GENERAL STRUCTURE OF $osp(1,2)$–COVARIANT QUANTIZATION OF REDUCIBLE GAUGE THEORIES

In general gauge theories a set of gauge (as well as matter) fields $A^i$ with Grassmann parity $\epsilon(A^i) = \epsilon_i$ is considered for which the classical action $S_{cl}(A)$ is invariant under the gauge transformations

$$\delta A^i = R^i_{\alpha_0} \xi^{\alpha_0}, \quad \alpha_0 = 1, \ldots, n_0, \quad S_{cl} R^i_{\alpha_0} = 0,$$

(1)
where \( \xi^{\alpha_0} \) are the parameters of these transformations and \( R^i_{\alpha_0} (A) \) are the gauge generators having Grassmann parity \( \epsilon(\xi^{\alpha_0}) = \epsilon_{\alpha_0} \) and \( \epsilon(R^i_{\alpha_0}) = \epsilon_i + \epsilon_{\alpha_0} \), respectively; by definition \( X_{,j} = \delta X/\delta A^j \).

For \textit{general gauge theories} the (open) algebra of generators has the form [2]:

\[
R^i_{\alpha_0} R^j_{\beta_0} - (-1)^{\epsilon_{\alpha_0}\epsilon_{\beta_0}} R^j_{\alpha_0} R^i_{\beta_0} = -R^i_{\gamma_0} F^\gamma_{\alpha_0\beta_0} - M^{ij}_{\alpha_0\beta_0} S_{\gamma_0 j},
\]

where \( F^\gamma_{\alpha_0\beta_0} (A) \) are the field dependent structure functions and \( M^{ij}_{\alpha_0\beta_0} (A) \) is graded antisymmetric with respect to \( (ij) \) and \( (\alpha_0\beta_0) \). In the case \( M^{ij}_{\alpha_0\beta_0} = 0 \) the algebra is closed.

If the set of generators \( R^i_{\alpha_0} \) are \textit{linearly independent} then the theory is \textit{irreducible} [10]. The \( Sp(2) \)– and \( osp(1,2) \)–covariant quantization of these theories have been considered in Ref. [2, 1]. If the generators \( R^i_{\alpha_0} \) are \textit{linearly dependent} then, according to the following characterization, the theory under consideration is called \textit{L–stage reducible} [11, 3]: There exists a chain of field dependent on–shell zero–modes \( Z^{-1}_{\alpha_s} (A) \),

\[
R^i_{\alpha_s} Z^{\alpha_s}_{\alpha_1} = S_{\gamma_0 j} K^{\gamma j}_{\alpha_1}, \quad K^{ij}_{\alpha_1} = - (-1)^{\epsilon_i\epsilon_j} K^{jj}_{\alpha_1},
Z^{\alpha_{s-1}}_{\alpha_s} Z^{\alpha_s}_{\alpha_s} = S_{\gamma_0 j} K^{\gamma j}_{\alpha_s}, \quad \alpha_s = 1, \ldots, n_s, \quad s = 2, \ldots, L,
\]

where the stage \( L \) of reducibility is defined by the lowest value \( s \) for which the matrix \( Z^{\alpha_{L-1}}_{\alpha_L} (A) \) is no longer degenerated. The \( Z^{\alpha_{s-1}}_{\alpha_s} \) are the on–shell zero modes for \( Z^{\alpha_{s-2}}_{\alpha_s} \) with \( \epsilon(Z^{\alpha_{s-1}}_{\alpha_s}) = \epsilon_{\alpha_{s-1}} + \epsilon_{\alpha_s} \), where \( \epsilon_{\alpha_s} \) is the parity of the \( s \)–stage gauge transformation associated with the index \( \alpha_s \). In the following, if not otherwise stated, we assume \( s \) to take on the values \( s = 0, \ldots, L \), thereby including also the case of irreducible theories.

The whole space of (anti)fields and sources together with their Grassmann parities (modulo 2) is characterized by the following sets

\[
\phi^A = (A^i, B^{\alpha_s|a_1 \ldots a_s}, C^{\alpha_s|a_0 \ldots a_s}), \quad \epsilon(\phi^A) \equiv \epsilon_A = (\epsilon_i, \epsilon_{\alpha_s} + s, \epsilon_{\alpha_s} + s + 1),
\]

\[
\bar{\phi}_A = (\bar{A}^i, \bar{B}_{\alpha_s|a_1 \ldots a_s}, \bar{C}_{\alpha_s|a_0 \ldots a_s}), \quad \epsilon(\bar{\phi}_A) = \epsilon_A,
\]

\[
\phi^s_{Aa} = (A^a_1, B^{\alpha_s a|a_1 \ldots a_s}, C^{\alpha_s a|a_0 \ldots a_s}), \quad \epsilon(\phi^s_{Aa}) = \epsilon_A + 1,
\]

and

\[
\eta_A = (D^i, E_{\alpha_s|a_1 \ldots a_s}, F_{\alpha_s|a_0 \ldots a_s}), \quad \epsilon(\eta_A) = \epsilon_A,
\]

where the pyramids of auxiliary fields \( B^{\alpha_s|a_1 \ldots a_s} \) and (anti)ghosts \( C^{\alpha_s|a_0 \ldots a_s} \) \( (s = 0, \ldots, L) \) are \( Sp(2) \)–tensors of rank \( s \) and \( s + 1 \), respectively, \textit{symmetric} with respect to the indices behind the stroke \(|\); and similarly for the antifields and sources. Of course, the totally symmetrized tensors are irreducible and have maximal \( Sp(2) \)–spin.
Raising and lowering of $Sp(2)$–indices is obtained by the invariant tensor of the group,
\[
\epsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \epsilon^{ac}\epsilon_{cb} = \delta^a_b.
\]
Let us point to the fact that in the $Sp(2)$–approach the internal $Sp(2)$ indices $a_0, \ldots , a_s$ of the component fields behind the stroke | are dummy ones, i.e. they are not affected by main operations like antibrackets ( , )\text{a}, operators $\Delta^a, V^a$ being introduced there.

Let us now repeat the general modifications of the $Sp(2)$–formalism introduced in Ref. [1] to obtain the $osp(1, 2)$–covariant quantization which also apply to $L$–stage reducible theories of massive fields whose bosonic action $S = S_{m}(\phi^A, \phi^{\alpha}_A, \phi^{\beta}_A, \eta_A)$ depends on the mass $m$ as a further independent parameter. In addition to the $m$–extended generalized quantum master equations which ensure (anti)BRST invariance, $S_m$ is required to obey the generating equations of $Sp(2)$–invariance, too:
\[
\bar{\Delta}_m \exp\{(i/\hbar)S_m\} = 0, \quad \bar{\Delta}_a \exp\{(i/\hbar)S_m\} = 0,
\]
or equivalently,
\[
\frac{1}{2}(S_m, S_m)^a + V_m^a S_m = i\hbar \Delta^a S_m, \quad \frac{1}{2}\{S_m, S_m\}_a + V_a S_m = i\hbar \Delta_a S_m;
\]
\[
\bar{\Delta}_m^a = \Delta^a + (i/\hbar)V_m^a \text{ and } \bar{\Delta}_a = \Delta_a + (i/\hbar)V_a \text{ are odd and even second–order differential operators, respectively; together with the brackets } (S_m, S_m)^a \text{ and } \{S_m, S_m\}_a \text{ they are defined below Eqs. (8)–(12). As long as } m \neq 0 \text{ the operators } \bar{\Delta}_m^a \text{ are neither nilpotent nor do they anticommute among themselves; instead, together with the operators } \bar{\Delta}_a \text{ they form the (super)algebra } osp(1, 2):
\]
\[
[\bar{\Delta}_a, \bar{\Delta}_\beta] = (i/\hbar)\epsilon_{a\alpha}^\beta \bar{\Delta}_\alpha, \quad [\bar{\Delta}_\alpha, \bar{\Delta}_m^a] = (i/\hbar)\bar{\Delta}_m^a (\sigma_a)_b^\alpha, \quad \{\bar{\Delta}_m^a, \bar{\Delta}_b^a\} = - (i/\hbar)m^2 (\sigma_a)^{ab} \bar{\Delta}_m^a.
\]
The matrices $\sigma_\alpha$ ($\alpha = 0, +, -$) generate $sl(2, R)$, the even part of $osp(1, 2)$, which is isomorphic to $sp(2, R)$,
\[
(\sigma_\alpha)_a^b (\sigma_\beta)_c^b = g_{\alpha\beta}^c \delta_a^b + \frac{1}{2}\epsilon_{\alpha\beta\gamma}(\sigma_\gamma)_a^b, \quad (\sigma_\alpha)_a^b = g^{\alpha\beta}(\sigma_\beta)_a^b,
\]
\[
g^{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}, \quad g^{\alpha\gamma}g_{\gamma\beta} = \delta^\alpha_\beta,
\]
and are expressed through the Pauli matrices $\tau_\alpha$ ($\alpha = 1, 2, 3$) as $(\sigma_0)_a^b = (\tau_3)_a^b, (\sigma_\pm)_a^b = - \frac{1}{2} (\tau_1 \pm i\tau_2)_a^b$. Here, $\epsilon_{\alpha\beta\gamma}$ is the antisymmetric tensor, $\epsilon_{0+} = 1$. As has been pointed out in Ref. [1] the ghost number operator is $(\hbar/\iota)\bar{\Delta}_0 = \Delta_{gh}$.
In writing Eqs. (4) we have introduced the (anti)brackets \((F,G)^a\) and \(\{F,G\}_a\) defining the well known odd graded and a new even graded algebraic structure on the space of fields and antifields, respectively,

\[
(F,G)^a = \frac{\delta F}{\delta \phi^A} \frac{\delta G}{\delta \phi^{A_a}} - (-1)^{(e(F)+1)(e(G)+1)}(F \leftrightarrow G),
\]

\[
\{F,G\}_a = (\sigma_a)_B \frac{\delta F}{\delta \phi^A} \frac{\delta G}{\delta \eta_B} + (-1)^{e(F)e(G)}(F \leftrightarrow G),
\]

whose properties were analyzed in Ref. [1]. The first–order differential operators \(V_m^a\) and \(V_a\) are given by

\[
V_m^a = \epsilon^{ab} \phi^*_A \frac{\delta}{\delta \phi^A} - \eta_A \frac{\delta}{\delta \phi^{A_a}} + m^2 (P_+)_m^B \phi^*_B \frac{\delta}{\delta \phi^*_A} - m^2 \epsilon^{ab} (P_-)_m^B \phi^*_c \frac{\delta}{\delta \phi^{A_c}},
\]

\[
V_a = \bar{\phi}^*_B (\sigma_a)_B \frac{\delta}{\delta \phi^A} + (\phi^*_A (\sigma_a)^b_A + \phi^*_B (\sigma_a)^B_A) \frac{\delta}{\delta \phi^{A_a}} + \eta_B (\sigma_a)_A \frac{\delta}{\delta \eta_A},
\]

and the second–order differential operators \(\Delta^a\) and \(\Delta_a\), whose structure is extracted from (8) and (9), are

\[
\Delta^a = (-1)^{e_A} \frac{\delta_L}{\delta \phi^A} \frac{\delta}{\delta \phi^*_A}, \quad \Delta_a = (-1)^{e_A} (\sigma_a)_B \frac{\delta_L}{\delta \phi^A} \frac{\delta}{\delta \eta_A},
\]

As in [1] the strategy to define the operators \(\bar{\Delta}_m^a = \Delta^a + (i/\hbar)V_m^a\), \(\bar{\Delta}_a = \Delta_a + (i/\hbar)V_a\) is governed by a specific realization of the (anti)BRST– and \(Sp(2)\)–transformations of the antifields. In accordance with (10) and (11) the action of \(V_m^a\) and \(V_a\) on the antifields is given by

\[
V_m^a \bar{\phi}^*_A = \epsilon^{ab} \phi^*_A \bar{\phi}^*_b, \quad V_a \bar{\phi}^*_A = \bar{\phi}^*_B (\sigma_a)_B,
\]

\[
V_m^a \phi^*_A = m^2 (P_+)_m^B \phi^*_B - \delta^a_b \eta_A, \quad V_a \phi^*_A = \phi^*_B (\sigma_a)^b_A + \phi^*_B (\sigma_a)^B_A,
\]

\[
V_m^a \eta_A = -m^2 \epsilon^{ab} (P_-)_m^B \phi^*_c, \quad V_a \eta_A = \eta_B (\sigma_a)_A,
\]

where the following abbreviations are used:

\[
(P_+)_m^B \equiv \eta_A (P_+)^B_m, \quad (P_-)_m^B \equiv \phi^*_A (P_-)_m^B, \quad (\sigma_a)_B \equiv \phi^*_B (\sigma_a)^B_A.
\]

The transformations (13) have the same form as in the irreducible case except for the matrix \((P_+)^B_m\) which obviously has to be generalized as follows:

\[
(P_+)^B_m \equiv \begin{cases} 
\delta^a_b \delta^b_c & \text{for } A = i, B = j; \\
\delta^a_b (s+1) S^b_{a_1 \cdots a_s} & \text{for } A = \alpha_s | a_1 \cdots a_s, B = \beta_s | b_1 \cdots b_s, \\
\delta^a_b (s+2) S^b_{a_1 \cdots a_s} & \text{for } A = \alpha_s | a_0 \cdots a_s, B = \beta_s | b_0 \cdots b_s, \\
0 & \text{otherwise}
\end{cases}
\]

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where the symmetrizer $S_{a_0 \cdots a_{s+b}}^{b_0 \cdots b_a}$ is defined as
\[
S_{a_0 \cdots a_{s+b}}^{b_0 \cdots b_a} \equiv \frac{1}{(s+2)!} \frac{\partial}{\partial X^{a_0}} \cdots \frac{\partial}{\partial X^{a_s}} \frac{\partial}{\partial X^{b_0}} \cdots \frac{\partial}{\partial X^{b_a}} X^{a_0} X^{b_0} \cdots X^{b_s},
\]
so that $S_{a_0 \cdots a_{s+b}}^{b_0 \cdots b_a} S_{a_0 \cdots a_{s+b}}^{b_0 \cdots b_a} = S_{a_0 \cdots a_{s+b}}^{b_0 \cdots b_a}$, $X^a$ being independent bosonic variables; it possesses the properties
\[
S_{a_0 \cdots a_{s+b}}^{b_0 \cdots b_a} = \frac{1}{s+2} \left( \sum_{r=0}^{s} \delta_{a_0}^{b_r} S_{a_{1+1} \cdots a_{s+b}}^{b_0 \cdots b_{r+1} \cdots b_{a+b}} + \frac{1}{s+1} \sum_{r=0}^{s} \delta_{a_0}^{b_r} S_{a_{1+1} \cdots a_{s+b}}^{b_0 \cdots b_{r-1} \cdots b_{a+b}} \right),
\]
\[
S_{a_0 \cdots a_{s+b}}^{b_0 \cdots b_a} = \frac{1}{s+1} \sum_{r=0}^{s} \delta_{a_0}^{b_r} S_{a_{1+1} \cdots a_{s+b}}^{b_0 \cdots b_{r+1} \cdots b_{a+b}}.
\]
The matrices $(P_-)_A^B \equiv \delta_a^b (P_-)_{A}^B_{a b}$ and $(\sigma_a)_A^B$ act nontrivially on the components of the (anti)fields having (dummy) internal $Sp(2)$ indices. For example,
\[
(P_-)_A^B \phi_B = (0, -sB \alpha_{a_1 \cdots a_s}, -(s+1)C \alpha_{a_0 \cdots a_s}),
\]
\[
\bar{\phi}_B (\sigma_a)_A^B = (0, \sum_{r=1}^{s} \bar{B} \alpha_{a_1 \cdots a_r a_{r+1} \cdots a_{s+b}} (\sigma_a)_a^c \sum_{r=0}^{s} \bar{C} \alpha_{a_0 \cdots a_r a_{r+1} \cdots a_{s+b}} (\sigma_a)_a^c).
\]
Therefore, $V_\alpha$ acts only on the (anti)ghost part of the antifields, and $V_m^a$ is partly of that kind (a componentwise notation of the transformations (13) is given in Appendix B).

In order to prove that the transformations (13) obey the $osp(1, 2)$--superalgebra
\[
[V_\alpha, V_\beta] = \epsilon_{\alpha \beta} \gamma V_7, \quad [V_\alpha, V_m^a] = V_m^b (\sigma_a)_b^c, \quad \{V_m^a, V_m^b\} = -m^2 (\sigma_a)_a^b V_m^a
\]
one needs the following two equalities:
\[
\epsilon^{ad} (P_+)_{Ad}^B \epsilon^{bd} (P_+)_{Ad}^B = - (\sigma_a)_a^b (\sigma_a)_B^A, \quad \epsilon^{ad} (P_+)_A^B \epsilon^{bd} (P_+)_A^B = - (\sigma_a)_a^b (\sigma_a)_c^d \epsilon (P_+)_{Ad}^B + \delta^d (\sigma_a)_A^B,
\]
and the relation $(P_-)_A^B \delta (P_-)_A^B = 0$ (remember that for $A = a_0 \cdots a_s$, $B = b_0 \cdots b_s$ the indices $a_0 \cdots a_s$, $b_0 \cdots b_s$ are completely symmetric). The first one is equivalent to
\[
\epsilon^{ad} \delta^b + \epsilon^{bd} \delta^a = - (\sigma_a)_a^b (\sigma_a)_c^d,
\]
whereas the second one equals
\[
\sum_{r=0}^{s} \delta_{a_0}^{b_0} \cdots \delta_{a_r-1}^{b_r-1} \epsilon^{ad} (\delta_{a_r}^{\delta_{a_c}}^{b_r} + \delta_{a_r}^{\delta_{a_c}}^{b_r}) + \epsilon^{bd} (\delta_{a_r}^{\delta_{a_c}}^{b_r} + \delta_{a_r}^{\delta_{a_c}}^{b_r}) \delta_{a_{r+1}}^{b_r} \cdots \delta_{a_s}^{b_s} + (\sigma_a)_a^b \sum_{r=0}^{s} \delta_{a_0}^{b_0} \cdots \delta_{a_r-1}^{b_r-1} (s+1) (\sigma_a)_a^c \delta_{a_r}^{b_r} - (\sigma_a)_a^c \delta_{a_r}^{b_r} \right) \delta_{a_{r+1}}^{b_r} \cdots \delta_{a_s}^{b_s} \]
\[
= - (\sigma_a)_a^b (\sigma_a)_a^c \delta_{a_0}^{b_0} \cdots \delta_{a_s}^{b_s} + \delta^d \sum_{r=0}^{s} \delta_{a_0}^{b_0} \cdots \delta_{a_r-1}^{b_r-1} (\sigma_a)_a^b \delta_{a_{r+1}}^{b_r+1} \cdots \delta_{a_s}^{b_s}.
\]
It is easily proven that every of the equalities (15) is satisfied for \( s = 1, \ldots, L \), provided the same is true for \( s = 0 \). Indeed, by virtue of (14), the equations for the reducible case can be cast into the form

\[
(\sigma^a)^{ab} \sum_{r=0}^s \delta^b_{a_0} \cdots \delta^b_{a_{r-1}} ((\sigma^a)^b_{c} \delta^c_{d_a} + \delta^b_{c} (\sigma^a)^d_{a_r}) \delta^b_{a_{r+1}} \cdots \delta^b_{a_s} = (\sigma^a)^{ab} ((s+1)(\sigma^a)^d_{a_0} \cdots \delta^b_{a_s} + \delta^b_{d_a} \sum_{r=0}^s \delta^b_{a_0} \cdots \delta^b_{a_{r-1}} (\sigma^a)^b_{a_r} \delta^b_{a_{r+1}} \cdots \delta^b_{a_s})
\]

and reduce to the one for the irreducible case [1],

\[
(\sigma^a)^{ab} ((\sigma^a)^b_{c} \delta^c_{d_a} + \delta^b_{c} (\sigma^a)^d_{a_0}) = (\sigma^a)^{ab} ((\sigma^a)^d_{a_0} + \delta^d_{c} (\sigma^a)^b_{a_0}). \quad (16)
\]

The last relation (16) can be established by the following two equalities:

\[
\epsilon^{ab} \delta^c_{d_a} + \epsilon^{bc} \delta^a_{d_b} + \epsilon^{ca} \delta^b_{d_c} = 0, \quad \epsilon^{ab} (\delta^c_{d_a} - \delta^d_{c} = \epsilon^{cd} (\delta^a_{d_a} - \delta^b_{c} = \delta^f_{f})).
\]

Let us recall that the relations (14)–(16) hold for matrices \( \sigma_a \) build up from the Pauli ones \( \tau_a, (\alpha = 1, 2, 3) (\sigma_0)^b_a = (\tau_3)^b_a, (\sigma_\pm)^b_a = -\frac{1}{2} (\tau_1 \pm i \tau_2)^b_a \). In this way all definitions of Ref. [1] are generalized to \( L \)-stage reducible gauge theories. Thus, the general results established in Ref. [1] remain valid also in this case.

The quantum action \( S_m \), being a solution of Eqs. (3), (4) with the boundary condition \( S_m|_{\phi^*_A=\eta=\bar{h}=0} = S_{cl}(A) \), suffers from the gauge degeneracy. To remove this degeneracy an \( Sp(2) \)-invariant, gauge-fixing bosonic functional \( F = F(\phi^A) \) has to be introduced such that the gauge fixed action \( S_{m,ext} = S_{m,ext}(\phi^A, \phi^*_A, \bar{\phi}_A, \eta_A) \) satisfies Eqs. (3), (4) as well. As has been shown in Ref. [1], it is defined by

\[
\exp\{(i/\hbar)S_{m,ext}\} = \hat{U}_m(F) \exp\{(i/\hbar)S_m\},
\]

\[
\hat{U}_m(F) = \exp\left\{ \frac{\delta F}{\delta \phi^A} \left( \frac{\delta}{\delta \phi^A} - \frac{i}{2} m^2 (P_-)^A_B \frac{\delta}{\delta \eta_B} \right) - \frac{(\hbar/i)^{1/2}}{2} \epsilon_{ab} \delta \frac{\delta F}{\delta \phi^A_B} \frac{\delta \phi^B}{\delta \phi^A_B} \frac{\delta}{\delta \eta_B} + \frac{(\hbar/i) m^2 F}{} \right\}.
\]

where the \( \eta \)-dependence of \( S_m \) is restricted by the (first) condition

\[
(\sigma^a)^A_B (\delta F / \delta \phi^A_B = 0, \quad (\sigma^a)^A_B (\delta F / \delta \phi^B_A \phi^A_B = 0, \quad (17)
\]

such that \( [\hat{\Delta}^*, \hat{U}_m(F)] \exp\{(i/\hbar)S_m\} = 0 \) and \( [\hat{\Delta}_\alpha, \hat{U}_m(F)] \exp\{(i/\hbar)S_m\} = 0 \). The second condition in (17) reveals the \( Sp(2) \)-invariance of \( F \). One of the natural solution of the conditions (17) is

\[
\frac{\delta S_m}{\delta \eta_A} = \phi^A,
\]
i.e. $S_m$ is restricted to be linear in $\eta_A$ (see Ref. [1]). Then, as a consequence of that restriction and the tracelessness of $\sigma_\alpha$, the second equation (4) simplifies into

$$(\sigma_\alpha)_B^A \frac{\delta S_m}{\delta \phi^A} \phi^B + V_\alpha S_m = 0. \quad (18)$$

Furthermore, let us introduce the operator

$$\hat{U}_m(Y) = \exp\{(\hbar/i)\hat{T}_m(Y)\}, \quad \hat{T}_m(Y) = \frac{1}{2}\epsilon_{ab}\{\hat{\Delta}_m^b, [\hat{\Delta}_m^a, Y]\} + (i/\hbar)^2 m^2 Y,$$

with $Y = Y(\phi^A, \bar{\phi}_A, \phi^*_{Aa})$ being an arbitrary (local) bosonic $Sp(2)$–scalar independent on $\eta_A$. Then, the operator $\hat{U}_m(Y)$ converts any (local) solution $S_m$ of Eqs. (3) into another (local) solution $\tilde{S}_m$,

$$\exp\{(i/\hbar)\tilde{S}_m\} = \hat{U}_m(Y) \exp\{(i/\hbar)S_m\},$$

provided it holds [1]

$$\frac{\delta S_m}{\delta \eta_A} = \phi^A, \quad \frac{\delta Y}{\delta \eta_A} = 0, \quad (\sigma_\alpha)_B^A \frac{\delta Y}{\delta \phi^A} \phi^B + V_\alpha Y = 0.$$

Thus, the gauge itself is realized through the use of a special transformation of this kind, namely by the operator $\hat{U}_m(Y)$ with the special choice of $Y$ in the form $Y = F(\phi^A)$.

By the same way as in Ref. [1] it can be proven that the vacuum functional

$$Z_m(0) = \int d\phi^A \exp\{(i/\hbar)S_m,_{\text{eff}}\}, \quad (19)$$

where $S_m,_{\text{eff}}(\phi^A) = S_m,_{\text{ext}}(\phi^A, \phi^*_{Aa}, \bar{\phi}_A, \eta_A)|_{\phi^* = \bar{\phi} = \eta = 0}$, is not independent on the choice of the gauge–fixing functional $F$ since the mass term $m^2 F$ in the action $S_m,_{\text{eff}}$ violates its gauge independence. However, this gauge dependence disappears in the limit $m = 0$ (the same is true for the $S$–matrix). By introducing the auxiliary fields $\pi^{Aa}, \lambda^A$ and $\zeta^A$ the functional (19) can be represented in the form [1]

$$Z_m(0) = \int d\phi^A \ d\eta_A \ d\zeta^A \ d\phi^*_{Aa} \ d\pi^{Aa} \ d\bar{\phi}_A \ d\lambda^A \ \exp\{(i/\hbar)(S^\zeta_m + W^\zeta_F - W_X)\}, \quad (20)$$

with

$$W_F = -\frac{\delta F}{\delta \phi^A}(\lambda^A + \frac{1}{2}m^2 (P_+)_B^A \bar{\phi}_B) - \frac{1}{2}\epsilon_{ab}\pi^{Aa} \frac{\delta^2 F}{\delta \phi^A \delta \bar{\phi}^B} \pi^{Bb} + m^2 F,$$

$$W_X = (\eta_A - \frac{1}{2}m^2 (P_+)_A^B \bar{\phi}_B)\phi^A - \phi^*_{Aa} \pi^{Aa} - \bar{\phi}_A(\lambda^A - \frac{1}{2}m^2 (P_-)_B^A \bar{\phi}^B),$$

where $S^\zeta_m$ and $W^\zeta_F$ are obtained from $S_m$ and $W_F$, respectively, by carrying out the replacement $\phi^A \rightarrow \phi^A + \zeta^A$. 

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Consider the extended generating functional of the Green’s functions:

\[ Z_m(J_A; \phi_{Aa}^*, \bar{\phi}_A, \eta_A) = \int d\phi^A \exp\{(i/\hbar)(S_{m,\text{ext}}(\phi^A, \phi_{Aa}^*, \bar{\phi}_A, \eta_A) + J_A\phi^A)\} \]

the generating functional of the vertex functions as usual is defined according to

\[ \Gamma_m(\phi^A; \phi_{Aa}^*, \bar{\phi}_A, \eta_A) = (\hbar/i) \ln Z_m(J_A; \phi_{Aa}^*, \bar{\phi}_A, \eta_A) - J_A\phi^A, \]

\[ \phi^A = (\hbar/i) \frac{\delta \ln Z_m(J_A; \phi_{Aa}^*, \bar{\phi}_A, \eta_A)}{\delta J_A}. \]

As consequence of the generating equations (4) for \( \Gamma_m \) one gets the Ward identities:

\[ \frac{1}{2}(\Gamma_m, \Gamma_m)^a + V^a \Gamma_m = 0, \quad \frac{1}{2}\{\Gamma_m, \Gamma_m\}_\alpha + V_\alpha \Gamma_m = 0. \]  

Moreover, if \( S_m \) is restricted to be linear in \( \eta_A \), then \( \Gamma_m \) possesses the same property. In this case, according to Eq. (18), the second identity (21) simplifies into

\[ (\sigma_\alpha)_B^A \frac{\delta \Gamma_m}{\delta \phi^A} \phi^B + V_\alpha \Gamma_m = 0, \quad \frac{\delta \Gamma_m}{\delta \eta_A} = \phi^A. \]

This finishes the general introduction of the \( osp(1,2) \) covariant approach of quantizing \( L \)-stage reducible general gauge theories.

**III. MASSIVE FIRST–STAGE REDUCIBLE THEORIES WITH A CLOSED GAUGE ALGEBRA**

To illustrate the generalized \( osp(1,2) \)-quantization rules, we consider first–stage reducible massive theories with closed algebra. Such theories are characterized by the fact, first, that because \( M_{\alpha_0\beta_0}^{ij} = 0 \), the algebra of generators, Eq. (2), reduces to

\[ R^i_{\alpha_0,\beta_0} R^j_{\beta_0,\gamma_0} R^i_{\alpha_0} = -R^i_{\alpha_0} F^\gamma_{\alpha_0} \]

(22)

here, for the sake of simplicity, we assume that the \( A^i \) are bosonic fields. Secondly, due to the condition of first–stage reducibility,

\[ R^i_{\alpha_0} Z^\alpha_{\alpha_1} = 0, \]

(23)

any equation of the form \( R^i_{\alpha_0} X^{\alpha_0} = 0 \) has the solution \( X^{\alpha_0} = Z^\alpha_{\alpha_1} Y^{\alpha_1} \) (for irreducible theories Eq. (23) has only the solution \( X^{\alpha_0} = 0 \)). In the case of field–dependent structure functions the Jacobi identity takes the form

\[ R^j_{\beta_0} \left( F^\delta_{\beta_0 \alpha_0} F^\gamma_{\beta_0 \gamma_0} - R^i_{\alpha_0} F^\delta_{\beta_0 \gamma_0, i} + \text{cyclic perm}(\alpha_0, \beta_0, \gamma_0) \right) = 0, \]

(24)
where the expression in the parenthesis vanishes only for irreducible theories. It should be noted that the generators $R_{\alpha_0}^i$ and the zero modes $Z_{\alpha_1}^{a_0}$ are not uniquely defined. By taking nonsingular linear combinations of them they can be transformed into the so-called standard basis defined in Ref. [3]. But in the following we will choose an arbitrary basis without any restriction and proceed along the lines of Ref. [8].

Let us restrict our considerations to solutions $S_m$ of the classical master equations

$$\frac{1}{2}(S_m, S_m)^a + V_m^a S_m = 0, \quad \frac{1}{2}\{S_m, S_m\}_\alpha + V_\alpha S_m = 0,$$

being linear in the antifields. These equations, because of the linearity with respect to the antifields, may be expressed also by $s_m^a S_m = 0$ and $d_\alpha S_m = 0$, where the symmetry operators being denoted by $s_m^a = s_m^a \phi^A \delta_\alpha^A / \delta \phi^A + V_m^a$ and $d_\alpha = d_\alpha \phi^A \delta_\alpha^A / \delta \phi^A + V_\alpha$, where

$$s_m^a \phi^A = (-1)^{\epsilon_A} \delta S_m / \delta \phi^A \text{ and } d_\alpha \phi^A = (-1)^{\epsilon_A}(\sigma_\alpha) B^A \delta S_m / \delta \eta_B, \text{ are required to fulfill the } osp(1,2)\text{–superalgebra:}$$

$$[d_\alpha, d_\beta] = \epsilon_{\alpha\beta}\gamma_i d_i, \quad [d_\alpha, s_m^a] = s_m^b(\sigma_\alpha)_b^a, \quad \{s_m^a, s_m^b\} = -m^2(\sigma^a)_{ab}d_\alpha. \quad (25)$$

In Ref. [1] it has been shown that such solutions can be written in the form

$$S_m = S_{cl} + (\frac{1}{2}\epsilon_{ab} s_m^a s_m^b + m^2)X, \quad (26)$$

where for the first–stage reducible case the $Sp(2)$–scalar $X$ has to be chosen as $X = \tilde{A}_i A^i + \tilde{B}_{\alpha_0} B^{a_0} + \tilde{B}_{\alpha_1 a} B^{\alpha_1 a} + \tilde{C}_{\alpha_0 a} C^{\alpha_0 a} + \tilde{C}_{\alpha_1 ab} C^{\alpha_1 ab}$. Let us emphasize that $s_m^a$ and $d_\alpha$ are not related to the first–order differential operators $Q_m^a = (S_m, )^a - i\hbar \Delta_m^a, \Delta_m^a = \Delta^a + (i/\hbar)V_m^a$ and $Q_\alpha = \{S_m, \}_{\alpha} - i\hbar \Delta_\alpha, \Delta_\alpha = \Delta_\alpha + (i/\hbar)V_\alpha$ at the lowest order approximation of $\hbar$, which was also introduced in Ref. [1], rather they are (nonlinear) realizations of the $osp(1,2)$–superalgebra in terms of fields and antifields. A realization of the (anti)BRST– and $Sp(2)$–transformations of the antifields already has been given (see Appendix B). Thus, we are left with the problem to determine the corresponding transformations for the fields $A^i, B^{a_0}, B^{\alpha_1 a}, C^{\alpha_0 a}, C^{\alpha_1 ab}$.

To begin with let us cast the Jacobi identity (24) into a more practical form. Owing to (23) the expression in paranthesis must be proportinal to the zero–modes $Z_{\alpha_1}^{a_0}$,

$$F_{\beta_0, \gamma_0}^{00} - R_{\alpha_0}^i F_{\beta_0, \gamma_0, i}^{00} + \text{cyclic perm}(\alpha_0, \beta_0, \gamma_0) = 3Z_{\alpha_1}^{a_0} H_{\alpha_0, \beta_0, \gamma_0}^{a_0}, \quad (27)$$

where $H_{\alpha_0, \beta_0, \gamma_0}^{a_0}(A)$ are some new structure functions, being totally antisymmetric with respect to the indicies $\alpha_0, \beta_0, \gamma_0$ and depending, in general, on the gauge fields $A^i$. For later use we need an expression for the combination $R_{\beta_0, \gamma_0}^i Z_{\alpha_1}^{a_0}$. Multiplying (22) by $Z_{\alpha_1}^{a_0}$
and using the relation $R_{\alpha_0,j}^\beta Z_{\alpha_1}^{\alpha_0} = -R_{\alpha_0}^\beta Z_{\alpha_1,j}^{\alpha_0}$, which follows from (23), we get

$$R_{\alpha_0,j}^\beta (Z_{\alpha_1}^{\alpha_0} R_{\beta_0}^j + F_{\beta_0}^{\alpha_0} Z_{\alpha_1}^{\gamma_0}) = 0.$$ 

Introducing additional new structure functions $G_{\beta_0}^{\alpha_1}(A)$ the solution of the previous relation can be written in the form

$$Z_{\alpha_1,j}^{\alpha_0} R_{\beta_0}^j + F_{\beta_0}^{\alpha_0} Z_{\alpha_1}^{\gamma_0} = -Z_{\alpha_1}^{\gamma_0} G_{\beta_0}^{\alpha_1},$$

which is a new gauge structure equation for the first–stage reducible case. Multiplying this equation by $Z_{\beta_1}$ and taking into account the reducibility condition (23),

$$F_{\beta_0}^{\alpha_0} Z_{\alpha_1}^{\gamma_0} Z_{\beta_1}^{\beta_0} = -Z_{\alpha_1}^{\gamma_0} Z_{\beta_1}^{\beta_0} G_{\alpha_0}^{\gamma_1},$$

for $G_{\beta_0}^{\gamma_1}$ we obtain the useful equality

$$Z_{\beta_1}^{\alpha_0} G_{\alpha_0}^{\gamma_1} = -Z_{\alpha_1}^{\alpha_0} G_{\alpha_0}^{\gamma_1}.$$ (30)

Moreover, using the relation (28), by virtue of (22) and (27) we are able to establish two further new gauge structure relations for the first–stage reducible case (see Appendix A):

$$(G_{\beta_0}^{\alpha_1} G_{\gamma_0}^{\beta_1} + R_{\beta_0}^{\alpha_1} G_{\gamma_0}^{\beta_1} + \text{antisym}(\beta_0 \leftrightarrow \gamma_0)) + G_{\alpha_0}^{\alpha_1} F_{\beta_0}^{\alpha_0} + 3Z_{\alpha_1}^{\alpha_0} H_{\alpha_0}^{\alpha_1} = 0 \quad (31)$$

and the total antisymmetric expression in $(\alpha_0, \beta_0, \gamma_0, \delta_0)$,

$$\left( H_{\alpha_0}^{\alpha_1} F_{\gamma_0}^{\gamma_0} - H_{\alpha_0}^{\alpha_1} F_{\delta_0}^{\gamma_0} + \text{cyclic perm}(\alpha_0, \beta_0, \gamma_0) \right) + \left\{ R_{\beta_0}^{\gamma_0} H_{\alpha_0}^{\alpha_1} - G_{\alpha_0}^{\alpha_1} H_{\alpha_0}^{\beta_1} \right. \right.$$ 

$$\left. + \text{antisym}(\delta_0 \leftrightarrow (\alpha_0, \beta_0, \gamma_0)) \right\} = 0.$$ (32)

The first one agrees with Eq. (14) in Ref. [5], but the second one differs from Eq. (15) in Ref. [5] by terms arising from antisymmetrization. As a consequence of this difference quartic (anti)ghost terms do not disappear in the (anti)BRST transformations (see Eq. (39) below). From (31) and (32) it follows that the new tensors $H_{\alpha_0}^{\alpha_1}$ and $G_{\alpha_0}^{\alpha_1}$ are not independent of each other. The gauge commutator relation (22), the Jacobi identity (27) and the new gauge structure relations (28), (31) and (32) are the key equations for the following considerations.

Let us now derive the (anti)BRST transformations of the fields under consideration. Imposing the $osp(1,2)$–superalgebra (25) on the gauge fields $A^i$, owing to $d_A A^i = 0$, this yields $\{s^a_m, s^b_m\} A^i = 0$. Then, with

$$s^a_m A^i = R_{\alpha_0}^i C^{\alpha_0 a},$$

(33)
and by virtue of (27), we find
\[ R^a_\alpha \left( s_m^a C^{a0b} + s_m^b C^{a0a} + F^{ao}_{\beta \gamma \delta} C^{\beta \gamma a} C^{\gamma \delta b} \right) = 0. \]

The general solution of this equation is
\[ s_m^a C^{a0b} = Z^{a0}_{\alpha \beta} C^{\alpha \beta 0} + \epsilon^{ab} B^{ao}_{\gamma \delta} - \frac{1}{2} F^{ao}_{\beta \gamma \delta} C^{\beta \gamma a} C^{\gamma \delta b}, \]  
(34)

where the (bosonic) ghosts \( C^{\alpha \beta \gamma} \) can be taken to be symmetric, \( C^{\alpha \beta \gamma} = C^{\gamma \beta \alpha} \), because its antisymmetric part enters into the definition of \( B^{ao} \).

Imposing the superalgebra (25) on the (anti)ghosts \( C^{\alpha \beta \gamma} \) and taking into account
\[ d_a C^{a0b} = C^{a0c} (\sigma_a)^b \]  
it gives \( \{ s_m^a, s_m^b \} C^{\alpha \beta \gamma} = -m^2 (\sigma^a)^{ab} C^{ao0d} (\sigma_d)^c \). The right–hand side of this restriction can be rewritten by means of the relations \( (\sigma_a)_d^c = \epsilon_{de} \epsilon^{ef} (\sigma_a)^e f \) and \( (\sigma^a)^{ab} (\sigma_b)^c f = -\epsilon^{ae} \delta^b_c + \epsilon^{bc} \delta^a_f \) as \( \{ s_m^a, s_m^b \} C^{\alpha \beta \gamma} = -m^2 (\epsilon^{ac} C^{a0b} + \epsilon^{bc} C^{aoa}) \). Then, with (34), by virtue of (27), we obtain
\[ \{ Z^{a0}_{\alpha \beta} (s_m^a C^{\alpha \beta \gamma} + \frac{1}{2} H^{\alpha \beta \gamma \delta \epsilon \delta_{\gamma \delta}} C^{\beta \gamma a} C^{\gamma \delta b} C^{\delta \epsilon c}) + \epsilon^{bc} (s_m^b B^{ao} + m^2 C^{aoa} - \frac{1}{2} F^{ao}_{\beta \gamma \delta} B^{\beta \gamma a} C^{\gamma \delta b} - \frac{1}{12} \epsilon_{de} (F^{ao}_{\eta \beta \gamma \delta} F^{\eta \gamma \delta \epsilon} + 2 R^a_{\beta \gamma \delta \epsilon}) C^{\alpha \beta 0} C^{\gamma \delta a} C^{\delta \epsilon b}) + \frac{1}{2} F^{ao}_{\beta \gamma \delta} Z^{ao}_{\alpha \beta} C^{\delta \epsilon \alpha \beta} + (R^a_{\beta \gamma \delta} Z^{ao}_{\alpha \beta} + \frac{1}{2} F^{ao}_{\beta \gamma \delta} Z^{ao}_{\alpha \beta}) C^{\delta \epsilon \alpha \beta} + \text{sym}(a \leftrightarrow b) = 0. \]

Replacing \( R^a_{\beta \gamma \delta} Z^{ao}_{\alpha \beta} \) according to (28) and using the relation
\[ \frac{1}{2} F^{ao}_{\beta \gamma \delta} \{ Z^{ao}_{\alpha \beta} C^{\delta \epsilon \alpha \beta} C^{\alpha \beta 0} - C^{\beta \gamma 0} C^{\gamma \delta a} C^{\delta \epsilon b} \} + \text{sym}(a \leftrightarrow b) = 0; \]
this leads to
\[ \{ Z^{ao}_{\alpha \beta} (s_m^a C^{\alpha \beta \gamma} - C^{\alpha \beta 0}_{\beta \gamma \delta}) C^{\beta \gamma a} C^{\gamma \delta b} + \frac{1}{2} H^{ao}_{\beta \gamma \delta \epsilon \delta_{\gamma \delta}} C^{\beta \gamma a} C^{\gamma \delta b} C^{\delta \epsilon c}) + \epsilon^{bc} (s_m^a B^{ao} + m^2 C^{aoa} - \frac{1}{2} F^{ao}_{\beta \gamma \delta} B^{\beta \gamma a} C^{\gamma \delta b} - \frac{1}{12} \epsilon_{de} (F^{ao}_{\eta \beta \gamma \delta} F^{\eta \gamma \delta \epsilon} + 2 R^a_{\beta \gamma \delta \epsilon}) C^{\alpha \beta 0} C^{\gamma \delta a} C^{\delta \epsilon b}) + \frac{1}{2} F^{ao}_{\beta \gamma \delta} Z^{ao}_{\alpha \beta} C^{\delta \epsilon \alpha \beta} + (R^a_{\beta \gamma \delta} Z^{ao}_{\alpha \beta} + \frac{1}{2} F^{ao}_{\beta \gamma \delta} Z^{ao}_{\alpha \beta}) C^{\alpha \beta 0} C^{\gamma \delta a} C^{\delta \epsilon b} + \text{sym}(a \leftrightarrow b) = 0. \]

Here, \( s_m^a B^{ao} \) can give a local contribution to \( s_m^a C^{\alpha \beta \gamma} \) if and only if it is proportional to \( Z^{ao}_{\alpha \beta} \). Therefore, if we introduce with \( s_m^a B^{ao} \) the new (fermionic) auxiliary field \( B^{ao} \) according to
\[ s_m^a B^{ao} = Z^{ao}_{\alpha \beta} B^{ao \alpha} + \frac{1}{2} F^{ao}_{\beta \gamma \delta} (B^{\beta \gamma a} - \epsilon_{de} Z^{ao}_{\alpha \beta} C^{\gamma \delta a} C^{\delta \epsilon b}) + \frac{1}{12} \epsilon_{de} (F^{ao}_{\eta \beta \gamma \delta} F^{\eta \gamma \delta \epsilon} + 2 R^a_{\beta \gamma \delta \epsilon}) C^{\gamma \delta a} C^{\gamma \delta b} C^{\gamma \delta \epsilon b} - m^2 C^{\gamma \delta a} \]  
(35)

for \( s_m^a C^{\alpha \beta \gamma} \) we will get the equation
\[ Z^{ao}_{\alpha \beta} \{ s_m^a C^{\alpha \beta \gamma} + \epsilon^{bc} B^{ao \alpha} - C^{\alpha \beta 0}_{\beta \gamma \delta} C^{\beta \gamma a} C^{\gamma \delta b} + \frac{1}{2} H^{ao}_{\beta \gamma \delta \epsilon \delta_{\gamma \delta}} C^{\beta \gamma a} C^{\gamma \delta b} C^{\delta \epsilon c}) + \text{sym}(a \leftrightarrow b) = 0. \]
Because the ghosts $C^{a_1bc}$ are symmetric with respect to $b$ and $c$ the general solution of this equation is of the form

$$s_m^a C^{a_1bc} = -\epsilon^{ac} B^{a_1b} - \epsilon^{ab} B^{a_1c} + C_{a_0 b_1}^{a_1} C^{a_0 a} C^{b_1 bc} - \frac{1}{2} H_{a_0 b_1 \gamma_0}^{a_1} C^{\gamma_0 a} C^{b_1 b} C^{\gamma_0 c}. \quad (36)$$

The expression for $s_m^a B^{a_1b}$ can be found by applying the superalgebra (25) on $B^{a_0}$. Due to $d_a B^{a_0} = 0$, this leads to the requirement $\{s_m^a, s_m^b\} B^{a_0} = 0$. After a somewhat involved algebraic calculation this gives

$$\begin{align*}
Z_{a_1}^0 \{s_m^a B^{a_1b} - m^2 C^{a_1ab} - C_{a_0 b_1}^{a_1} C^{a_0 a} B^{b_1 b} \\
+ H_{a_0 b_1 \gamma_0}^{a_1} (\frac{1}{2} B^{a_0} C^{b_0 a} C^{\gamma_0 b} - \epsilon_{cd} C^{b_0 a} Z_{\beta_1}^{\gamma_0} C^{b_1 bc} C^{\gamma_0 d}) \\
- \frac{1}{8} \epsilon_{cd} (C_{\delta_0 b_1}^{a_1} H_{a_0 b_1 \gamma_0}^{a_1} - R_{\delta_0}^{a_1} H_{a_0 b_1 \gamma_0}^{a_1} C^{\gamma_0 a} C^{b_1 bc} C^{\gamma_0 d}) \\
+ \frac{1}{8} \epsilon_{cd} H_{\delta_0 a_0 b_0}^{a_1} F_{\gamma_0 \delta_0}^{a_1} C^{\gamma_0 a} C^{b_1 bc} C^{\gamma_0 d} + \frac{1}{2} \epsilon_{cd} Z_{a_1}^{a_0} C^{a_1 bc} C^{b_1 bd} \} + \text{sym}(a \leftrightarrow b) = 0,
\end{align*}$$

and further, by virtue of (29),

$$\begin{align*}
Z_{a_1}^0 \{s_m^a B^{a_1b} - m^2 C^{a_1ab} - C_{a_0 b_1}^{a_1} C^{a_0 a} B^{b_1 b} \\
+ H_{a_0 b_1 \gamma_0}^{a_1} (\frac{1}{2} B^{a_0} C^{b_0 a} C^{\gamma_0 b} - \epsilon_{cd} C^{b_0 a} Z_{\beta_1}^{\gamma_0} C^{b_1 bc} C^{\gamma_0 d}) \\
- \frac{1}{8} \epsilon_{cd} (C_{\delta_0 b_1}^{a_1} H_{a_0 b_1 \gamma_0}^{a_1} - R_{\delta_0}^{a_1} H_{a_0 b_1 \gamma_0}^{a_1} C^{\gamma_0 a} C^{b_1 bc} C^{\gamma_0 d}) \\
+ \frac{1}{8} \epsilon_{cd} H_{\delta_0 a_0 b_0}^{a_1} F_{\gamma_0 \delta_0}^{a_1} C^{\gamma_0 a} C^{b_1 bc} C^{\gamma_0 d} + \frac{1}{2} \epsilon_{cd} Z_{a_1}^{a_0} C^{a_1 bc} C^{b_1 bd} \} + \text{sym}(a \leftrightarrow b) = 0.
\end{align*} \quad (37)$$

In deriving the cubic (anti)ghost terms in this equation the following equality was used:

$$\epsilon_{ab} \delta_c^d + \epsilon_{bc} \delta_a^d + \epsilon_{ac} \delta_b^d = 0.$$

Let us point out that in (37) the quartic (anti)ghost terms cannot be dropped due to our modification of relation (32).

Another equation for $s_m^a B^{a_1b}$ can be obtained by applying the superalgebra (25) on $C^{a_1cd}$. Due to $d_a C^{a_1cd} = C^{a_1cd}(\sigma_a)_c^e + C^{a_1ce}(\sigma_a)_e^d$, this leads to $\{s_m^a, s_m^b\} C^{a_1cd} = -m^2 (a^{\alpha} C^{a_1bd} + \epsilon^{ad} C^{a_1bc} + \epsilon^{bc} C^{a_1ad} + \epsilon^{bd} C^{a_1ac})$. Then, with (34), taking into account the relation (31), we obtain

$$\begin{align*}
\{\epsilon^{bc} (s_m^a B^{a_1d} - m^2 C^{a_1bd} - C_{a_0 b_1}^{a_1} C^{a_0 a} B^{b_1 d} + \frac{1}{2} H_{a_0 b_1 \gamma_0}^{a_1} B^{a_0} C^{b_1 bd} C^{\gamma_0 d}) + \text{sym}(c \leftrightarrow d) \\
- \frac{1}{2} H_{a_0 b_1 \gamma_0}^{a_1} Z_{\gamma_0 a}^{a_0} C^{\gamma_1 ab} C^{\gamma_0 d} + \frac{3}{2} H_{a_0 b_1 \gamma_0}^{a_1} Z_{\gamma_0 b}^{a_0} C^{\gamma_1 ad} C^{b_1 bc} C^{\gamma_0 b} + C^{b_1 ac} C^{b_1 bd} C^{\gamma_0 d} + C^{b_1 ad} C^{b_1 bd} C^{\gamma_0 c} \} + \text{sym}(a \leftrightarrow b) = 0,
\end{align*} \quad (38)$$
where again, for the same reason as before, quartic (anti)ghost cannot be dropped.

It is not very difficult to see, by virtue of (32), that the general solution of (37) and (38) reads

\[
S^a_{m}B^{\alpha_1 b} = C^{\alpha_1 \alpha_0}_{\alpha_0 \beta_1} (C^{\alpha_0 a} B^{\beta_1 b} - \frac{1}{2} \epsilon_{cd} Z_{\gamma_1}^{\alpha_0} C^{\gamma_1 ac} C^{\beta_1 bd}) - \frac{1}{2} H_{\alpha_0 \beta_0 \gamma_0}^{\alpha_1} B^{\alpha_0} C^{\beta_0 a} C^{\gamma_0 b} + \frac{1}{4} \epsilon_{cd} H_{\alpha_0 \beta_0 \gamma_0}^{\alpha_1} Z_{\gamma_1}^{\alpha_0} (3 C^{\beta_1 \alpha c} C^{\beta_1 \alpha c} C^{\gamma_0 d} + C^{\beta_0 b} C^{\beta_1 \alpha c} C^{\gamma_0 d}) + \frac{1}{8} \epsilon_{cd} (G_{\delta_0 \beta_1}^{\alpha_1} H_{\alpha_0 \beta_0 \gamma_0}^{\alpha_1} - R_{\delta_0}^{i} H_{\alpha_0 \beta_0 \gamma_0}^{\alpha_1}) C^{\gamma_0 a} C^{\delta_0 b} C^{\delta_0 d} - \frac{1}{16} \epsilon_{cd} H_{\delta_0 \beta_0 \gamma_0}^{\alpha_1} F_{\gamma_0 \delta_0} (C^{\gamma_0 a} C^{\delta_0 b} + C^{\gamma_0 b} C^{\delta_0 a}) C^{\delta_0 d} + m^2 C^{\alpha_1 ab}.
\]

Because in (39) no new auxiliary fields had to be introduced one would expect that the condition \( \{s^a_m, s^b_{m}\} B^{\alpha_1 c} = -m^2 (\sigma_0)^{ab} B^{\alpha_1 d} (\sigma_0)^1 c \) should be fulfilled identically as a consequence of the previous formalae. Corresponding direct calculations require the same tedious algebraic work but it can be proved that this relation is indeed satisfied.

The relations (33)–(36) and (39) specify the transformations of the \( osp(1, 2) \)–symmetry for first–stage reducible massive theories with closed gauge algebra. By using the method of Ref. [3] it can be shown that the solution \( S_m \), Eq. (26), is the most general one of the classical master equations with vanishing new ghost number, i. e., \( ngh(S_m) = 0 \).

Finally, let us determine the action \( S_{m, eff} \) in the vacuum functional (19) for the class of \( minimal \) gauges \( F \) depending only on the fields \( A^i \), the (anti)ghosts \( C^{\alpha_0 a}, C^{\alpha_1 ab} \) and the auxiliary fields \( B^{\alpha_0}, B^{\alpha_1 a} \). Inserting into (20) for \( S_m \) the action

\[
S_m = S_{cl} + A^*_a (s^a_{m} A^i) + \bar{A}_i (\frac{1}{2} \epsilon_{ab} s^b_{m} s^a_{m} A^i) + B^*_a (s^a_{m} B^{\alpha_0}) + \bar{B}^{\alpha_0} (\frac{1}{2} \epsilon_{ab} s^b_{m} s^a_{m} B^{\alpha_0}) + F_{\alpha_0 c} C^{\alpha_0 c} - C^{\alpha_0 a} C^{\alpha_0 c} + \bar{C}_{\alpha_0 c} (\frac{1}{2} \epsilon_{ab} s^b_{m} s^a_{m} C^{\alpha_0 c}) - m^2 \bar{C}_{\alpha_0 c} C^{\alpha_0 c} + E_{\alpha_1 A B} (s^a_{m} B^{\alpha_1 b}) + \bar{B}_{\alpha_1 c} (\frac{1}{2} \epsilon_{ab} s^b_{m} s^a_{m} B^{\alpha_1 c}) - \frac{1}{2} m^2 \bar{B}_{\alpha_1 c} B^{\alpha_1 c} + F_{\alpha_1 c d} C^{\alpha_1 c d} + C^{d_1 a d} s^a_{m} C^{\alpha_1 c d} + \bar{C}_{\alpha_1 c d} (\frac{1}{2} \epsilon_{ab} s^b_{m} s^a_{m} C^{\alpha_1 c d}) - m^2 \bar{C}_{\alpha_1 c d} C^{\alpha_1 c d}
\]

and performing the integration over antifields and auxiliary fields we get the following expression for \( S_{m, eff} \) (at the lowest order of \( \hbar \)):

\[
Z_m(0) = \int dA^i dB^{\alpha_0} dC^{\alpha_0 a} dB^{\alpha_1 a} dC^{\alpha_1 ab} \exp \{(i/\hbar)S_{m, eff}\}, \quad S_{m, eff} = S_{cl} + W_F,
\]
where $W_F$ is given by

$$W_F = \frac{1}{2} \epsilon_{ab}(\frac{\delta F}{\delta A^i} s^b_m s^a_i - s^a_m A^i \frac{\delta^2 F}{\delta A^i \delta A^j} s^b_m A^j)$$

$$+ \frac{1}{2} \epsilon_{ab} (\frac{\delta F}{\delta B^c_{e} m} s^b_m s^a_c C^a_{m c} - s^a_m C^a_{m c} \frac{\delta^2 F}{\delta C^a_{m c} \delta B^c_{e} m} s^b_m B^e_{m c})$$

$$+ \frac{1}{2} \epsilon_{ab} (\frac{\delta F}{\delta C^a_{c d} m} s^b_m s^a_c B^c_{d} - s^a_m B^c_{d} \frac{\delta^2 F}{\delta C^a_{c d} \delta C^a_{e f} m} s^b_m B^e_{m c} B^f_{m c})$$

$$- \frac{1}{2} \epsilon_{ab} s^a_m C^a_{m c} \frac{\delta^2 F}{\delta C^a_{m c} \delta B^c_{m d} m} s^b_m B^e_{m c} + m^2 F.$$

This gauge–fixing term can be rewritten as

$$W_F = (\frac{1}{2} \epsilon_{ab}s^b_m s^a_m + m^2) F,$$

showing that the action $S_{m, eff}$ is in fact $osp(1, 2)$–invariant and that the method of gauge fixing suggested in Section II will actually remove the degeneracy of the classical action.

VI. EXAMPLES

As a first example let us give the $osp(1, 2)$–symmetric generalization of the Chapline–Manton model [7] which describes the unified $N = 1$ supersymmetric Yang–Mills theory and $N = 1$ supergravity in ten dimensions. A striking feature of this model is that the Yang–Mills part of the classical action (the dots $\cdots$ indicate the supergravity part and additional terms of the super–Yang–Mills part)

$$S_{CM} = -\frac{3}{4} (\partial [\rho A_{\mu \nu}] - X_{\rho \mu \nu}) (\partial [\rho A^{\mu \nu}] - X^{\rho \mu \nu}) + \cdots ,$$

$$X_{\mu \nu} \equiv A^{\alpha}_{[\rho G^{\alpha}_{\mu \nu}]} - \frac{1}{3} F^{\alpha \beta \gamma} A^{\alpha}_{[\mu A^{\beta}_{\nu}]} A^{\gamma}_{\nu],} G^{\alpha}_{\mu \nu} \equiv \partial [\mu A^{\alpha}_{\nu}] + F^{\alpha \beta \gamma} A^{\beta}_{\mu} A^{\gamma}_{\nu},$$

exhibits a new type of (mixed) gauge invariance; here $X_{\rho \mu \nu}$ is the Chern–Simons 3–form, $G^{\alpha}_{\mu \nu}$ the ordinary Yang–Mills field strength and $F^{\alpha \beta \gamma}$ are the totally antisymmetric structure constants. The non–abelian gauge transformation of the Yang–Mills potential $A^{\alpha}_{\mu}$ is accompanied by an abelian gauge transformation of the skew symmetric supergravity potential $A_{\mu \nu}$:

$$\delta A^{\alpha}_{\mu} = D^{\alpha \beta}_\mu \theta^\beta(x), \quad D^{\alpha \beta}_\mu \equiv \delta^{\alpha \beta} \partial_\mu - F^{\alpha \beta \gamma} A^{\gamma}_{\mu}, \quad \delta A_{\mu \nu} = \partial_{[\mu} A^\alpha_{\nu]} (x) + \theta^\alpha(x) \partial_\mu A^{\alpha}_{\nu]} ,$$

where we have dropped the gauge and supergravity coupling constant; in addition $A^{\alpha}_{\mu}$ and $A_{\mu \nu}$ undergo supersymmetric transformations. This theory is a first–stage reducible one.
with closed gauge algebra. Its complete spectrum of (anti)ghosts and auxiliary fields, $C^{\alpha a}$, $B^{\alpha}$ and $C_{\mu}^{\alpha}$, $B_{\mu}$, $C_{ab}^{\alpha}$, $B_{a}^{\alpha}$, has been constructed in Ref. [12]. In order to obtain the $osp(1,2)$–symmetric generalization of the corresponding massive theory the gauge–fixed action will be written as

$$S_{m,\text{eff}} = S_{\text{CM}} + (\frac{1}{2} \epsilon_{ab} b_{m} s_{m}^{a} + m^{2}) F$$

with gauge fixing functional

$$F = \frac{1}{2} (A_{\mu}^{\alpha} A^{\mu \alpha} + \xi_{ab} C^{\alpha a} C^{\alpha b}) + \frac{1}{2} \tau (A_{\mu \nu} A^{\mu \nu} + \rho (\epsilon_{cd} C^{\mu \nu} + \frac{1}{2} \sigma \epsilon_{ef} C^{\mu e} C^{\nu f})) + \cdots,$$

$\xi$, $\rho$, $\sigma$ and $\tau$ being the gauge parameters, where the dots $\cdots$ stand for all usually necessary terms for fixing the supergravity gauge [13]. For the fields $A_{\mu}^{\alpha}$ the extended BRST transformations has been given in Ref. [1]:

- $s_{m}^{a} A_{\mu}^{\alpha} = D_{\mu}^{\beta a} C^{\beta a}$,
- $s_{m}^{a} C^{ab} = e^{ab} B^{\alpha} - \frac{1}{2} F^{\alpha \beta \gamma} C^{\beta a} C^{\gamma b}$,
- $s_{m}^{a} B^{\alpha} = -m^{2} C^{aa} + \frac{1}{2} F^{\alpha \beta \gamma} B^{\beta} C^{\gamma a} + \frac{1}{12} \epsilon_{cd} F^{\alpha \eta \gamma} F^{\gamma \delta \rho} C^{\rho a} C^{\delta c} C^{\beta d}$

and for the fields $A_{\mu \nu}$ the procedure outlined in (22)–(39) yields (in accordance with [5] up to $m$–dependent terms)

- $s_{m}^{a} A_{\mu \nu} = \partial_{[\mu} C_{\nu]}^{a} + C^{aa} \partial_{[\mu} A_{\nu]}^{a}$,
- $s_{m}^{a} C^{ab} = \partial_{[\mu} C_{\nu]}^{ab} + \epsilon^{ab} B_{\mu} - \frac{1}{2} F^{\alpha \beta \gamma} A_{\mu}^{\alpha} C^{\beta a} C^{\gamma b}$,
- $s_{m}^{a} B_{\mu} = -m^{2} C_{\mu}^{aa} + \partial_{\mu} B^{a} + \frac{1}{6} \epsilon_{cd} F^{\alpha \beta \gamma} C^{aa} C^{\beta c} D_{\mu}^{\delta a} C^{\delta d}$
- $+ \frac{1}{2} F^{\alpha \beta \gamma} A_{\mu}^{\alpha} B^{\beta} C^{\gamma a} + \frac{1}{12} \epsilon_{cd} F^{\alpha \eta \gamma} F^{\gamma \delta \rho} A_{\mu}^{\alpha} C^{\gamma a} C^{\delta c} C^{\beta d}$,
- $s_{m}^{a} C^{bc} = -\epsilon^{ab} B_{c} - \epsilon^{ac} B_{b} + \frac{1}{6} F^{\alpha \beta \gamma} C^{aa} C^{\beta b} C^{\gamma c}$,
- $s_{m}^{a} B_{b} = m^{2} C^{ab} + \frac{1}{6} F^{\alpha \beta \gamma} B^{\alpha} C^{\beta a} C^{\gamma b}$.

The improvement of the results in [5] through Eq. (39) does not matter here since the corresponding symmetry is abelian.

For the corresponding gauge–fixing terms one gets

$$\frac{1}{4} \epsilon_{ab} b_{m} s_{m}^{a} (A_{\mu}^{\alpha} A^{\mu \alpha}) = A_{\mu}^{\alpha} \partial^{\mu} B^{a} + \frac{1}{2} \epsilon_{ab} (\partial^{\mu} C^{\alpha b}) D_{\mu}^{\beta a} C^{\beta a},$$

$$\frac{1}{4} \epsilon_{ab} b_{m} s_{m}^{a} (\epsilon_{cd} C^{ac} C^{cd}) = \frac{1}{2} m^{2} \epsilon_{cd} C^{aa} C^{cd} + B^{a} B^{a} - \frac{1}{24} \epsilon_{ab} \epsilon_{cd} F^{\rho \alpha \beta} F^{\gamma \delta \rho} C^{aa} C^{\beta c} C^{\gamma b} C^{\delta d}$$
and

\[ \frac{1}{2} \epsilon_{abc} b^{a}_{m}(A_{\mu}A_{\nu}A_{\mu}) = A_{\mu}\partial^{[\mu}B^{\nu]} + \frac{1}{2} \epsilon_{ab}(\partial_{[\mu}C_{\nu]} + C^{ab}\partial_{[\mu}A_{\nu]^a} + C^{aa}\partial_{[\mu}A_{\nu]^a}, \]

\[ \frac{1}{2} \epsilon_{abc} b^{a}_{m}(C_{\mu}C_{\nu}C_{\mu}) = \frac{1}{2} \epsilon_{cd}(C_{\mu}C_{\nu}C_{\mu}B_{\mu} - 2\epsilon_{cd}(\partial_{[\mu}B_{\nu]}C_{\mu} + \epsilon_{cd}F^{[\mu_{a}\beta_{\gamma}}A^{\mu_{b}]B_{c}}C_{\nu}^{cd}C_{\mu}C_{\nu}B_{\mu}^d) + \frac{1}{2} \epsilon_{cd}F^{[\mu_{a}\beta_{\gamma}}C_{\nu}^{cd}C_{\mu}C_{\nu}B_{\mu}^d + \frac{1}{2} \epsilon_{cd}F^{[\mu_{a}\beta_{\gamma}}C_{\nu}^{cd}C_{\mu}C_{\nu}B_{\mu}^d). \]

The elimination of \( B^a \), \( B^\mu \) and \( B^a \) can be performed by gaussian integration; it provides the gauge–fixing terms \( \frac{1}{2} \epsilon_{a} \rho_{\nu_{a}}(\partial^{a} A_{\nu_{a}} + \rho_{\nu_{a}}(\partial^{a} A_{\nu_{a}})A_{\mu_{a}}^{\beta_{\gamma}}A_{\mu_{b}}^{\beta_{\gamma}}C_{\mu}C_{\nu}B_{\mu}^d) \) and \( \rho_{\nu_{a}}(\partial^{a} A_{\nu_{a}})^2 \) as well as higher–order (anti)ghost interaction terms. Thus, the degeneracy of the classical action is indeed removed. If compared with Ref. [12], here the complete spectrum of (anti)ghosts \( C^{\alpha_{a}}, C^{\nu_{a}}, C^{ab} \) and auxiliary fields \( B^a, B^\mu, B^a \) is produced in a direct manner and, due to the \( Sp(2) \)–invariance, also a much more compact notation is obtained which simplifies all the formulas.

As a second example let us give the \( osp(1, 2) \)–symmetric generalization of topological Yang–Mills theory [8] in four dimensions (the interest in such a theory is its connection to Donaldson theory [14]). The classical action is proportional to the Pontryagin index

\[ S_{TYM} = \frac{1}{4} G_{\mu_{1}}^{\nu_{1}}G_{\mu_{2}}^{\nu_{2}} \]

\[ G_{\mu_{1}}^{\nu_{1}} \equiv \partial_{[\mu_{1}}A_{\nu_{1}]}^{\alpha_{a}}, \quad F_{\mu_{1}}^{\alpha_{a}}A_{\mu_{2}}^{\beta_{b}}, \quad \tilde{G}_{\mu_{1}}^{\nu_{1}} \equiv \frac{1}{2} \epsilon_{\mu_{1}\nu_{1}\rho_{1}\sigma_{1}}G_{\rho_{1}\sigma_{1}}^{\alpha_{a}}, \]

where \( \tilde{G}_{\mu_{1}}^{\nu_{1}} \) is the dual field stength. Since the Pontryagin index is a group invariant the action is invariant under two types of gauge transformations

\[ \delta A_{\mu_{1}}^{\alpha_{a}} = D_{\mu_{1}}^{\alpha_{a}} + \theta_{\mu_{1}}^{\alpha_{a}}(x), \quad \delta A_{\mu_{2}}^{\beta_{b}} = \theta_{\mu_{2}}^{\beta_{b}}, \]

which form a closed algebra: the commutator of two gauge transformations with parameters \( \theta^{\alpha_{a}}, \theta_{\mu_{2}}^{\beta_{b}} \) and \( \sigma_{\mu_{2}}, \sigma_{\mu_{2}}^{\beta_{b}} \) corresponds to a gauge transformation with parameters \( (F_{\mu_{1}}^{\alpha_{a}}(x) + \theta_{\mu_{1}}^{\alpha_{a}})) \). This theory is a first–stage reducible one since obviously both gauge transformations are not independent. Its complete spectrum of (anti)ghosts and auxiliary fields, \( C^{\alpha_{a}}, B_{\alpha}^{a}, C_{\mu_{1}}^{\nu_{2}}, B_{\mu_{1}}^{a}, C^{\nu_{a}}, B_{\nu_{a}}^{a}, C^{\nu_{a}}, B_{\nu_{a}}^{a}, C^{\nu_{a}}, B_{\nu_{a}}^{a} \), has been constructed in Ref. [15]. In order to obtain the \( osp(1, 2) \)–symmetric generalization of the corresponding massive theory the gauge–fixed action will be cast into the form

\[ S_{m.eff} = S_{TYM} + (\frac{1}{2} \epsilon_{ab} s_{m} b^{a}_{m} s_{m}^{a} + m^{2})F \]
with gauge fixing functional

\[ F = \frac{1}{2}(A^\alpha_\mu A^{\mu \alpha} + \xi_{ab} C^{\alpha a} C^{\alpha b} + \rho(\epsilon_{cd} C^{\alpha c} C^{\mu d} + \frac{1}{2}\sigma \epsilon_{ef} F^{\alpha e \gamma} C^{\alpha c} C^{\alpha d})) , \]

\(\xi, \rho\) and \(\sigma\) being the gauge parameters.

For the extended BRST transformations the procedure outlined in (22)–(39) yields

\[ S_m^a A^\alpha_\mu = D^\alpha_\mu C^{\beta a} + C^{\alpha a} , \]
\[ S_m^{ca} C^{ab} = C^{caab} + \epsilon^{ab} B^{ba} - \frac{1}{2} F^{\alpha \beta \gamma} C^{\beta a} C^{\gamma b} , \]
\[ S_m^{ca} B^{a} = -m^2 C^{aa} + B^{aa} + \frac{1}{2} F^{\alpha \beta \gamma} B^{\beta} C^{\gamma a} - \frac{1}{2} \epsilon_{cd} F^{\alpha \beta} C^{\beta a} C^{\gamma d} + \frac{1}{12} \epsilon_{cd} F^{\alpha \beta} F^{\gamma \delta} C^{\gamma a} C^{\delta b} , \]
\[ S_m^{ca} C^{\gamma a} = -D^{\alpha}_\mu C^{\beta ab} + \epsilon^{ab} B^{\mu a} - F^{\alpha \beta \gamma} C^{\beta a} C^{\gamma b} , \]
\[ S_m^{ca} B^{\mu a} = -m^2 C^{ca} - D^{\alpha}_\mu B^{\beta a} + F^{\alpha \beta \gamma} B^{\beta a} C^{\gamma a} - \epsilon_{cd} F^{\alpha \beta \gamma} C^{\beta a} C^{\gamma d} , \]
\[ S_m^{ca} C^{\gamma a} = -D^{\gamma}_a B^{ab} - \epsilon^{ab} B^{ac} - F^{\alpha \beta \gamma} C^{\beta a} C^{\gamma c} , \]
\[ S_m^{ca} B^{bc} = m^2 C^{caab} - F^{\alpha \beta \gamma} C^{\beta a} B^{\gamma b} + \frac{1}{2} \epsilon_{cd} F^{\alpha \beta \gamma} C^{\beta c} C^{\gamma d} \]

and for the corresponding gauge–fixing terms one obtains

\[ \frac{1}{2} \epsilon_{ab} S_m^{ca} (A^\mu_\alpha A^\alpha_\mu) = A^\mu_\alpha (\partial_\mu B^{aa} + B^{\mu a}) + \frac{1}{2} \epsilon_{ab} C^{caab} (\partial_\mu C^{\alpha a} + C^{\mu a}) \]
\[ + \frac{1}{2} \epsilon_{ab} (\partial_\mu C^{\alpha ab}) (D^\alpha_\mu C^{\beta a} + C^{\alpha a}) , \]
\[ \frac{1}{2} \epsilon_{ab} S_m^{ca} (\epsilon_{cd} C^{\alpha ac} C^{\mu d}) = \frac{1}{2} m^2 \epsilon_{cd} C^{\alpha ac} C^{\mu d} + B^{\alpha a} B^{\mu a} - \frac{1}{2} \epsilon_{ab} \epsilon_{cd} F^{\alpha \beta \gamma} C^{\beta a} C^{\gamma d} \]
\[ + \frac{1}{2} \epsilon_{ab} \epsilon_{cd} C^{\alpha ac} C^{\mu d} + \frac{1}{2} \epsilon_{ab} \epsilon_{cd} C^{\alpha c} C^{\gamma d} , \]
\[ \frac{1}{2} \epsilon_{ab} S_m^{ca} (\epsilon_{cd} C^{\alpha ac} C^{\mu d}) = \frac{1}{2} m^2 \epsilon_{cd} C^{\alpha ac} C^{\mu d} + B^{\alpha a} B^{\mu a} + \frac{1}{2} \epsilon_{ab} \epsilon_{cd} (D^\mu_\alpha C^{\beta a} C^{\gamma d}) \]
\[ + 2 \epsilon_{cd} (D^\mu_\alpha B^{\beta a}) C^{\gamma d} + \epsilon_{ab} \epsilon_{cd} C^{\alpha ac} C^{\gamma d} , \]
\[ \frac{1}{2} \epsilon_{ab} S_m^{ca} (\epsilon_{cd} C^{\alpha ac} C^{\mu d}) = \frac{1}{2} m^2 \epsilon_{cd} C^{\alpha ac} C^{\mu d} - \frac{3}{2} \epsilon_{cd} B^{\alpha a} B^{\mu a} + \frac{1}{2} \epsilon_{ab} \epsilon_{cd} F^{\alpha \beta \gamma} B^{\alpha a} C^{\gamma d} \]
\[ - \frac{1}{2} \epsilon_{cd} \epsilon_{ef} F^{\alpha \beta \gamma} B^{\alpha c} C^{\gamma d} + \frac{1}{2} \epsilon_{ab} \epsilon_{cd} \epsilon_{ef} F^{\alpha \beta \gamma} C^{\alpha a} C^{\beta d} C^{\gamma c} . \]

The elimination of \(B^{a}, B^{\alpha}_\mu\) and \(B^aa\) by means of gaussian integration provides the gauge–fixing terms \(\frac{1}{2} \xi^{-1}(\partial^\mu A^{\alpha}_\mu)^2\) and \(\frac{1}{2} \rho^{-1}(A^{\mu a})^2\) as well as higher–order \((anti)\)ghost interaction terms. In contrast to Ref. [15], here the complete spectrum of \((anti)\)ghosts \(C^{\alpha a}, C^{\alpha a}, C^{\alpha ba}\) and auxiliary fields \(B^{a}, B^{\alpha}_\mu, B^aa\) is produced in a straightforward manner. Another advantage consists in the \(Sp(2)\)–invariant formulation of the theory which simplifies all the formulas.

V. CONCLUDING REMARKS

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We have shown that the osp(1, 2)–symmetric quantization developed for irreducible massive gauge theories in Ref. [1] can be applied also to the case of $L$–stage reducible theories by an appropriate generalization of the matrix $(P_+)^{Ba}_{Ab}$. This formalism establishes the well–known fact that mass terms violate gauge independence of the $S$–matrix so that after performing BPHZL renormalization, one has to take the limit of vanishing mass; after that gauge independence should be restored.

Proceeding in the same manner as in Ref. [5] we have built solutions of the quantum master equations for massive first–stage reducible theories with linearly dependent gauge generators and we found the osp(1, 2)–symmetric realization of the ghost spectrum for general closed gauge algebra. Thereby, if compared with the massless case, no extra fields had to be introduced. As a consequence of the improved gauge structure equation (32) also quartic (anti)ghost terms appear in the extended BRST transformations.

The restriction to theories with closed gauge algebra simplifies the problem of finding the full spectrum of (anti)ghosts and auxiliary fields, and the corresponding symmetry transformations, in so far as it can be done without introducing the explicit form of the gauge–fixing terms. Otherwise, for theories with open gauge algebra this is no longer possible. The important question whether it is possible also in this case to find the full spectrum of (anti)ghosts and auxiliary fields together with the corresponding symmetry transformations in a straightforward manner requires a more detailed consideration.

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APPENDIX A. PROOF OF THE IDENTITIES (31) AND (32)

In order to verify the relation (31) we multiply the Jacobi identity (24) with $Z^{a_0}_{\beta_1}$, by virtue of $R^i_{\alpha_0} Z^{a_0}_{\beta_1} = 0$, this yields

\[
(F^i_{\mu_0\alpha_0} Z^{a_0}_{\beta_1}) F^q_{\beta_0\gamma_0} + F^i_{\mu_0\beta_0} (F^q_{\gamma_0\alpha_0} Z^{a_0}_{\beta_1}) - F^i_{\mu_0\gamma_0} (F^q_{\beta_0\alpha_0} Z^{a_0}_{\beta_1}) - R^i_{\beta_0} (F^q_{\gamma_0\alpha_0}, Z^{a_0}_{\beta_1}) + R^i_{\gamma_0} (F^q_{\beta_0\alpha_0}, Z^{a_0}_{\beta_1}) - Z^a_{\alpha_1} (3 Z^{a_0}_{\beta_1} H^{a_1}_{\alpha_0\beta_0\gamma_0}) = 0.
\]
After replacing all terms of the form $F_{\gamma_0\alpha_0}^\delta Z_{\beta_1}^{\alpha_0}$ according to the relation (28) this gives

$$Z_{\beta_1,i}^{\delta_0}(R_{\alpha_0}^i F_{\beta_0\gamma_0}^{\alpha_0}) + Z_{\alpha_1}(G_{\alpha_0\beta_1} F_{\beta_0\gamma_0}^{\alpha_0} + 3 Z_{\beta_1}^{\alpha_0} H_{\alpha_0\beta_0\gamma_0}^{\alpha_0}) + \{ R_{\beta_0}^i (F_{\gamma_0\alpha_0,i}^{\alpha_0} Z_{\beta_1}^{\alpha_0} - F_{\beta_0\gamma_0}^{\alpha_0} Z_{\beta_1,i}^{\alpha_0}) - (F_{\delta_0}^{\alpha_0} Z_{\alpha_1}^{\alpha_0}) G_{\beta_0\beta_1}^{\alpha_0} + \text{antisym}(\beta_0 \leftrightarrow \gamma_0) \} = 0,$$

and, using the same relation once more,

$$Z_{\beta_1,i}^{\delta_0}(R_{\alpha_0}^i F_{\beta_0\gamma_0}^{\alpha_0}) + Z_{\alpha_1}(G_{\alpha_0\beta_1} F_{\beta_0\gamma_0}^{\alpha_0} + 3 Z_{\beta_1}^{\alpha_0} H_{\alpha_0\beta_0\gamma_0}^{\alpha_0}) + \{ R_{\beta_0}^i ((F_{\gamma_0\alpha_0} Z_{\beta_1}^{\alpha_0})_i + Z_{\alpha_1}^{\alpha_0} G_{\gamma_0}^{\alpha_1} G_{\gamma_0\beta_1}^{\alpha_1} + Z_{\alpha_1}^{\alpha_0} G_{\gamma_0}^{\alpha_1} G_{\gamma_0\beta_1}^{\alpha_1} + \text{antisym}(\beta_0 \leftrightarrow \gamma_0) \} = 0.$$

Here, the left-hand side can be rewritten as

$$Z_{\beta_1,i}^{\delta_0}(R_{\alpha_0}^i F_{\beta_0\gamma_0}^{\alpha_0}) + Z_{\alpha_1}(G_{\alpha_0\beta_1} F_{\beta_0\gamma_0}^{\alpha_0} + 3 Z_{\beta_1}^{\alpha_0} H_{\alpha_0\beta_0\gamma_0}^{\alpha_0}) + \{ R_{\beta_0}^i (Z_{\gamma_0}^{\alpha_1} G_{\gamma_0\beta_1}^{\alpha_1})_i + Z_{\alpha_1}^{\alpha_0} (G_{\gamma_0}^{\alpha_1} G_{\gamma_0\beta_1}^{\alpha_1} + R_{\gamma_0}^i G_{\beta_0\beta_1}^{\alpha_0}) + \text{antisym}(\beta_0 \leftrightarrow \gamma_0) \} = 0.$$

and further, once again using relation (28),

$$Z_{\beta_1,i}^{\delta_0}(R_{\alpha_0}^i F_{\beta_0\gamma_0}^{\alpha_0}) + Z_{\alpha_1}(G_{\alpha_0\beta_1} F_{\beta_0\gamma_0}^{\alpha_0} + 3 Z_{\beta_1}^{\alpha_0} H_{\alpha_0\beta_0\gamma_0}^{\alpha_0}) - \{ R_{\beta_0}^i (Z_{\gamma_0}^{\alpha_1} G_{\gamma_0\beta_1}^{\alpha_1})_i - Z_{\alpha_1}^{(0)} (G_{\gamma_0}^{\alpha_1} G_{\gamma_0\beta_1}^{\alpha_1} + R_{\gamma_0}^i G_{\beta_0\beta_1}^{\alpha_0}) + \text{antisym}(\beta_0 \leftrightarrow \gamma_0) \} = 0. \quad (40)$$

Because of first-stage reducibility and since the gauge algebra (22) is closed,

$$Z_{\beta_1,i}^{\delta_0}(R_{\alpha_0}^i F_{\beta_0\gamma_0}^{\alpha_0}) = Z_{\beta_1,i}^{\delta_0}(R_{\beta_0}^i R_{\gamma_0}^i R_{\beta_0}^i - R_{\gamma_0}^i R_{\beta_0}^i) = R_{\beta_0}^i (Z_{\beta_1,j}^{\delta_0} R_{\gamma_0}^i) - R_{\gamma_0}^i (Z_{\beta_1,j}^{\delta_0} R_{\beta_0}^i),$$

from (40) we get

$$(G_{\gamma_0}^{\alpha_1} G_{\gamma_0\beta_1}^{\alpha_1} + R_{\gamma_0}^i G_{\gamma_0\beta_1}^{\alpha_1}) + \text{antisym}(\beta_0 \leftrightarrow \gamma_0) + G_{\alpha_0\beta}^{\alpha_0} F_{\beta_0\gamma_0}^{\alpha_0} + 3 Z_{\beta_1}^{\alpha_0} H_{\alpha_0\beta_0\gamma_0}^{\alpha_0} = 0$$

which is just the gauge structure relation (31).

In order to prove that the relation (32) is satisfied we have to consider the identity

$$\{(Z_{\alpha_0}^{\lambda_0} H_{\gamma_0\alpha_0\beta_0}^{\lambda_0}) F_{\gamma_0\delta_0}^{\gamma_0} + \text{cyclic perm}(\alpha_0, \beta_0, \gamma_0)\} + 2 R_{\delta_0}^i (Z_{\alpha_0}^{\lambda_0} H_{\alpha_0\beta_0\gamma_0}^{\lambda_0})_i + 2 F_{\delta_0}^{\alpha_0} (Z_{\gamma_0}^{\alpha_1} H_{\alpha_0\beta_0\gamma_0}^{\alpha_1}) \equiv 0,$$

which can be verified by a direct calculation by using the Jacobi identity (24). Taking into account the relation (28) one obtains

$$Z_{\alpha_1}^{\lambda_0} \{(H_{\gamma_0\alpha_0\beta_0}^{\alpha_0} F_{\gamma_0\delta_0}^{\alpha_0} + \text{cyclic perm}(\alpha_0, \beta_0, \gamma_0)) + 2 R_{\delta_0}^i H_{\alpha_0\beta_0\gamma_0}^{\delta_0} - 2 G_{\beta_1}^{\alpha_0} H_{\alpha_0\beta_0\gamma_0}^{\beta_1} \} + \text{antisym}(\delta_0 \leftrightarrow (\alpha_0, \beta_0, \gamma_0)) = 0. \quad (41)$$
After factoring out the zero modes $Z_{\alpha_0}$, and by using the identity

$$(H_{\gamma\alpha_0\beta_0}^{\alpha_0} F_{\gamma\delta_0}^{\gamma_0} + \text{cyclic perm}(\alpha_0, \beta_0, \gamma_0)) + \text{antisym}(\delta_0 \leftrightarrow (\alpha_0, \beta_0, \gamma_0))$$

$$\equiv 2(H_{\gamma\alpha_0\beta_0}^{\alpha_0} F_{\gamma\delta_0}^{\gamma_0} - H_{\gamma\delta_0\alpha_0}^{\alpha_0} F_{\beta_0\gamma_0}^{\beta_1} + \text{cyclic perm}(\alpha_0, \beta_0, \gamma_0)),$$

the equation (41) acquires the form

$$(H_{\gamma\alpha_0\beta_0}^{\alpha_0} F_{\gamma\delta_0}^{\gamma_0} - H_{\gamma\delta_0\alpha_0}^{\alpha_0} F_{\beta_0\gamma_0}^{\beta_1} + \text{cyclic perm}(\alpha_0, \beta_0, \gamma_0))$$

$$+ \{ R_{\delta_0} H_{\alpha_0\beta_0\gamma_0}^{\alpha_0} + G_{\delta_0\beta_0} H_{\alpha_0\beta_0\gamma_0}^{\beta_1} + \text{antisym}(\delta_0 \leftrightarrow (\alpha_0, \beta_0, \gamma_0)) \} = 0,$$

which is the gauge structure relation (32). Note, that the left–hand side of this relation is still a total antisymmetric expression in $(\alpha_0, \beta_0, \gamma_0, \delta_0)$.

**APPENDIX B. COMPONENTWISE NOTATION OF THE TRANSFORMATIONS (13)**

In componentwise notation the extended BRST– and $Sp(2)$–transformations (13) of the antifields reads as follows ($s = 0, \ldots, L$) (the first component $D_i$ in $\eta_A$ is put equal to zero):

$$V^a_m A^*_i = \epsilon^{ab} A^{*_i}_{ab},$$

$$V^a_m A^*_b = m^2 \delta^a_b A^*_b,$$

$$V^a_m B^\alpha_{a_1 \ldots a_s} = \epsilon^{ab} B^\alpha_{a_1 \ldots a_s},$$

$$V^a_m E^\alpha_{a_1 \ldots a_s} = m^2 \epsilon^{ab} (s B^\alpha_{a_1 \ldots a_s} - \sum_{r=1}^s B^\alpha_{a_r\bar{a}_{r-1} \bar{a}_{r+1} \ldots a_s}),$$

$$V^a_m B^*^\alpha_{a_1 \ldots a_s} = m^2 (\delta^a_b \bar{B}^\alpha_{a_1 \ldots a_s} + \sum_{r=1}^s \delta^a_{a_r} \bar{B}^\alpha_{a_1 \ldots a_{r-1} a_{r+1} \ldots a_s}) - \delta^a_b E^\alpha_{a_1 \ldots a_s},$$

$$V^a_m C^\alpha_{a_1 \ldots a_s} = \epsilon^{ab} C^\alpha_{a_1 \ldots a_s},$$

$$V^a_m F^\alpha_{a_1 \ldots a_s} = m^2 \epsilon^{ab} ((s + 1) C^\alpha_{a_1 \ldots a_s} - \sum_{r=0}^s C^\alpha_{a_1 \ldots a_{r-1} a_{r+1} \ldots a_s}),$$

$$V^a_m C^*^\alpha_{a_1 \ldots a_s} = m^2 (\delta^a_b \bar{C}^\alpha_{a_1 \ldots a_s} + \sum_{r=0}^s \delta^a_{a_r} \bar{C}^\alpha_{a_1 \ldots a_{r-1} a_{r+1} \ldots a_s}) - \delta^a_b F^\alpha_{a_1 \ldots a_s}$$
with respect to all \( Sp \) sources \( s \) satisfy the \( osp \) and \( E \) where the additional sources \( C \) antifields \( B \) can be realized without introducing restriction than satisfying this algebra by the help of (anti)BRST transformations which

Let us emphasize that expressing this algebra through operator identities is a stronger restriction than satisfying this algebra by the help of (anti)BRST transformations which can be realized without introducing \( E_{\alpha_{a_1}a_{a_2}} \) and \( F_{\alpha_{a_0}a_{a_2}} \) namely by choosing also the antifields \( B_{\alpha_{a_0}b_{a_1}a_{a_2}} \) and \( C_{\alpha_{a_0}b_{a_0}a_{a_2}} \) as irreducible representations, i.e. totally symmetric with respect to all \( Sp(2) \)-indices, \( B_{\alpha_{a_0}b_{a_1}a_{a_2}} = B_{\alpha_{a_0}a_{a_2}b_{a_1}} \) for \( r = 1, \ldots, s \) and \( C_{\alpha_{a_0}b_{a_0}a_{a_2}} = C_{\alpha_{a_0}b_{a_0}a_{a_2}b_{a_1}} \) for \( r = 0, \ldots, s \) (\( s = 0, \ldots, L \)).

Let us also write down the componentwise notation of the operators \( V_a^a \), \( V_a \) and \( \Delta a \),

\[
V_a \bar{A}_a = 0, \quad V_a A_b^a = A_b^a (\sigma_a)^c, \quad V_a \bar{B}_{a| a_{a_1} \ldots a_s} = \sum_{r=1}^s \bar{B}_{a| a_{a_1} \ldots a_{a_r-1}a_{a_r+1} \ldots a_s} (\sigma_a)^c, \quad V_a B_{a| a_{a_1} \ldots a_s} = B_{a| a_{a_1} \ldots a_s} (\sigma_a)^c + \sum_{r=1}^s B_{a| a_{a_1} \ldots a_{a_r-1}a_{a_r+1} \ldots a_s} (\sigma_a)^c, \\
V_a \bar{C}_{a| a_{a_0} \ldots a_s} = \sum_{r=0}^s \bar{C}_{a| a_{a_0} \ldots a_{a_r-1}a_{a_r+1} \ldots a_s} (\sigma_a)^c, \quad V_a C_{a| a_{a_0} \ldots a_s} = C_{a| a_{a_1} \ldots a_s} (\sigma_a)^c + \sum_{r=0}^s C_{a| a_{a_0} \ldots a_{a_r-1}a_{a_r+1} \ldots a_s} (\sigma_a)^c, \\
V_a E_{a| a_{a_1} \ldots a_s} = \sum_{r=1}^s E_{a| a_{a_1} \ldots a_{a_r-1}a_{a_r+1} \ldots a_s} (\sigma_a)^c, \quad V_a F_{a| a_{a_0} \ldots a_s} = \sum_{r=0}^s F_{a| a_{a_0} \ldots a_{a_r-1}a_{a_r+1} \ldots a_s} (\sigma_a)^c,
\]

where the additional sources \( E_{\alpha_{a_1}a_{a_2}} \) and \( F_{\alpha_{a_0}a_{a_2}} \) have to be introduced in order to satisfy the \( osp(1,2) \)-superalgebra

\[
[V_a, V_{\beta}] = \epsilon_{\alpha \beta} \gamma^7 V_7, \quad [V_a, V_m^a] = V_m^b (\sigma_a)^b, \quad \{V_m^a, V_m^b\} = -m^2 (\sigma_a)^{ab} V_a.
\]

Let us emphasize that expressing this algebra through operator identities is a stronger restriction than satisfying this algebra by the help of (anti)BRST transformations which can be realized without introducing \( E_{\alpha_{a_1}a_{a_2}} \) and \( F_{\alpha_{a_0}a_{a_2}} \), namely by choosing also the antifields \( B_{a| a_{a_1} \ldots a_s} \) and \( C_{a| a_{a_0} \ldots a_s} \) as irreducible representations, i.e. totally symmetric with respect to all \( Sp(2) \)-indices, \( B_{a| a_{a_1} \ldots a_s} = B_{a| a_{a_1} \ldots a_{a_s-1}b_{a_s+1} \ldots a_s} \) for \( r = 1, \ldots, s \) and \( C_{a| a_{a_0} \ldots a_{a_s}} = C_{a| a_{a_0} \ldots a_{a_s-1}b_{a_s+1} \ldots a_s} \) for \( r = 0, \ldots, s \) (\( s = 0, \ldots, L \)).
\(\Delta_a\), Eqs. (10)–(12). They are given by

\[
V_m = \epsilon^{ab} A^*_b \frac{\delta}{\delta A^*_a} + m^2 \Delta t_a \frac{\delta}{\delta A^*_a} + \sum_{s=0}^L \left\{ \epsilon^{ab} B^s_{\alpha s b[a_1 \ldots a_s]} \frac{\delta}{\delta B^s_{\alpha s [a_1 \ldots a_s]}}, \right.
\]

\[
+m^2 \left( \delta^a_b D^s_{\alpha s[a_1 \ldots a_s]} + \delta^a_{a_1 \ldots a_s} D^s_{\alpha s[a_1 \ldots a_r b_{a_1 \ldots a_s}]}, \delta \right) \frac{\delta}{\delta B^s_{\alpha s b[a_1 \ldots a_s]}},
\]

\[
-E_{\alpha s[a_1 \ldots a_s]} \frac{\delta}{\delta B^s_{\alpha s[a_1 \ldots a_s]}}, \frac{\delta}{\delta E_{\alpha s[a_1 \ldots a_s]}},
\]

\[
+ \epsilon^{ab} C^s_{\alpha s b[a_0 \ldots a_s]} \frac{\delta}{\delta C^s_{\alpha s [a_0 \ldots a_s]}} + m^2 \left( \delta^a_b C^s_{\alpha s [a_0 \ldots a_s]} + \delta^a_{a_0 \ldots a_s} C^s_{\alpha s [a_0 \ldots a_r b_{a_0 \ldots a_s}]}, \delta \right) \frac{\delta}{\delta C^s_{\alpha s b[a_0 \ldots a_s]}},
\]

\[
-F_{\alpha s[a_0 \ldots a_s]} \frac{\delta}{\delta C^s_{\alpha s[a_0 \ldots a_s]}}, \frac{\delta}{\delta F_{\alpha s[a_0 \ldots a_s]}},
\]

\[
V_a = A^c_{ic}(\sigma_a) \frac{\delta}{\delta A^c_{ib}} + \sum_{s=0}^L \left\{ \sum_{r=1}^s \left( \delta^a_b B^s_{\alpha s b[a_1 \ldots a_r b_{a_1 \ldots a_s}]}, \delta \right) \frac{\delta}{\delta B^s_{\alpha s b[a_1 \ldots a_s]}}, \right.
\]

\[
+ \sum_{r=1}^s \frac{\delta}{\delta E_{\alpha s[a_1 \ldots a_s]}}, \frac{\delta}{\delta E_{\alpha s[a_1 \ldots a_s]}},
\]

\[
+ \left( B^s_{\alpha s c[a_1 \ldots a_s]}(\sigma_a) + \sum_{r=1}^s \left( \delta^a_b C^s_{\alpha s [a_0 \ldots a_r b_{a_0 \ldots a_s}]}, \delta \right) \frac{\delta}{\delta C^s_{\alpha s b[a_0 \ldots a_s]}}, \right.
\]

\[
+ \sum_{r=0}^s \frac{\delta}{\delta F_{\alpha s[a_0 \ldots a_s]}}, \frac{\delta}{\delta F_{\alpha s[a_0 \ldots a_s]}},
\]

\[
+ \left( C^s_{\alpha s c[a_0 \ldots a_s]}(\sigma_a) + \sum_{r=0}^s \left( \delta^a_b D^s_{\alpha s [a_0 \ldots a_r b_{a_0 \ldots a_s}]}, \delta \right) \frac{\delta}{\delta D^s_{\alpha s b[a_0 \ldots a_s]}}, \right.
\]

\[
\left. \right\} \frac{\delta}{\delta C^s_{\alpha s b[a_0 \ldots a_s]}}, \frac{\delta}{\delta C^s_{\alpha s b[a_0 \ldots a_s]}},
\]

and

\[
\Delta^a = (-1)^{s+1} \frac{\delta L}{\delta A^v} + \frac{\delta}{\delta A^t_{1s}}, \frac{\delta}{\delta A^t_{1s}}, \sum_{s=0}^L \left\{ (-1)^{s+1} \frac{\delta L}{\delta B^s_{\alpha s [a_1 \ldots a_s]}}, \frac{\delta}{\delta B^s_{\alpha s [a_1 \ldots a_s]}}, \right.
\]

\[
+ (-1)^{s+1} \frac{\delta L}{\delta C^s_{\alpha s [a_0 \ldots a_s]}}, \frac{\delta}{\delta C^s_{\alpha s [a_0 \ldots a_s]}},
\]

\[
\Delta_a = \sum_{s=0}^L \left\{ (-1)^{s+1} \frac{\delta L}{\delta B^s_{\alpha s [a_1 \ldots a_s]}}, \frac{\delta}{\delta B^s_{\alpha s [a_1 \ldots a_s]}}, \right.
\]

\[
+ (-1)^{s+1} \frac{\delta L}{\delta C^s_{\alpha s [a_0 \ldots a_s]}}, \frac{\delta}{\delta C^s_{\alpha s [a_0 \ldots a_s]}},
\]

\[
\right\}.
\]
References