HARD PHOTONS BEYOND PROTON-NEUTRON BREMSSTRAHLUNG IN HEAVY-ION COLLISIONS ¹

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Abstract

We report on the study of extremely high energy photons, pions and etas, produced in intermediate energy heavy-ion collisions. Possibility of imaging the final-state phase space in these collisions by the Bose-Einstein correlations for photons is critically examined.

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1 Introduction

Measurement in heavy-ion (HI) collisions of hard photons of energies several times higher than the beam energy per nucleon, has stimulated large interest in the development of an appropriate theoretical framework for the description of rare energy fluctuations which could account for the measured properties of particles. In this context, the success of the stochastic multiplicative processes[1] in describing the subthreshold particle yields came as a surprise. Nucleon in the transient state of the HI collision is described by the stochastic equation: \( \frac{dp}{dt} = F + G(p) \lambda \), where \( \lambda \) is a random Gaussian process \( \langle \lambda \rangle = 0, \langle \lambda^2 \rangle = \sigma^2 \) accounting for fluctuating nuclear environment, and \( G(p) = g_0 + g_1 p + \cdots \) is a smooth function of the particle momentum. In the leading order, the time evolution of the relative momentum of two particles is: \( d(\delta p)/dt \sim \alpha(\delta p) \lambda \) and, hence, the relative kinetic energy evolves as: \[
\frac{d}{dt} E_{NN} \sim 2\alpha E_{NN} \lambda.
\] (1)

The relative energy of the nucleon-nucleon pair in Eq. (1) is a stochastic, short-correlation quantity driven by the multiplicative noise[1, 2]. Taking \( \varepsilon(t = 0) = \delta(E_{NN} - E_{CM}/A) \) as the initial condition, one may derive the limiting probability distribution of the relative energy field: \[
P(\varepsilon) = \frac{N}{\varepsilon} \exp \left( - \frac{(\log \varepsilon - \mu)^2}{2s} \right),
\] (2)
where \( N \) is the normalization factor, \( \sqrt{s} \) is the width of the Gaussian variable and \( \mu \equiv \log(E_{CM}/2A) + s/2 \). The production cross section of a particle ‘x’ in the NN collisions is then: \[
\sigma^{(x)} = \int_{E_{th}^{(x)}}^{E_{tot}} d\varepsilon P(\varepsilon)\sigma_{NN}^{(x)}(\varepsilon - E_{th}^{(x)}),
\] (3)
where \( E_{th}^{(x)} \) is the energy threshold in the NN collision and \( E_{tot} \) is the total energy available for the production of a particle in the HI collision. Assuming that the time-duration of instabilities is approximately independent of bombarding energy in a broad energy interval and that the parameters of random, short-correlated medium depend weakly both on energy and on the kind of produced particle, it was shown that all data on total and differential cross sections of produced pions at \( E_{CM} < 100 MeV/A \) is remarkably well fitted by a distribution (2) with 2 parameters: \( N \) and \( s \) in the whole region[1]. Also the spectrum of deeply subthreshold \( \eta \)’s at \( E_{CM}/A = 180 MeV \) is reproduced.
The above stochastic multiplicative process is an attempt to characterize the global features of relative energy fluctuations in the off-equilibrium media without entering into details of elementary processes involved in the production of particle $x$. Alternatively, one can start from the cascade picture of particle-particle collisions, including detailed informations about all elementary processes in the medium and, in this way, one can try to reconstruct the particle production scenario in HI reactions. Below, we shall use for this purpose the Dubna Cascade Model (DCM) [3] which has been recently extended by including various radiative capture processes such as: \( np \rightarrow d(\gamma, \pi, \eta \cdots) \), as well as the off-shell evolution of nucleons and $\Delta$'s [4].

2 Subthreshold pion dynamics as a source of hard photons, pions and etas

We shall omit discussion of ingredients concerning elementary processes considered in DCM. Interesting reader can find further informations in Refs. [4, 5]. In the calculations of the photon spectrum, we consider the $pn$ bremsstrahlung process in the one-boson approximation [6] and we omit the $pp$ bremsstrahlung. We include also the radiative capture process: \( pn \rightarrow d\gamma \), which can be important in the description of high energy part of the photon spectrum. This process requires the existence of bound state of final deuteron, i.e., the deuteron momentum must be above the Mott momentum if the influence of the surrounding nucleons is taken into account [7, 8]. The process \( pn \rightarrow d\gamma \) is most important at \( E/A \simeq 150 \text{MeV} [4, 8] \). The decay processes: \( \pi^0 \rightarrow \gamma\gamma \) and \( \Delta \rightarrow N\gamma \), which could give rise to hard photons, have been included as well.

It turns out that the most important source of hard photons is the process: \( \pi N \rightarrow N\gamma [5] \). Pions in the DCM are produced either directly: \( NN \rightarrow NN\pi \), or in two steps involving intermediate $\Delta$ or off-shell nucleon [9]: \( NN \rightarrow \Delta N \) or \( NN \rightarrow N^{(off)}N \) and then \( \Delta \rightarrow N\pi \), \( \Delta N \rightarrow NN\pi \) or \( N^{(off)}N \rightarrow NN\pi \). Formation of $\Delta$ is calculated so that the effective mass of the $\pi N$ system follows the $\Delta$ mass distribution, its width being dependent on the pion momentum in the $\pi N$ system. The yield of the primordial pions is subsequently modified by the absorption and rescattering processes in the nuclear medium [5]. The same two-step mechanism as well as the process \( \pi N^{(off)} \rightarrow N\pi(\eta \cdots) \) have been considered as a source of hard pions and deeply subthreshold etas.

Fig. 1 shows the comparison of the experimental data for the $\pi^+$ production in $p + Ar$ reaction [10] in the broad range of energies with the results of DCM calculations. The agreement between theory and experiment becomes even more impressive if one notices that the experimental data contains a systematic
error of about 40 \% [10].

Fig. 1 compares calculated and measured photon spectrum (full points) in the reaction $^{181}\text{Ta} + ^{197}\text{Au}$ at 39MeV/A [5] The dominant contributions: $pn \rightarrow pn\gamma$, $pn \rightarrow d\gamma$ and $N\pi \rightarrow N\gamma$, are shown for the comparison. The contribution from the decay $\pi^0 \rightarrow \gamma\gamma$ is insignificant at this bombarding energy. The agreement between DCM calculations of the photon spectrum and the experimental data is excellent up to the highest photon energies.

Finally, Fig. 1 presents measured [11] (full triangles) and calculated (full points) transverse mass spectrum of pions for the reaction $^{40}\text{Ar} + ^{40}\text{Ca}$ at 180MeV/A. One may notice the deficit of hard pions ($m_t > 0.4\text{GeV}$) in DCM calculations. The measured spectrum decreases slower than exponentially, similarly as in the log-normal behaviour. The open points in Fig. 1 show the contribution from processes involving off-shell nucleons. The crosses give the calculated $\eta$-production cross section. The estimated meson ratio $\sigma_\eta/\sigma_{\pi^0}$ is $\leq 3 \cdot 10^{-6}$ in the data as compared to $3 \cdot 10^{-7}$ in the DCM [11].

3 Imaging the final-state phase space in HI collisions by HBT correlations

Here we want to examine the status of HBT measurements [12] as a tool to study the spatio-temporal extension of the emitting source at intermediate energy HI collisions. The two-particle correlation function is defined as:

$$C^\text{ex}_2(p_1,p_2) \equiv \frac{P_2(p_1,p_2)}{P_1(p_1)P_1(p_2)} = 1 + \frac{| \int \exp(iq x) S(x,K)d^4x|^2}{[\int S(x_1,p_1)d^4x_1] \cdot [\int S(x_2,p_2)d^4x_2]}$$

where $S$ is the source term (the Wigner transform of the single particle density matrix), $q \equiv p_2 - p_1$ and $K \equiv (1/2)(E_1 + E_2, p_1 + p_2)$. $C^\text{ex}_2$ is positive definite even though $S(x,p)$ may take both positive and negative values. Eq. (4) constitutes a formal basis for incorporating the Bose-Einstein symmetrization in the cascade codes of HI reactions. However, in the actual applications, one replaces this exact result by [13]:

$$C^\text{app}_2(p_1,p_2) = \frac{| \int \exp(iq x_1) S(x_1,p_1)d^4x_1 \int \exp(iq x_2) S(x_2,p_2)d^4x_2 |}{[\int S(x_1,p_1)d^4x_1] \cdot [\int S(x_2,p_2)d^4x_2]}$$

which corresponds to weighting the pair at space-time points $(x_1,p_1),(x_2,p_2)$ with $1 + \cos[(p_1 - p_2)(x_1 - x_2)]$, and is equivalent to the assumption of smoothness of the source function. This assumption may lead to certain pathological results (see, e.g., the discussion in Ref. [14]).
The advantage of photons over other probes is that they escape from the interaction region unperturbed. Let us take the photon source in the form:

\[
S(x, K) = N \exp(-E/T) \exp(-r^2/R_0^2) \exp(-t^2/\tau^2),
\]

which approximates well the source function extracted from the DCM evolution [4]. The correlation function corresponding to (6) is:

\[
C_{ex}^2(Q, q_0) = 1 + 2\Lambda \exp \left(-\frac{Q^2 R_0^2}{2}ight) \exp \left(-\frac{q_0^2}{2}(\tau^2 + R_0^2)\right), \tag{7}
\]

where \( z \equiv Q/T \), \( Q^2 \equiv q^2 - q_0^2 \) and \( K_2(z) \) is the McDonald function. The polarization factor [16]: \( \Lambda = (1/4)(1 + \cos^2 \Theta_{12}) \), where \( \Theta_{12} \) is the angle between photon momenta, has been taken equal 1/2. Using (5) one finds:

\[
C_{app}^2(Q, q_0) = 1 + \Lambda \exp \left(-\frac{Q^2 R_0^2}{2}ight) \exp \left(-\frac{q_0^2}{2}(\tau^2 + R_0^2)\right). \tag{8}
\]

Note the absence of the pre-exponential function, which is a measure of importance of the off-shell effects and disappears if \( Q \to 0 \) or \( T \to \infty \).

Both exact (7) and approximate (8) correlation functions, are plotted in Fig. 3 for two source sizes: \( R_0 = 4\text{fm} \) and \( 8\text{fm} \), and for the 'apparent temperature' \( T = 15\text{MeV} \), which is a typical value in HI reactions at \( E/A < 100\text{MeV} \). \( C_2^e \) for \( R_0 = 4\text{fm} \) shows a strong increase due to the fact that a typical photon wavelength in this low-\( T \) regime is comparable to the source size. For both values of \( R_0 \) the approximate correlation function provides a poor approximation of the exact one, i.e., the cascade calculations using the smoothness approximation (5) are not reliable. These calculations could be improved using the smearing prescription [15] where the 'classical' source is convoluted with a minimum uncertainty Gaussian wave-packet: \( g(x, p) \sim \exp[-x^2/(2\sigma^2) + 2p^2\sigma^2)] \). However, in this case results both for \( C_2 \) and for the energy spectra depend on the value of the smearing parameter \( \sigma^2 \).

In the experimental analysis, the source size could be extracted more reliably using Eq. (7) in the fitting procedure. One should however notice that for small source sizes, the rise of the correlation function can be easily confused with a Gaussian peak at \( Q \sim m_\pi \) from the decay: \( \pi^0 \to \gamma\gamma \). In this unfavorable case, the experimental extraction of \( R_0 \) may be very difficult.

4 Conclusions

We have shown that cascade dynamics with the schematic account of mean-field dynamics, is successful in reproducing spectra of extremely high energy
photons and hard pions produced in the deeply subthreshold regime of HI reactions. The significant deviations between the DCM calculations and the data begins to be visible only at extremely large $m_t$’s. More data in this domain, which may provide a stringent test of the time-evolution of energy fluctuations in the off-equilibrium nuclear medium, are badly needed.

In principle, the same cascade scenario could be tested also in HBT measurements. However, the rapid shape variations of the $\gamma\gamma$- correlation function with the source radius and the importance of off-shellness ($Q \neq 0$) in the intermediate energy HI reactions may pose serious problems in extracting the quantitative information. HBT measurements using pions in this domain are also not free from these defects but the corrections in this case are somewhat easier to introduce both in the calculations and in the experimental analysis[4].
References


Fig. 1. Experimental energy and angle integrated yield of $\pi^+$ from $p + Ar$ reactions is compared with the results of DCM (stars).
Fig. 2. Measured photon spectrum in the reaction $^{181}\text{Ta} + ^{197}\text{Au}$ at 39 MeV/A is compared with the DCM calculations (the solid line). The calculated spectrum is decomposed into fractions corresponding to the following elementary processes: $pn \rightarrow pn\gamma$, $N\pi \rightarrow N\gamma$, $pn \rightarrow d\gamma$. 
Fig. 3. Experimental pion transverse mass distribution in the reaction $^{40}Ar + ^{40}Ca$ at 180 MeV/A is compared with those obtained in the DCM. For details see the description in the text.
Fig. 4. Four different projections of the correlation function calculated analytically for the source distribution (6). The solid lines show results obtained exactly from the definition (4) for the two source sizes: $R_0 = 4$ and 8fm, whereas the dashed and dotted curves have been obtained in the smoothness approximation (5) for $R_0 = 4$ and 8fm respectively.