NEXT-TO-LEADING ORDER EFFECTS ON A TRANSVERSITY BOUND[†]A. P. Contogouris^{a,b}, G. Veropoulos^b, Z. Merebashvili^{a,*} and G. Grispos^b

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ABSTRACT

The effect of next-to-leading order corrections (NLOC) on the transversity bound proposed by Soffer is reexamined. With proper definition of NLOC it is shown that the effect is significant, substantially suppressing the bound for all $q=u,d$ and s quarks ; the reason is explained. Assuming that the bound is saturated at $Q_0^2 = 4 \text{ GeV}^2$ and evolving the next-to-leading order unpolarized, helicity and transversity distributions to $Q^2 > Q_0^2$ via 2-loop anomalous dimensions we find that the bound is preserved.

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From the viewpoint of Operator Product Expansion the transverse structure function $h_1(x, Q)$ is on equal footing with the unpolarized $f_1(x, Q)$ and the longitudinally polarized $g_1(x, Q)$: all three are leading twist [1,2]. Correspondingly, the transversely polarized distribution, or transversity, $\Delta_T F_q(x, Q)$, the unpolarized $F_q(x, Q)$ and the longitudinally polarized, or helicity, $\Delta_L F_q(x, Q)$ are of equal importance in determining the structure of the hadron. However, f_1 and g_1 are chiral-even and naturally appear in deep inelastic scattering (DIS); so, on F_q there is plethora of information, and on $\Delta_L F_q$ data from polarized DIS are accumulating. In contrast, h_1 is chiral-odd and it properly appears in Drell-Yan (DY) lepton pair production by transversely polarized hadrons [1,2]. For this reaction specific experiments are planned [3] or have been proposed [4], but so far experimental information is lacking, and theoretical predictions on transversally polarized reactions are severely hampered. Thus the bound

$$2|h_1| \leq f_1 + g_1 \quad (1)$$

proposed by Soffer [5] has been very welcome. The bound originates from inequalities between similar helicity amplitudes in nucleon-nucleon reactions and is derived [5,6] via quark-nucleon scattering. Ref.[6] indicates that within QCD its status is not very clear; nevertheless, [6] does offer a derivation in a parton model approximation. Then, and in view of its usefulness within QCD, a currently adopted attitude [7-11] is to accept the inequality

$$2|\Delta_T F_q(x, Q_o)| \leq F_q(x, Q_o) + \Delta_L F_q(x, Q_o) \quad (2)$$

for some $Q_o^2 = O(1 \text{ GeV}^2)$; this should hold for each quark species q [5,6]. Subsequently we write

$$\Delta_T F_{q,max}(x, Q_o) \equiv \left\{ F_q(x, Q_o) + \Delta_L F_q(x, Q_o) \right\} / 2 \quad (3)$$

However, with the bound (3), in the context of perturbative QCD, two questions naturally arise :

- (A) What is the effect of higher order corrections (HOC) on the quantity $\Delta_T F_{q,max}$?
- (B) Does the evolution at $Q^2 > Q_o^2$ of the distributions $[\Delta_{L,T}]F_q$ preserve the bound ?

Regarding (A), an answer was given in Ref [8]: With the HOC defined in a specified manner the effect is not very significant (see, though, below).

Regarding (B), in leading order (LO), with the evolution of the (twist-2) transversities determined via 1-loop anomalous dimensions (split functions), it was shown simply and nicely that the answer is affirmative [10]. Recently the corresponding 2-loop anomalous

dimensions have been determined [9,11-14], and in next to-leading order (NLO) to answer (B) is less simple. Nevertheless, in certain usual factorization schemes including $\overline{MS}$, the answer seems again to be affirmative. This was shown for the valence transversities in a numerical way [11]. The question is also discussed in [15] in a general way.

Regarding the question (A), in [8], the HOC for *all* $[\Delta_{L,T}]F_q$, i.e. not only for the transversities but also for the unpolarized distributions and helicities, were defined via DY. As we make clear later, *precisely this* was the reason that the effect of HOC on $\Delta_T F_{q,max}$ was rather small. Regarding $\Delta_T F_q$, defining the HOC via DY is natural; however, regarding F_q and $\Delta_L F_q$, it is more natural to define the HOC via DIS, as is also suggested in [6]. The main purpose of this work is to show that the latter procedure *significantly suppresses* the bound $\Delta_T F_q$. The reason for this will also become clear.

Furthermore, regarding question (B), working with our final distributions, we proceed as follows : First, we evolve $F_q(x, Q)$ and $\Delta_L F_q(x, Q)$ from $Q_o^2 = 4 \text{ GeV}^2$ to some much higher $Q^2 = Q_j^2$ ($j=1,2$, say $Q_1^2 = 100$, $Q_2^2 = 10^4 \text{ GeV}^2$) and determine $(F_q + \Delta_L F_q)/2$ at Q_j^2 . Then assuming saturation of the bound at Q_o^2 , we evolve $\Delta_T F_q(x, Q)$ to Q_j^2 and examine whether the bound is respected. We hope that in what follows our procedure is made clear.

As in [8], denote by $[\Delta_L]F_q^{NLO}(x, Q)$ the unpolarized momentum distributions and helicities determined via 2-loop anomalous dimensions. Specifically, in all below, for F_q^{NLO} we use the set CTEQ4M [16] and for $\Delta_L F_q^{NLO}$ the set A of [17](GS-A); then we determine the bound

$$\Delta_T F_{q,max}^{(o)}(x, Q) \equiv \left\{ F_q^{NLO}(x, Q) + \Delta_L F_q^{NLO}(x, Q) \right\} / 2; \quad (4)$$

this will be our LO bound^(a). The set CTEQ4M is in the $\overline{MS}$ scheme and GS-A in an $\overline{MS}$ -equivalent (t'Hooft-Veltman modified) scheme, so that in n dimensions $\Delta P_{qq}^n(z) = P_{qq}^n$; thus it can said that in (4) everything is in $\overline{MS}$.

To $O(\alpha_s)$, perturbation theory leads to the distributions

$$\begin{aligned} [\Delta_L] \overline{F}_q(x, Q) &= [\Delta_L] F_q^{NLO}(x, Q) + \alpha_s(Q) \int_x^1 \frac{dx'}{x'} \left\{ [\Delta_L] C_q\left(\frac{x}{x'}\right) [\Delta_L] F_q^{NLO}(x', Q) \right. \\ &\quad \left. + [\Delta_L] C_g\left(\frac{x}{x'}\right) [\Delta_L] F_g^{NLO}(x', Q) \right\}. \end{aligned} \quad (5)$$

Here $[\Delta_L] F_g^{NLO}$ is the gluon distribution and $[\Delta_L] C_q(z), [\Delta_L] C_g(z)$ the HOC, all determined in the same scheme ($\overline{MS}$). Then, having determined $[\Delta_L] \overline{F}_q$, we write :

$$\Delta_T \bar{F}_{q,max}(x, Q) \equiv \left\{ \bar{F}_q(x, Q) + \Delta_L \bar{F}_q(x, Q) \right\} / 2. \quad (6)$$

Note that $\Delta_T \bar{F}_{q,max}$ is *not* the final bound to be compared with the initial one ($= \Delta_T F_{q,max}^{(o)}$) but, as it will become clear, it serves a useful purpose.

Regarding transversities, the perturbative calculation gives :

$$\Delta_T \bar{F}_q(x, Q) = \Delta_T F_q(x, Q) + \alpha_s(Q) \int_x^1 \frac{dx'}{x'} \Delta_T C_q\left(\frac{x}{x'}\right) \Delta_T F_q(x', Q); \quad (7)$$

again. $\Delta_T C_q(z)$ is the HOC properly determined; of course, no term $\sim \Delta_T C_g$ is present. Now, $\Delta_T \bar{F}_{q,max}$ as given by (6), is introduced in the l.h.s. of (7); the solution of the resulting integral equation, denoted by $\Delta_T F_{q,max}(x, Q)$, is the physically important bound to be compared with $\Delta_T F_{q,max}^{(o)}(x, Q)$.

We work throughout in $\overline{MS}$. According to previous discussion and the suggestion of [6], the HOC $[\Delta_L]C_q$ and $[\Delta_L]C_g$ to the momentum distributions in Eq.(5) are determined via DIS: they turn out to be [18,19]:

$$\begin{aligned} C_q(x) = \Delta_L C_q(x) &= \frac{C_F}{2\pi} \left\{ -\left(\frac{9}{2} + \frac{\pi^2}{3}\right) \delta(1-x) \right. \\ &+ \left. x \left[(1+x^2) \left(\frac{\ln(1-x)}{1-x} \right)_+ - \frac{3}{2} \frac{1}{(1-x)_+} - \frac{1+x^2}{1-x} \ln x + 3 + 2x \right] \right\} \end{aligned} \quad (8)$$

where $1/(1-x)_+$, $(\ln(1-x)/(1-x))_+$ well-known distributions and $C_F = 4/3$. Also

$$C_g = \frac{x}{2\pi} \left\{ \frac{x^2 + (1-x)^2}{2} \ln \frac{1-x}{x} + 3x(1-x) \right\} \quad (9)$$

and

$$\Delta_L C_g = \frac{x}{2\pi} \left\{ \left(x - \frac{1}{2}\right) \ln \frac{1-x}{x} + 3(1-x) \right\} \quad (10)$$

Note that in $n = 4 - 2\epsilon$ dimensions the split functions involved in the determination of $[\Delta_L]C_g(x)$ are

$$P_{qg}^n(x, \epsilon) = \frac{x^2 + (1-x)^2}{2} - \epsilon x(1-x) \quad (11)$$

and

$$\Delta P_{qg}^n(x, \epsilon) = x - \frac{1}{2} - \epsilon(1-x) \quad (12)$$

The HOC $\Delta_T C_q(x)$ will be defined via DY. These have been determined in detail in [8,20]. However, in [8,20] the HOC were presented in dimensional reduction (RD); and consistency demands that here be given in the scheme of e.g. Ref.[9], in which the 2-loop

transversity anomalous dimensions have been determined ($\overline{MS}$). The way to obtain the conversion terms is well-known : Denoting the n-dimensional split functions by

$$P_{ab}^n(z, \epsilon) = P_{ab}(z) + \epsilon P_{ab}^\epsilon(z) \quad (13)$$

the conversion terms are determined from the difference of the parts P_{ab}^ϵ in the two schemes. In RD one has $P_{ab}^\epsilon(z) \equiv 0$, hence (with obvious notations):

$$\Delta_T C_q^{\overline{MS}} = \Delta_T C_q^{RD}(z) - \frac{z}{\pi} \Delta_T^{\overline{MS}} P_{qq}^\epsilon(z) \quad (14)$$

We find :

$$\Delta_T^{\overline{MS}} P_{qq}^n = C_F \left\{ \frac{2z}{(1-z)_+} + \epsilon(1-z) + \frac{3+\epsilon}{2} \delta(1-z) \right\} \quad (15)$$

Then it follows that^(b) :

$$\begin{aligned} \Delta_T C_q^{\overline{MS}}(z) &= \frac{C_F}{2\pi} \left\{ \left(\frac{2\pi^2}{3} - 8 \right) \delta(1-z) + 8z^2 \left(\frac{\ln(1-z)}{1-z} \right)_+ \right. \\ &\quad \left. - 4z^2 \frac{\ln z}{1-z} + 4z(1-z) \right\} \end{aligned} \quad (16)$$

At a given Q^2 , $\Delta_T \overline{F}_{q,max}$ can be parametrized by the form

$$\Delta_T \overline{F}_{q,max}(x) = \overline{C} x^{\overline{a}} (1-x)^{\overline{b}} \quad (17)$$

Then the integral of equation (7) can be solved for $\Delta_T F_{q,max}(x)$ by taking the Mellin transform moments and using the convolution theorem. We have determined in detail the moments of terms in (15) and found them in accord with Ref.9 (Appendix B). In solving Eq.(7) as well as in evolving transversities via 2-loop anomalous dimensions we have extensively used the routine DINLAP of IMSL MATH/LIBRARY.

To see the effect of HOC, in Fig.1 we present results at $Q_o^2 = 4 GeV^2$ for $\Delta_T F_{q,max}^{(o)}$ (dashed line, denoted $F^{(o)}$), $\Delta_T \overline{F}_{q,max}$ (dotted, denoted $\overline{F}$), and $\Delta_T F_{q,max}$ (solid, F), and for $q=u, d$ and $s^{(c)}$. We use thruout the 2-loop expression of $\alpha_s(Q)$ with $\Lambda = 0.2 GeV$ and 4 flavors. The effect is seen by comparing F to $F^{(o)}$. Clearly, F is significantly smaller, and this is best seen for $q=u$. A comparison with Ref. [8] makes evident that now the HOC suppress the bound much more.

In Figs 2 and 3 the above procedure is repeated at $Q_1^2 = 100$ and $Q_2^2 = 10^4 GeV^2$; again F is significantly smaller than $F^{(o)}$. However, as Q^2 increases the differences decrease. This is because the effect of HOC, being proportional to $\alpha_s(Q)$, decreases as Q^2 increases.

Now assume that $\Delta_T F_q(x, Q_o) = \Delta_T F_{q,max}(x, Q_o)$, i.e. that at $Q^2 = Q_o^2$, $\Delta_T F_q$ saturates the bound. Using 2-loop anomalous dimensions [9,12-14] we evolve it to $Q^2 = Q_j^2$.

The result is also shown in Figs 2 and 3 with dash-dotted line. We see that for all $q=u,d,s$ the dash-dotted line lies below the solid, i.e that the bound is preserved.

Regarding the choice of scales in the above, for DIS: $Q^2 = -q^2$, where q =gauge boson's 4-momentum; for DY: $Q^2 = M_{l+l-}^2$, where M_{l+l-} =lepton pair's invariant mass (subsequently we call *physical* such scales); for α_s and $[\Delta_{L,T}]F_q$ the same scale was chosen. Departures from such choices can be studied easily; and the necessary changes are indicated in [8]. Also, different sets of distributions $[\Delta_L]F_q$ (e.g. GS-B for $\Delta_L F_q$) can be employed easily.

Finally we wish to explain why the present definition of HOC much suppresses the bound, in particular relative to Ref. [8]. First some preliminaries : The subsequent line of reasoning holds for scales physical or near – physical^(d). Then, relative to LO (Born), in DY, the NLO terms are known to introduce a *large* HOC. This is easily seen from Eq.(16): At least for x not too small, the HOC are dominated by the contributions of terms involving distributions. There are three such terms^(e) : (a) $(2\pi^2/3) \delta(1-z)$, (b) $-8 \delta(1-z)$ and (c) $8 (\ln(1-z)/(1-z))_+$, and their contributions are comparable and large in size. Roughly, the contribution of the term (c) cancels that of (b), so the term (a) remains as large HOC. In contrast, in DIS, the NLO terms are known to introduce a *small* HOC. Again, this is seen from Eqs (8)-(10): First, $[\Delta_L]C_g$ contain no distributions and lead to rather small HOC. Second, $[\Delta_L]C_q$ do contain distributions, but, roughly, their contributions cancel.

The subsequent argument, although essentially valid for all flavours $q=u,d,s$, is best illustrated by looking to $q=u$, at $Q^2 = Q_o^2$ (Fig.1). Now, both here and in [8], the effect of HOC is seen by first comparing $\bar{F}$ and $F^{(o)}$. Here, due to the smallness of DIS HOC, $\bar{F}$ and $F^{(o)}$ differ little. In [8], due to the large DY HOC, $\bar{F}$ and $F^{(o)}$ differ much. Next, compare the final bound F with $\bar{F}$ [Eq.(7)]: Both here and in [8], $\Delta_T C_q$ is determined via DY, so it gives a large contribution; hence F is suppressed relative to $\bar{F}$. It follows that here F is much smaller than $F^{(o)}$, i.e. that the bound is significantly suppressed by the HOC.

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Footnotes

(a) The presence of the NLO quantities $\Delta_L F_q^{NLO}$ in the r.h.side of(4) should not cause confusion: keep in mind that the sets CTEQ4M and GS-A are determined via 2-loop anomalous dimensions.

(b) Note that here and in [8], $\Delta_T C_q(z)$ is defined in terms of the projection of the contributing amplitudes to the hadronic tensor [8]. The above corrections $[\Delta_L]C_q$ and $[\Delta_L]C_g$ are defined similarly. Note that expression of the cross section contains an additional term $\sim -6z^2 \ln^2 z/(1-z)$ [20].

(c) Note that the whole calculation of [8] is carried at $Q_o^2 = 4 \text{ GeV}^2$. Hence there is no question of evolution, and statements to the effect that [8] does not use 2-loop transversity anomalous dimension are irrelevant.

(d) We have not considered optimal scales, as e.g. those of the Principle of Minimal Sensitivity or of Fastest Apparent Convergence. Some educated guess (A.P.Contogouris, N.Mebarki and H. Tanaka. Phys. Rev. D37(1988) 2458; A.P. Contogouris and N. Mebarki, ibid D39 (1989) 1464) suggests that the final overall effect of the HOC will be qualitatively similar.

(e) Such terms arise from soft, collinear and virtual gluons.

Figure Captions

Fig.1. The leading order upper bound $\Delta_T F_{q,max}^{(o)}(x, Q)$ (dashed line), the next-to-leading order $\Delta_T F_{q,max}(x, Q)$ (solid line) and the function $\Delta_T \bar{F}_{q,max}(x, Q^2)$ (dotted line) at $Q^2 = Q_o^2 = 4 \text{ GeV}^2$ for $q = u, d$ and s .

Fig.2. The same as Fig.1, but at $Q^2 = Q_1^2 = 100 \text{ GeV}^2$. In addition, dash-dotted line gives $\Delta_T F_{q,max}(x, Q_o)$ evolved to $Q^2 = Q_1^2$ via 2-loop transversity anomalous dimensions.

Fig.3. The same as Fig.1, but at $Q^2 = Q_2^2 = 10^4 \text{ GeV}^2$. Dash-dotted line gives $\Delta_T F_{q,max}(x, Q_o)$ evolved to $Q^2 = Q_2^2$ via 2-loop transversity anomalous dimensions.

$$Q^2 = Q_0^2 = 4 \text{ GeV}^2$$

$$Q^2 = Q_1^2 = 100 \text{ GeV}^2$$

Fig.2

$$Q^2=Q_2^2=10^4 \text{ GeV}^2$$

Fig.3