X-ray iron line reverberation from black hole accretion disks

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ABSTRACT

The relativistically broad X-ray iron line seen in many AGN spectra is thought to originate from the central regions of the putative black hole accretion disk. Both the line profile and strength will vary in response to rapid variability of the primary X-ray continuum source. The temporal response of the line contains information on the accretion disk structure, the X-ray source geometry, and the spin of the black hole. Since the X-ray source will have a size comparable to the fluorescing region of the accretion disk, the general reverberation problem is not invertible. However, progress can be made since, empirically, AGN light curves are seen to undergo dramatic short timescale variability which presumably corresponds to the creation of a single new active region within the distributed X-ray source. The iron line response to these individual events can be described using linear transfer theory.

We consider the line response to the activation/flaring of a new X-ray emitting region. Most of our detailed calculations are performed for the case of an X-ray source on the symmetry axis and at some height above the disk plane around a Kerr black hole. We also present preliminary calculations for off-axis flares. We suggest ways in which future, high-throughput X-ray observatories such as XMM and the Constellation X-ray Mission may use these reverberation signatures to probe both the mass and spin of AGN black holes, as well as the X-ray source geometry.

Subject headings: accretion, accretion disks — black hole physics — galaxies: active — galaxies: Seyfert — X-rays: galaxies
1. Introduction

Recent X-ray spectroscopy has provided the first direct observational probe of the innermost regions of black hole accretion disks. X-ray irradiation of the surface layers of the inner accretion disk by a disk-corona results in the emission of the prominent fluorescent Kα line of iron in the X-ray regime (George & Fabian 1991; Matt, Perola & Piro 1991). The combined action of mildly-relativistic Doppler shifts (due to the orbital motion of the accretion disk material) and strong gravitational redshifts broaden and skew this line in a dramatic and very characteristic manner (Fabian et al. 1989; Laor 1991). Spectroscopy of bright Seyfert nuclei with the Advanced Satellite for Cosmology and Astrophysics (ASCA) confirms that such lines do indeed possess exactly this broadened/skewed profile (Tanaka et al. 1995; Nandra et al. 1997a). Moreover, alternative broadening/skewing mechanisms (i.e., those not involving black hole accretion disks) appear to fail (Fabian et al. 1995). This is the first direct evidence for a relativistic accretion disk in an AGN. Furthermore, this emission line provides us with a “clock” in orbit about a black hole with which we can study strong gravity in detail.

The observed line profile depends upon the space-time geometry, the accretion disk structure, and the pattern of X-ray illumination across the surface of the disk. It is well known that the X-ray source in many AGN is highly variable on short timescales. For example, during ASCA performance verification observations of the bright Seyfert galaxy MCG−6-30-15, the ASCA band X-ray flux was observed to undergo a step-like doubling during a period of $\sim 100$ s or less (Reynolds et al. 1995). The timescale of this event is probably comparable to, or less than, the light crossing time of the black hole event horizon (see Section 7). Physically, this event most likely corresponds to the activation of a new flaring region within the X-ray emitting disk-corona. The subsequent change in the X-ray illumination pattern on the accretion disk, coupled with the finite light travel times within the system, will lead to temporal changes in the iron line profile and strength. Such changes are known as reverberation effects.
Even when observing a bright Seyfert galaxy, ASCA (which to this date represents the state-of-the-art in X-ray spectrometers) can only achieve a count rate in the iron line of $10^{-2} \text{photon s}^{-1}$. The long integration times required to define the line strength and profile given this count rate (~1 day) will average over the reverberation effects described in the above paragraph. While still providing extremely exciting results, these *time-averaged* studies will always have limitations. First, the length scales relevant to the time-averaged lines are expressible purely in terms of the gravitational radius $r_g = Gm/c^2$, where $m$ is the mass of the central black hole. Thus, time-averaged line profiles alone cannot determine the absolute value of $r_g$ and hence the mass of the black hole. Second, fitting time-averaged models to time-averaged line profiles results in a degeneracy which prevents one from disentangling the geometry of the X-ray source, the structure of the accretion disk, and the spin of the black hole. The ‘very-broad’ state of the iron line in MCG−6-30-15 as found by Iwasawa et al. (1996) provides an excellent illustration of this problem. By assuming that the X-ray source is a thin corona over the surface of the disk with an X-ray flux proportional to the viscous dissipation in the underlying disk (Page & Thorne 1974), the models of Dabrowski et al. (1997) suggest that the black hole must be rapidly spinning with a dimensionless spin parameter of $a > 0.94$. However, Reynolds & Begelman (1997; hereafter RB97) present an alternative scenario in which a high latitude X-ray source, displaced somewhat from the disk, excites appreciable fluorescence from inside the radius of marginal stability. They show that the MCG−6-30-15 result is compatible with a line emitted from Schwarzschild geometry (i.e., a non-rotating black hole). Young et al. (1998) demonstrated that a significant Compton reflection component (primarily a strong iron edge) is predicted by the RB97 scenario. However, it is a subject of debate whether current data are able to rule out the RB97 picture via the non-detection of the reflected continuum/edges.

Reverberation effects hold the key to unlocking the full diagnostic power of broad iron lines. Iron line reverberation was first explicitly discussed in detail by Stella (1990). He calculated the detailed line variability from a disk around a Schwarzschild black hole given a step-function
doubling in the flux of the X-ray source which was assumed to be at the exact geometric center of the disk. This work was extended by Matt & Perola (1992) and Campana & Stella (1993, 1995), who calculated the temporal behavior of moments of the line (considered by these authors to be more observationally convenient), as well as some alternative source geometries. The principal goal of these studies was to suggest how iron line reverberation can be used to measure the mass of the black hole.

In this paper we complement and extend this previous work in the light of current observational issues. In particular, our model for computing iron line reverberation assumes a black hole with an arbitrary spin parameter (assumed to be consistent with the cosmic censorship hypothesis, i.e., $|a| < 1$) and we search for diagnostics of this spin in the observed reverberation signatures. In Section 2 we discuss the general structure of the iron line reverberation problem and show it to be qualitatively more difficult than reverberation mapping of the broad line region (BLR; see Peterson 1993 for a review of BLR reverberation mapping techniques). In Section 3 we outline our assumptions and discuss their immediate consequences. In Section 4 we discuss the computation of (time-dependent) iron line profiles. Detailed reverberation calculations for on-axis sources are presented in Section 5. In Section 6 we briefly address the issue of off-axis flares and show two preliminary off-axis calculations. To illustrate the use of these techniques, we discuss the determination of the X-ray source geometry and black hole spin for the case of MCG–6-30-15 in Section 7. Our conclusions are drawn together in Section 8.

2. The general nature of the problem

The techniques and formalism of reverberation have been applied extensively to the problem of mapping the broad line region (BLR) of AGN. The size of the BLR (i.e., light weeks to light months) is very large compared to the source of ionizing UV/X-ray photons. Hence the problem that these studies address is the following: what geometry of line-emitting plasma is required to
reproduce the observed line variability given excitation by a (variable) point source of radiation at the center. Assuming that the local line emission is linearly proportional to the instantaneous ionizing flux, one can write a linear transfer equation

\[ F_{l}(\nu, t) = \int_{-\infty}^{\infty} F_{c}(t - t') \psi(\nu, t') \, dt' \]  

(1)

where \( F_{l}(\nu, t) \) is the observed line flux at frequency \( \nu \) and time \( t \), \( F_{c}(t) \) is the observed ionizing continuum flux at time \( t \), and \( \psi(\nu, t) \) is known as the 2-dimensional transfer function. In writing this equation, it has been assumed that the ionizing continuum possesses a fixed spectral form and varies only in normalization. Integrating \( \psi(\nu, t) \) over frequency gives the corresponding 1-dimensional transfer function, \( \psi_{1d}(t) \). Given well-sampled data of sufficient quality, this equation can be inverted in order to obtain the 2-d transfer function. Much of the attention in BLR reverberation studies has concentrated upon measuring this transfer function from large datasets and comparing with theoretical calculations in order to probe the BLR geometry.

Iron line reverberation mapping is a qualitatively different endeavor. Unlike the BLR studies, the geometry of the reprocessor (i.e., the accretion disk) is relatively well understood. Instead, the unknowns are the geometry of the X-ray source and the spin of the black hole (i.e., the geometry of spacetime). The fact that the X-ray source is very likely distributed destroys the linear nature of the reverberation problem. To see this, suppose that we divide the (distributed) X-ray source into a number of sub-units, each of which is small enough to be considered as a point source. Suppose that the iron line response to the continuum emission from each subunit is linear in the sense that it obeys equation (1). Summing up the contribution from all subunits, the overall observed line flux can be written as

\[ F_{l}(\nu, t) = \int_{-\infty}^{\infty} F_{c}(t - t') \sum_{n} f^{(n)}(t - t') \psi^{(n)}(\nu, t') \, dt' \]  

(2)

where \( f^{(n)}(t) \) is the fraction of the observed continuum coming from the \( n \)th source subunit at time \( t \), and \( \psi^{(n)}(\nu, t) \) is the 2-d transfer function for the \( n \)th subunit. Since different source subunits are
in spatially different locations, the functions $\psi^{(n)}$ will generally be different for different $n$. Thus, the only situation in which eqn (2) can be cast into the form of eqn (1) is when all of the $f^{(n)}$ are time-independent. This corresponds to the acausal, and hence unphysical, scenario in which all parts of the distributed X-ray source vary simultaneously and in direct proportion to one another. Counting the number of degrees of freedom, it is clear that equation (2) cannot be inverted to give $f^{(n)}(t)$ and $\psi^{(n)}(\nu, t)$ in terms of $F_l(\nu, t)$ and $F_c(t)$.

Despite the horrors described above, progress is possible. We know that the X-ray light curves of AGN often show extremely rapid flare and step-like events. Some of these events occur on times comparable to, or shorter than, the lightcrossing time of the central black hole (see discussion in Section 1). If these events correspond to the activation of single isolated regions of the X-ray corona, the iron line response to these events might be describable in reasonably simple terms (i.e., using linear response theory). This dictates the approach that we shall take in this paper. Rather than addressing the general problem of extracting information from general forms of $F_l(\nu, t)$ and $F_c(t)$, we shall examine the line response in specific scenarios with the aim of creating specific observational diagnostics.

3. The disk-flare model

In this section, we describe the basic model of the accretion disk and X-ray flare that is utilized throughout the rest of this paper. Our assumptions are tailored to the case of Seyfert galaxies since this will be the first class of systems in which iron line reverberation will be accessible observationally. In essence this model is the Kerr geometry generalization of the model used by RB97. However, it must be stressed that whilst RB97 focussed on line emission from within the radius of marginal stability ($r = r_{ms}$), that is not the principal aim in this work. Line emission from within $r = r_{ms}$ is included in this model using the simple prescription of RB97 but is only of importance in certain extreme regimes of parameter space. Thus, the unmodelled
physical complexities of this region such as the effect of strong shear on radiative transfer or
the possible presence of a dynamically important magnetic field, do not represent a significant
limitation of our model.

3.1. The accretion disk

We consider an accretion disk in prograde orbit about a rotating black hole which has mass
\( m \) and dimensionless spin-parameter \( a \). The disk is assumed to be geometrically-thin and is taken
to lie in the symmetry plane of the rotating black hole. The mass of the disk is assumed to be
negligible compared to the black hole, allowing the space-time of the region to be described by
the Kerr metric.

A crucial physical quantity is the velocity field of the disk material. We use the following
approximation for the accretion disk velocity field which is valid due to our assumption of a
geometrically-thin disk. Outside of the radius of marginal stability (\( r > r_{\text{ms}} \)), we assume that the
disk material lies on circular free-fall orbits. The small inflow inherent to the accretion process
is not important for our current purposes (i.e. determining iron line diagnostics) and shall be
ignored. Thus, our model velocity field is given by \( \dot{r} = 0, \dot{\theta} = 0 \) together with equations (A5),
(A6), (A8) and (A9). Within the radius of marginal stability, such circular orbits are no longer
stable. We assume that material spirals into the hole on ballistic orbits with the energy and angular
momentum of the disk material at \( r = r_{\text{ms}} \). Then, the velocity field is given by \( \dot{\theta} = 0 \) together
with equations (A5), (A6), (A7) and the conditions that the energy is \( E = E(r_{\text{ms}}) \) and the angular
momentum is \( l = l(r_{\text{ms}}) \).

To produce appreciable X-ray reflection and iron line fluorescence upon X-ray illumination,
the disk must be optically thick to electron scattering. For any reasonable set of AGN parameters,
this condition is easily satisfied outside of the radius of marginal stability (e.g., see Frank, King
& Raine 1992). Even within the radius of marginal stability, mass continuity reveals that the disk is still Thomson thick unless the accretion rate is rather small (i.e., less that a few per cent of the Eddington rate; RB97). In this paper, we shall assume that the disk is optically-thick at all radii of interest.

3.2. The X-ray flare

As motivated by the discussion at the end of Section 2, we will be considering iron line reverberation due to an isolated X-ray flare. The flare is assumed to be a point source, emitting an instantaneous flash, with Boyer-Lindquist coordinates \((r_s, \theta_s, \phi_s)\). The flare is assumed to be an isotropic emitter in the locally non-rotating frame.

We calculate the illuminating X-ray flux as seen by the disk material, \(F_X(r, \phi)\), as follows. For a particular source position \((r_s, \theta_s, \phi_s)\) we numerically integrate an isotropic distribution of photons through the Kerr geometry from the source to the accretion disk, event horizon, or ‘infinity’ (operationally defined at \(r = 1000m\)). We use the expressions of Karas, Vokrouhlický & Polnarev (1992), quoted in the Appendix, to enforce this isotropy. Thus, we can calculate the illuminating X-ray flux as seen in the inertial frame of the disk material, taking into account the gravitational focusing effects and the Doppler effects caused by the orbital motion of the disk material. To facilitate our reverberation calculations, we keep track of the time taken \(t_s(r, \phi)\) for photons to travel from the source to position \((r, \phi)\) on the disk.

3.3. Ionization of the disk material

The ionization state of the disk has an important bearing on its X-ray reflection properties and hence must be considered. Motivated by ASCA observations of Seyfert galaxies (Tanaka et al. 1995, Reynolds 1997, Nandra et al. 1997a), and theoretical studies (Matt, Fabian & Ross 1993,
1996) we assume that the disk outside of the radius of marginal stability is ‘cold’ in the sense that it produces a 6.4 keV fluorescent iron line upon X-ray illumination\(^4\).

Inside the radius of marginal stability, the density drops precipitously and hence photoionization can lead to significant ionization of the disk material. We define the X-ray ionization parameter by

\[
\xi(r, \phi) = \frac{4\pi F_X(r, \phi)}{n(r) \cos \alpha},
\]

where \(F_X\) ionizing flux striking the disk (assumed to be a power-law of photon index \(\Gamma = 2\) between 13.6 eV and 100 keV), \(n(r)\) is the electron number density, and \(\alpha\) is the angle that the incoming rays make with the disk normal (all quantities measured in the local inertial frame of the disk material). The electron density \(n(r)\) is determined from the continuity equation. It is readily shown that the expressions of RB97 generalize to the Kerr metric, giving

\[
n = \frac{\dot{m}}{4\pi r h_{\text{disk}} (-u^r)m_p},
\]

where \(\dot{m}\) is the mass accretion rate and \(h_{\text{disk}}\) is the half thickness of the disk (see the Appendix for a sketch of the relativistic derivation of this equation). We take \(h_{\text{disk}}/r = 10^{-2}\) to be representative (Muchotrzeb & Paczyński 1982).

Once the ionization parameter has been determined, the response of the line emission to the ionization state is computed using a slightly modified form of the simple prescription of RB97 (based on the detailed calculations of Matt, Fabian & Ross 1993, 1996). In this prescription, each point of the disk is treated as being in one of four ionization zones. For \(\xi < 100 \text{ erg cm s}^{-1}\), we assume a cold iron fluorescence line at 6.4 keV with the strength given by the standard neutral slab

\(^4\)In higher luminosity AGN, which are operating closer to the Eddington limit, even the region outside of the radius of marginal stability may be appreciably ionized. These effects have been considered theoretically by Matt, Fabian & Ross (1993, 1996), and may have been observed by Nandra et al. (1997b).
calculations (e.g., George & Fabian 1991). For ξ in the range of 100 erg cm s\(^{-1}\) to 500 erg cm s\(^{-1}\), we assume no line emission due to the efficient resonance trapping and Auger destruction of these line photons. For ξ in the range 500 erg cm s\(^{-1}\) to 5000 erg cm s\(^{-1}\), we assume a blend of helium-like and hydrogen-like iron line emission with a rest-frame energies of 6.67 keV and 6.97 keV, respectively, with an effective fluorescent yield for each line the same as that for the neutral case. For ξ > 5000 erg cm s\(^{-1}\), the material is taken to be completely ionized and no line emission results.

3.4. Caveats and realities

The model described above captures the essentials necessary to perform our calculations. However, some of its approximations and simplifications deserve a brief mention. One major simplification is that we have chosen not to use a full transonic disk model, favoring instead the ballistic approximation made in RB97. However, this approximation is accurate to within a few percent due to the assumption that the disk is geometrically-thin and hence the material orbits are essentially ballistic. This is ample accuracy for the current purposes.

We have also neglected the vertical structure of the accretion disk, choosing to treat it as possessing a uniform vertical structure. In the cold region outside of the radius of marginal stability, this is not an important omission since the exact ionization balance is unimportant and the surface layers of gas are optically-thin to the fluorescent iron line photons. For most of the relevant regions of parameter space, this comparatively well understood region dominates the observed iron line properties. Within the radius of marginal stability, where ionization effects are important, the exact vertical structure will effect the relevant ionization state and hence the iron line emission. Equally important will be the complex radiative transfer effects as the reprocessed X-ray photons (including the fluorescent iron line photons) propagate through the ionized, strongly shearing, disk material. Such calculations are well beyond the scope of this work.
that the region inside the radius of marginal stability usually has a small observable influence, our simple scheme seems appropriate for application to all foreseeable iron line observations.

4. Calculation of observables

We suppose that the observer is situated at a large distance from the black hole system and is viewing the system with inclination $i$ (where $i = 0$ corresponds to viewing the accretion disk face-on). We shall define the Cartesian coordinate system $(x, y)$ on the observer’s image plane. The origin of this coordinate system is the place where a ‘radial’ ray from the black hole (i.e., $\phi = \theta = \text{constant}$ in Boyer-Lindquist coordinates) intercepts the image plane. Photon paths (i.e., null geodesics) are numerically integrated through the Kerr metric from points $(x, y)$ on the image plane (which for practical purposes we place at $r = 1000m$) to the disk plane\(^5\). The photon paths are integrated using the four constants of motion: the energy, azimuthal angular momentum, photon rest mass and the Carter constant. The relevant equations of motion governing these photon paths are reported explicitly in the Appendix. Note that this procedure assigns a unique point $(r, \phi)$ on the accretion disk to each point $(x, y)$ on the image plane.

From the results of these integrations, we can evaluate the following functions which are defined on the image plane.

1. $R(x, y)$ — the radius $r$ of the point in the accretion disk which is viewed at the point $(x, y)$ in the image plane.

2. $\Phi(x, y)$ — the azimuthal angle $\phi$ of the point in the accretion disk which is viewed at the

\(^5\)If the condition $r < 1.05r_{\text{evt}}$ (where $r_{\text{evt}}$ is the radius of the event horizon and is reported in the appendix) is satisfied at any point along the photon path, the photon is deemed to have entered the event horizon and the integration is terminated.
point \((x, y)\) in the image plane.

3. \(g(x, y)\) — the frequency boost factor of photons that are emitted from the accretion disk (with frequency \(\nu\) in the inertial frame of that point in the accretion disk) and pass through the point \((x, y)\) on the image plane (with observed frequency \(g(x, y)\nu\)).

4. \(t(x, y)\) — the time taken for a photon to be emitted from the accretion disk and pass through the point \((x, y)\) on image plane.

The time dependent line (photon) flux \(F_l(\nu, t)\) which is driven by the isolated X-ray flare can then be calculated by

\[
F_l(\nu, T) d\nu dT \propto \int \int_{T \to T + dT, \nu \to \nu + d\nu} A(x, y) F_X(R, \Phi) g^3 \, dx \, dy
\]

where \(T(x, y) = t(x, y) + t_s(R(x, y), \Phi(x, y))\), and the integration is over those regions of the image plane with \(T\) in the range \(T \to T + dT\) and \(\nu = g\nu_0\) (where \(\nu_0 \equiv 6.4\, \text{keV}\) is the rest-frame frequency of the emission line) in the range \(\nu \to \nu + d\nu\). \(A(x, y)\) is the proper area of the disk element subtended by the image plane pixel at position \((x, y)\). This factor accounts correctly for the general relativistic solid angle transformations.

5. **Reverberation signatures for on-axis flares**

First, we consider iron line reverberation with an X-ray source situated on the symmetry axis of the black-hole/disk system, at a height \(r = h\) above the disk plane. Physically, this would be an appropriate geometry if the X-ray flares occur at high latitude in a geometrically-thick disk-corona. Alternatively, X-ray flaring from the base of an axial jet would be well modelled by these calculations. Figure 1 gives the 2-dimensional transfer functions for the case of Schwarzschild geometry \((a = 0)\). We have assumed a source height of \(h = 10Gm/c^2\), a source efficiency of \(\eta_s = 0.01\), and have shown four observer inclinations \((i = 3^\circ, 30^\circ, 60^\circ, 80^\circ)\). The
zero of the time variable is defined as the moment when the direct radiation from the pulsing source reaches the observer. The 2-d transfer functions are most readily interpreted by noting that vertical ‘slices’ correspond to line profiles at some instant in time.

First, consider the almost face-on case (i.e., $i = 3^\circ$; note that we avoid the exactly face-on case so as to avoid the coordinate singularity present at the poles). There is an initial delay between the observed pulse and the response in the line which is simply due to light travel times. If one were to image the disk at subsequent times, the line emitting region would be an expanding ring centered on the disk. Initially, the line emission will come from the innermost regions of the disk and will be highly redshifted by gravitational redshifts and the transverse Doppler effect. As the line-emitting region expands, these effects lessen and the observed line frequency tends to the rest-frame frequency.

As one considers higher inclination systems, Doppler effects come into play and the line is broadened. The time delay between the observed pulse and the line response is also shortened due to the geometry. At moderate-to-high inclinations, a generic feature appears in the line response whose presence is a direct consequence of relativity. Soon after the line profile starts responding to the observed flare, the red wing of the line fades away. During these times, the red wing of the line is due to emission from the front and receding portions of the disk, with gravitational redshifts being the dominant effect. Some time later, the observed ‘echo’ of the X-ray flare reaches the back side of the disk, whose solid angle at the observer is enhanced by lensing around the black hole itself. When the echo reaches this region, the red-wing of the line dramatically recovers before finally fading away with the rest of the line response.

An interesting feature can be seen in the high inclination systems (i.e., $i = 60^\circ$ and $i = 80^\circ$). The transfer functions for these inclinations show a high-energy ‘double loop’ at $t \sim 20Gm/c^3$ corresponding to distinct high-energy peaks in the iron line profile. This is due to iron line emission from ionized helium and hydrogen-like iron which exists in a narrow annulus just inside
the innermost stable orbit. In time-averaged line profiles, this ionized emission would be seen as isolated high-energy peaks (e.g., see Fig. 5 of RB97). Conversely, the observation of such isolated high-energy peaks in a high inclination source would be evidence that we are witnessing a sharp change in the ionization structure of the disk. This, in turn, would be evidence that we are seeing the radius of marginal stability around a slowly rotating black hole.

Similarly, Fig. 2 shows the 2-d transfer functions for a maximally rotating Kerr black hole and the same source parameters. Again, four inclinations are shown ($i = 3^\circ, 30^\circ, 60^\circ, 80^\circ$). Note that we define a maximally rotating hole, following Novikov & Thorne (1973), to be one with a spin parameter of $a = 0.998$. The innermost stable orbit, and hence by assumption the Keplerian part of the accretion disk, is much smaller than in the Schwarzschild case ($r_{ms} = 1.23 Gm/c^2$ for $a = 0.998$). This immediately leads to an interesting phenomenon in the line reverberation which is most simply illustrated by considering the face-on case (Fig. 3a). At a time of $\sim 25 Gm/c^3$, the observer sees two rings of line emission — one is propagating outwards into the disk (as in the Schwarzschild case) and the other is propagating inwards towards the event horizon. This second ring corresponds to line photons that have been delayed due to their passage through the strongly curved space in the near vicinity of the hole (i.e., the Shapiro effect). This produces a small red bump in the observed line spectrum which moves progressively to lower energies as time proceeds. In the case of a truly-maximal Kerr hole (i.e., one with $a = 1$), our model would have the Keplerian disk extending exactly to the horizon and this inward moving ring would be seen to tend asymptotically to the event horizon (with ever increasing redshifts). This phenomenon is further illustrated in Fig. 3, where we show the observed line profiles at various times. This effect is more prominent for sources closer to the disk (i.e., smaller $h$). Our calculations show these redwards moving bumps to be generic features of line reverberation around near-extremal kerr holes, and hence may be considered direct observational signatures of near-extremal kerr geometry.
In the immediate future (i.e., prior to the launch of XMM), only the time-lag of the pulse and response is within observational reach. It is interesting to compare the minimum time-lags derived from our fully relativistic calculations with those expected in Euclidean geometry. In Euclidean geometry, it is readily shown that the minimum time lag (i.e., the time between the observed continuum pulse and the first observed response from the disk line) is

\[ t_{\text{delay}} = \frac{2h}{c} \cos i. \]  

(6)

This is plotted in Fig. 4 and compared to the results from our relativistic calculations. Note that for low inclinations, there is an extra time-lag due to the time taken for photons to propagate through the strongly curved space in the vicinity of the black hole. Noting that \( i \) can be obtained from time averaged line profiles, the observation of such a lag allows \( h \) to be determined.

Further in the future, high throughput instruments such as XMM and the Constellation X-ray Mission (formerly HTXS) will be able to measure the time response of several different bands covering the line profile. Measuring the response of the line profile to a dramatic event in the continuum light curve will allow the 2-d transfer function (for that event; see Section 2) to be mapped out. Again, noting that \( i \) can be constrained from the time-averaged line profiles, comparing such data to theoretical transfer functions will allow the mass and spin of the hole to be constrained. In a future publication, we will present detailed simulations of Constellation-X observations and determine the possible constraints that can be applied to the source parameters.

6. Reverberation signatures for off-axis flares

If the X-ray emission of accreting black holes does indeed originate in an accretion disk corona, we would expect most of the X-ray flares to occur significantly away from the symmetry axis. This clearly expands the parameter space that we must explore: the instantaneous motion of the flare as well as its latitude and azimuth must be specified in order to calculate the
corresponding illumination pattern on the disk. A detailed investigation of iron line reverberation from off-axis flares will form the basis for a future publication. However, for completeness we present a representative off-axis calculation to highlight the principal effects. In these preliminary calculations, the flare is assumed to be an isotropic emitter in the locally non-rotating frame.

In Fig. 5 we show the 2-d transfer functions for an off-axis flare with $r_s = 10r_g$ and $\theta_s = 70^\circ$ on both the receding side of the disk ($\phi_s = 90^\circ$; panel a) and the approaching side of the disk ($\phi_s = 270^\circ$; panel b). We assume an extreme-Kerr hole ($a = 0.998$) and an observer inclination of $i = 30^\circ$ (appropriate for a typical Seyfert 1 nucleus). These figures are to be contrasted with Fig. 3b, which shows the on-axis flare case for the same spin-parameter, inclination and value of $r_s$.

As expected, X-ray flares close to the disk tend to produce briefer and narrower line responses. In the limiting case of flares in a a thin, disk-hugging X-ray emitting corona (i.e., the limit $\theta \to 90^\circ$ of the above calculations) the line response to the continuum flare will be essentially immediate and monochromatic. In other words, the 2-d transfer function becomes a delta-function $\psi(\nu,t) = \phi_0 \delta(\nu - \nu_f) \delta(t)$ where the observed frequency of the line response $\nu_f$ is dependent on the radius and azimuth of the flare, the spin-parameter of the hole, and the observer inclination.

7. MCG–6-30-15 revisited

As an illustration of the use of iron line reverberation, we return to the case of MCG–6-30-15. As mentioned in the introduction, the very-broad state of the iron line in this Seyfert 1 galaxy as found by Iwasawa et al. (1996) can be reasonably modeled as either emission from a disk around a near-extremal Kerr hole (Iwasawa et al. 1996; Dabrowski et al. 1997), or emission from within the innermost stable orbit of a slowly rotating hole (RB97). A basic degeneracy exists...
insofar as both of these models can be made consistent with current time-averaged iron line data. Reverberation studies are the key to unambiguously distinguishing these effects.

A key ingredient of the RB97 model is an X-ray source which is displaced from the disk by heights of \( h \sim 4Gm/c^2 \) to \( h \sim 12Gm/c^2 \) (depending on exactly which state the system is in). Physically, this was imagined to be X-ray emission from near the base of a jet or an extended corona. The region inside the innermost stable orbit is then strongly illuminated by this emission due to gravitational focusing and blueshifting as the photons ‘fall’ onto the disk. Additionally, in order for this region not to be almost completely ionized by this flux, the X-ray efficiency of the source has to be rather low (\( \eta_x \sim 10^{-3} \)). Figure 6a shows the 2-d transfer function of the RB97 scenario for the very-broad state of the iron line in MCG–6-30-15 (\( i = 27^\circ, a = 0, h = 4Gm/c^2 \) and \( \eta_x = 10^{-3} \)).

On the other hand, the extremal-Kerr model for this line (Dabrowski et al. 1996) assumes a thin-coronal geometry. In this case, a flare from the continuum source will induce an immediate monochromatic response in the line as described in Section 6. This should be easily distinguishable from the scenario of the previous paragraph. Of course, a possible hybrid model for the very broad line is a displaced source above a disk around a rapidly-rotating black hole (Martocchia & Matt 1996). The transfer function for this case is shown in Fig. 6b. This will be rather more difficult to distinguish from the RB97 model. However, sufficiently detailed constraints on the 2-d transfer function, especially on the re-emergence of the red wing of the line, can distinguish these models.

A crucial parameter in these studies is the light crossing time of the gravitational radius (\( t_g = Gm/c^3 \)). For the case of MCG–6-30-15, the multiwaveband analysis of Reynolds et al. (1997) estimated a lower-limit on the bolometric luminosity of the AGN to be \( L \gtrsim 1 \times 10^{44} \text{erg s}^{-1} \). The fact that the disk (outside of the innermost stable orbit) is in a physical state capable of producing cold iron fluorescence suggests that the system is operating at no more that 10 per
cent of its Eddington rate. Thus, a lower-limit to the mass of the black hole in MCG−6-30-15 is $M \gtrsim 10^7 M_\odot$. The corresponding timescale is $t_g \gtrsim 50$ s. Given the nature of these limits, this timescale could easily be an order of magnitude larger. Taking the conservative figure of $t_g = 50$ s, the RB97 model for the ‘very-broad’ state of the system predicts a time lag of approximately $400$ s between a continuum flare and the response of the iron line.

8. Conclusions

We have examined how broad X-ray iron lines respond to variability in the primary X-ray source, i.e., we have addressed the reverberation of fluorescent lines from the innermost regions of an accretion disk around a black hole.

In any physical model for the X-ray source (e.g., disk-corona or jet), the source is likely to be extended with a size comparable to the region of the disk producing the line. In this case, there is no well-defined transfer function that relates the continuum light curve to the time-dependent line profile. Hence, it is not possible to perform a general inversion and constrain the source geometry or black hole spin-parameter from continuum/line light curves. The analogous problem in optical/UV BLR studies is invertible since the exciting UV source is very much smaller than the BLR. Despite this fundamental non-invertibility, progress is possible since AGN light curves are known to undergo dramatic events which presumably correspond to the activation of a new localized active region. The response of the iron line emission to the activation of such a region will possess clean reverberation signatures (and can be described in terms of a transfer function for that particular active region).

Our detailed calculations assume an X-ray source which is located on the symmetry axis of the black-hole/disk system and at a height $h$ above the disk plane. We have computed the line flux as a function of energy and time when the X-ray source emits a $\delta$-function pulse of hard X-ray
radiation. This amounts to computing the 2-dimensional transfer function for the problem. The
most basic consequence of having the X-ray source displaced from the disk is a time lag between
the observed continuum pulse and the first response of the line. This lag is slightly longer than,
but approximated by, the Euclidean result presented in equation (6). The ‘excess’ lag is due to the
passage of the photons through the strongly curved spacetime in the vicinity of the black hole.
The detection of this lag (which should be within reach of RXTE) will allow the height of the
source, \(h\), to be measured. We also briefly address the off-axis flare case.

From our computations of the general 2-d transfer functions, we note several interesting
features.

1. A generic feature of line variability in moderate-to-high inclination systems is a ‘re-
emergence’ of the red-wing some time after the observed X-ray flare. This is gravitationally
redshifted emission from the back-side of the disk that has been enhanced by gravitational
lensing around the hole. The exact nature and timing of this ‘re-emergence’ effect depends
upon the mass and spin of the black hole, and should be fairly readily observable by future
high-throughput spectrometers.

2. An observer viewing line reverberation from a disk around a rapidly rotating hole will
see a distinct bump in the line profile that progresses to lower energies as time proceeds.
If the observer could image the disk, this spectral feature would originate from a ring of
emission moving asymptotically towards the horizon. It corresponds to photons that have
been delayed by their progress through the strongly curved spacetime around the black
hole. Since the formation of such a feature requires a relatively cold disk close to the event
horizon, its detection would indicate the existence of a rapidly rotating black hole. Indeed,
our calculations show this feature to be a good signature of near extremal kerr geometry.
We note, however, that a detection of this feature would be challenging since it is weak and
would be largely swamped by the primary continuum emission.
3. A high inclination observer viewing a line from around a slowly rotating hole may observe a distinct high-energy peak on the iron line profile. This corresponds to line emission from ionized material within the radius of marginal stability. Although the signature of this emission is more clearly separated from the bulk of the line emission if one considers time-resolved line profiles, this feature is also observable in the time-averaged line profile. The detection of this distinct high-energy peak, which should be easily within reach of XMM and Constellation, would strongly argue for a Schwarzschild black hole.

Starting with Constellation, future instruments should be able to measure the detailed response of the iron line profile to rapid changes in the primary X-ray continuum. Given such data, it will be possible to place unprecedented constraints on the geometry of the system, the mass of the black hole, and even the black hole’s spin.

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A. Some properties of the Kerr metric

In this appendix, we collect together some standard results on the properties of the Kerr metric which we use in the main body of the paper. These results can be found in several standard texts, but we mostly shall follow the techniques and notation of Shapiro & Teukolsky (1983).

A.1. The metric and the orbital motion of accretion disk material

In standard Boyer-Lindquist coordinates, the Kerr metric is

\[
\begin{align*}
    ds^2 &= - \left(1 - \frac{2mr}{\Sigma}\right) dt^2 - \frac{4amr \sin^2 \theta}{\Sigma} dt d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2amr \sin^2 \theta}{\Sigma}\right) \sin^2 \theta d\phi^2 \\
\end{align*}
\]  

(A1)

where

\[
\begin{align*}
    \Delta &= r^2 - 2mr + a^2 \\
    \Sigma &= r^2 + a^2 \cos^2 \theta
\end{align*}
\]  

(A2) (A3)

This manuscript was prepared with the AAS \LaTeX\ macros v4.0.
and we have chosen units such as to set $G = c = 1$. The event horizon of the black hole is given by the outer root of $\Delta = 0$, i.e. $r_{\text{evt}} = m(1 + \sqrt{1 - a^2})$.

By assumption, the accretion disk lies in the equatorial plane of the rotating black hole and orbits in the prograde sense. Since the disk is assumed to be geometrically-thin, pressure forces are negligible and the disk material will follow free-fall paths through the metric. From the metric, we form the (restricted) Lagrangian $\mathcal{L}_d$ describing free-fall particle orbits in the equatorial plane,

$$\mathcal{L}_d = \frac{1}{2} \left( \frac{ds}{d\tau} \right)^2_{\theta = \pi/2} = \frac{1}{2} \left[ - \left( 1 - \frac{2m}{r} \right) \dot{t}^2 - \frac{4am}{r} \dot{t} \dot{\phi} + \frac{r^2}{\Delta} \dot{r}^2 + \left( r^2 + a^2 + \frac{2ma^2}{r} \right) \dot{\phi}^2 \right]$$

where $\tau$ is an affine parameter that can (and will) be taken to be proper time in the case of massive particles, and the dot denotes differentiation with respect to $\tau$. The Euler-Lagrange equations together with the fact that the Kerr metric is stationary and axisymmetric (i.e. $ds^2$ has no explicit dependence upon $t$ or $\phi$) imply that for a free-fall path

$$\dot{t} = \frac{(r^3 + a^2r + 2a^2m)E - 2aml}{r\Delta} \quad (A5)$$
$$\dot{\phi} = \frac{(r - 2m)l + 2amE}{r\Delta} \quad (A6)$$

where $E$ is the conserved energy ($E = -\partial \mathcal{L}_d / \partial \dot{t}$), and $l$ is the conserved aximuthal angular momentum ($l = \partial \mathcal{L}_d / \partial \dot{\phi}$). For massive particles, the conservation of rest-mass gives $\mathcal{L}_d = -\frac{1}{2}$ which leads to the following radial equation of motion:

$$r^3 \ddot{r} = E^2 (r^3 + a^2r + 2ma^2) - 4amEl - (r - 2m)l^2 - rm^2 \Delta. \quad (A7)$$

Equations (A5)–(A7) are the basic equations of motion for a massive particle in the equatorial plane of the Kerr metric.

Demanding that the orbits be circular (i.e. $\dot{r} = \ddot{r} = 0$) determines the energy and angular momentum as a function of radius. The result is

$$\frac{E(r)}{m} = \frac{r^2 - 2mr + a\sqrt{mr}}{r(r^2 - 3mr + 2a\sqrt{mr})^{1/2}} \quad (A8)$$
A very important result is that such circular orbits are only stable for $r > r_{\text{ms}}$, where the critical radius $r_{\text{ms}}$ is called the \textit{radius of marginal stability} and is given by

$$r_{\text{ms}} = m \left( 3 + Z_2 - [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2} \right)$$

(A10)

where

$$Z_1 = 1 + \left( 1 - \frac{a^2}{m^2} \right) \left[ \left( 1 + \frac{a}{m} \right)^{1/3} + \left( 1 - \frac{a}{m} \right)^{1/3} \right]$$

(A11)

$$Z_2 = \left( 3\frac{a^2}{m^2} + Z_1^2 \right)^{1/2}$$

(A12)

This marginally stable orbit is also sometimes referred to as the \textit{innermost stable orbit}. Hence $r_{\text{ms}} = 6m$ for a Schwarzschild hole ($a = 0$). For prograde orbits around a Kerr black hole, $r_{\text{ms}} < 6m$ and tends to $r_{\text{ms}} \to m$ as $a \to 1$. Note that $r_{\text{evt}} \to m$ also as $a \to 1$. In other words, in the limit of a maximally rotating black hole, the radius of marginal stability extends all of the way down to the event horizon.

\section*{A.2. Continuity equation and the density of the disk}

The fully-relativistic (baryon number) continuity equation is

$$\left( \rho u^\mu \right)_; \mu = 0,$$

(A13)

which, using the metric connection, can be expressed in terms of the determinant of the metric tensor ($\det g$) as

$$\left( \rho u^\mu \sqrt{-\det g} \right)_; \mu = 0.$$  

(A14)

By writing the metric tensor in matrix form, it is readily shown that for flows in the equatorial plane $\det g = r^4$, and so equation (A14) can be integrated to give

$$2\pi r \sigma(r) u^r = \dot{m}$$

(A15)
where \( \dot{m} \) is the mass accretion rate and \( \sigma(r) \) is the surface density. Assuming the disk to possess a uniform density structure in the vertical direction and have a half height \( h_{\text{disk}} \), the density of the disk at a given radius is

\[
n = \frac{\sigma}{2h_{\text{disk}}m_p} \dot{m} = \frac{\dot{m}}{4\pi rh_{\text{disk}}(-u^r)m_p}.
\]

This is equation (2) of the main text.

### A.3. Photon propagation in the Kerr metric

Generalizing the Lagrangian to include orbits not in the plane, we get

\[
2\mathcal{L} = -(1 - \frac{2mr}{\Sigma}) i^2 - \frac{4amr \sin^2 \theta}{\Sigma} i \phi + \frac{\Sigma}{\Delta} i^2 + \Sigma \dot{\theta}^2 + \left( r^2 + a^2 + \frac{2a^2mr \sin^2 \theta}{\Sigma} \right) \sin^2 \theta \dot{\phi}^2
\]

The Euler-Lagrange equations, together with stationarity and axisymmetry give

\[
\dot{t} = \frac{E \left[ \Sigma(r^2 + a^2) + 2mra^2 \sin^2 \theta \right] - 2amrl}{(\Sigma - 2mr)(r^2 + a^2) + 2mra^2 \sin^2 \theta} \tag{A18}
\]

\[
\dot{\phi} = \frac{2amrE \sin^2 \theta + (\Sigma - 2mr)l}{(\Sigma - 2mr)(r^2 + a^2) \sin^2 \theta + 2mra^2 \sin^4 \theta} \tag{A19}
\]

The conservation of photon rest mass gives \( \mathcal{L} = 0 \) which leads to the radial equation of motion

\[
\dot{r}^2 = \frac{\Delta}{\Sigma} (iE - \dot{\phi}l - \dot{\theta}^2 \Sigma) \tag{A20}
\]

The \( \theta \)-equation of motion is most readily obtained from the Carter constant

\[
Q = p_{\theta}^2 - a^2 E \cos^2 \theta + l^2 \cot^2 \theta \tag{A21}
\]

where \( p_{\theta} \) is the canonical \( \theta \)-momentum given by

\[
p_{\theta} = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \Sigma \dot{\theta} \tag{A22}
\]
Hence,
\[ \dot{\theta}^2 = \frac{Q + a^2 E \cos^2 \theta - l^2 \cot^2 \theta}{\Sigma^2} \]  
(A23)

Equations (A18)–(A20) and (A23) are differential equations of motion for a photon in the Kerr metric. Note that since the equations of motion only determine \( \dot{r}^2 \) and \( \dot{\theta}^2 \), the initial signs of \( \dot{r} \) and \( \dot{\theta} \) must be explicitly given. Further, the numerical integrator must search for turning points in \( r \) and \( \theta \) along the path and explicitly change the sign of \( \dot{r} \) and \( \dot{\theta} \) at such turning points.

**A.4. Enforcing isotropy in the X-ray source frame**

We assume the X-ray source to be in a locally non-rotating frame at \((r_s, \theta_s, \phi_s)\). Following Karas, Vokrouhlický & Polnarev (1992), we define \( \alpha_s \) and \( \beta_s \) to be the polar and azimuthal angles on the sky as seen by an observer situated at the X-ray source (see Fig. 1 of Karas, Vokrouhlický & Polnarev). Consider a photon emitted from this source. The constants of motion \( l \) and \( Q \) characterizing its subsequent path are given by

\[ l = \left( \frac{A^{1/2} \sin \theta \sin \alpha_s \sin \beta_s}{\Sigma \epsilon} \right)_{r=r_s, \theta=\theta_s}, \]  
(A24)

\[ Q = \left( \frac{(r^2 + a^2 - al)^2}{\Delta} - \frac{\Sigma \cos^2 \alpha}{\epsilon^2} - l^2 + 2al - a^2 \right)_{r=r_s, \theta=\theta_s}, \]  
(A25)

where

\[ A = (r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta, \]  
(A26)

\[ \epsilon = A^{1/2} \left( \Sigma^{1/2} \Delta^{1/2} + 2ar \Sigma^{-1/2} \sin \theta \sin \alpha_s \sin \beta_s \right). \]  
(A27)

To enable the integration of a given photon away from the source, it remains to determine the initial signs of \( \dot{r} \) and \( \dot{\theta} \). Substituting eqn (A24)-(A27) into (A18)-(A23), we can examine \( \dot{r}^2 \) and \( \dot{\theta}^2 \) as a function of \( \alpha_s \) and \( \beta_s \). The values of \( \alpha_s \) and \( \beta_s \) where \( \dot{r}^2 = 0 \) (or \( \dot{\theta}^2 = 0 \)) delineate the regions of the source’s local sky in which \( \dot{r} \) (or \( \dot{\theta} \)) is initially positive or negative. In fact, the assumption
of a locally non-rotating source leads to the Euclidean-like result:

\[ \dot{r} > 0 \text{ for } 0 \leq \alpha_s < \pi/2, \quad \text{(A28)} \]
\[ \dot{r} < 0 \text{ for } \pi/2 < \alpha_s \leq \pi, \quad \text{(A29)} \]
\[ \dot{\theta} > 0 \text{ for } |\beta_s| < \pi/2, \quad \text{(A30)} \]
\[ \dot{\theta} < 0 \text{ for } |\beta_s| > \pi/2. \quad \text{(A31)} \]