Subject headings: Other planets, solar system: general, comets

Squares detected brighter than a given magnitude \( M \) provide a cumulative sky density \( f(M) \). The observational function for the entire trans-Neptunian region is the same power-law function (estimated from the distribution of known trans-Neptunian objects). The number of squares with bright squares that are significantly brighter than the expected number of squares with objects per a given square is above the expected number of objects per a given square. The number of squares with objects per a given square is above the expected number of objects per a given square. The expected number of objects per a given square is above the expected number of objects per a given square.

ABSTRACT

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Pencil Beam Surveys for P lane Tr ans-Neptunian Objects

1. Introduction and Motivation

Since the discovery of the first so-called Edgeworth-Kuiper Belt object 1992 QB1 (Jewitt and Luu 1993), approximately 65 trans-neptunian objects (TNOs) have been catalogued, ranging in apparent magnitude from about 20 to 24.6 in R-band and in heliocentric distance from 30 to 50 AU. The dynamical structure and properties of objects in this region hold signatures of the outer planet-formation process (e.g., Malhotra 1995, Morbidelli and Valsecchi 1997, Weissman and Levison 1997). Assuming comet-like albedos of $p = 0.04$, the discovered objects have diameters $D$ ranging from $\sim 100$ to 800 km. Albedo uncertainties will effect this size range somewhat, with diameters scaling as $p^{-0.5}$.

The size distribution of TNOs is of great interest. Although originally it had been hoped that the population might be collisionless and thus might retain the signature of the planetesimal formation process, recent work (Davis and Farinella 1997, Stern and Cowell 1997) has shown that collisional effects cannot be neglected over the age of the solar system. However, knowledge of the size distribution is still important for understanding the link between the Kuiper Belt and both the short-period comets (Levison and Duncan 1997) and Pluto (Weissman and Levison 1996).

Dones (1997) provides an excellent review of the open problems in Kuiper Belt research. The HST results of Cochran et al. (1995) provided another strong observational motivation by statistically detecting a very large population of faint trans-neptunian objects near $R = 28$ AU. Our research was partially motivated by attempting to work at intermediate magnitudes.

Figure 1 shows a compilation of previous results, giving the cumulative surface density $\Sigma(m_R < R)$ of TNOs near the ecliptic brighter than a given limiting $R$ magnitude, that is, the luminosity function of the trans-neptunian belt. Linear relations on this plot correspond to power-law behaviour, of the form $\log \Sigma = \alpha (R - R_i)$, implying that the sky density of TNOs increases by a factor $10^{ \alpha}$ with each additional magnitude of depth. It is important to note that there is no consideration in this figure of either the ecliptic latitude of the surveys, nor their elongation relative to Neptune. The sky density of objects is expected to decline with increasing latitude (Jewitt et al. 1996) due to the expected concentration of the belt to the invariable plane of the solar system. A peak in $\Sigma$ is expected in a magnitude-limited sample near $90^\circ$ elongation from Neptune, because TNOs trapped in the 3:2 mean-motion resonance with Neptune have their perihelion concentrated near this elongation, and thus are brighter and more easily detected there (Malhotra 1995). This resonance contains $\sim 40\%$ of the multi-opposition objects, which due to the selection bias reflects an intrinsic population of only $\sim 10$–$20\%$ of the TNOs inside 50 AU (Jewitt et al. 1998). However, almost all of the catalogued objects were detected in surveys conducted within a few degrees of the ecliptic at elongations roughly $90^\circ$ from Neptune.

The goal of our program, begun in 1994, has been to find small TNOs rather than more objects with diameters larger than $\sim 100$ km. Instead of searching large areas of sky to limiting magnitudes of $R \sim 22$–24, we chose to concentrate on one or two fields for each observing run, and integrate for 4–6 hours to reach a limiting magnitude of $R \sim 25$–26 for each field. In essence, the idea is that a power law increase in the sky density of objects at fainter magnitudes will produce objects in the field if the search is faint enough. Thus, going deep enough will allow us to extend the range over which the luminosity function is determined. A previous negative result, covering 0.05 square degrees to a limiting magnitude of $m_R \simeq 25$ was reported in Gladman and Kavelaars (1997), and is represented as the point PAL96 in Fig. 1.

The initial work of Jewitt and Luu (1995) and Irwin et al. (1995) implied shallow slopes for the luminosity function at magnitudes of $R \sim 23$–25, with $\alpha \sim 0.3$ (dotted line in Fig. 1, maximum likelihood fit). This relatively shallow slope required an upper-size cutoff for TNOs to be consistent with the lack of observed objects at brighter magnitudes where various surveys had failed to find them (this information was incorporated into the Irwin et al. maximum likelihood fit). Extrapolation of this shallow slope implied relatively few very faint TNOs ($R \geq 28$), and yet a large population of such TNOs appears to be required to supply the short-period comets from a Kuiper Belt source (Duncan and Levison 1997). The Irwin et al. (1995) fit was in sharp disagreement with the HST result, which statistically detected objects near $m_R \simeq 28$, implying $\sim 30,000$ TNOs/au. There is a continuing controversy in the literature about the HST result (see Brown et al. 1997; Cochran et al. 1998), which will likely only be satisfactorily resolved by repeating the HST experiment. Additional work by Jewitt and Luu (1996) doubled the number of known objects, resulting in a steeper estimate for
the luminosity function slope $\alpha \approx 0.38$ (dashed line in Fig. 1, fitted to the hollow squares and I95 point).

More recently, Jewitt et al. (1998, J1T98 hereafter) completed a large area survey to relatively shallow depth ($R \approx 22.5$), and reported a luminosity function fitted by a single power law passing through $\Sigma(R < 23.3) = 1$ object/arcmin$^2$ with $\alpha \approx 0.58$; i.e., increasing by a factor of $\geq 3.8$ for each fainter magnitude (long-dash line in Fig. 1). The number of detected objects at $R < 22$ was far below that predicted by the Jewitt and Lu (1996) estimate of the luminosity function, implying the even steeper luminosity function. Note that in a flux-limited survey, this steeper rate of increase implies that one would expect three-quarters of all detected objects to be in the last magnitude above the limit, if the detection limit were a strict cut-off (in the form of a Heaviside function). In reality, since the discovery efficiency in a flux-limited sample typically falls from almost 100% to 0 over this faintest magnitude above the limit, only about half of all the objects in the final magnitude are discovered, and thus about 55–65% of all discovered objects would be expected in the last magnitude above the limit (mildly depending on $\alpha$). The recent survey of J1T98 (black squares in Fig. 1), corrected for incompleteness for the faintest magnitudes, shows exactly this behaviour; the previous surveys (summarized in Jewitt et al. 1996, hollow squares in Fig. 1) did not, and we will discuss the implications of this below.

Extrapolating the J1T98 luminosity function to $R = 26$ predicts $\sim 35$ objects/arcmin$^2$, which would imply $\sim 1$ object per $10^6 \times 10^6$ field at $R \approx 26$ (our target magnitude). Obviously given the steep increase of the surface density, most objects found by such a pencil-beam search will be too faint to be followed after the discovery observations in order to obtain an accurate orbit. Thus, the acquisition of a larger multi-opposition orbital database of TNOs is not the primary goal of our work.

Under the assumptions of a constant albedo for all objects and that all objects were at the same heliocentric distance $r$, then one could easily convert the apparent luminosity distribution into a size distribution. Of course, in reality the strong $1/r^4$ dependence of reflected light means one must include a model of the orbital distribution and correct for the effects of the magnitude limited sample. Jewitt and Lu (1996) modeled this, based on surveys with limiting magnitudes ranging from $m_R = 23.2$ to 24.8, and obtained a size distribution in the form of a differential power
### Table 1

<table>
<thead>
<tr>
<th>Dates (UT)</th>
<th>Place</th>
<th>$\theta''$</th>
<th>$A ($''$)</th>
<th>$T_{int}$</th>
<th>$\Delta T$</th>
<th>ZeroPt</th>
<th>50% limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 1995</td>
<td>Palomar</td>
<td>1.7</td>
<td>0.05</td>
<td>21×300s</td>
<td>3.03</td>
<td>25.4</td>
<td>24.8±0.2</td>
</tr>
<tr>
<td>Jan. 1996</td>
<td>Palomar</td>
<td>2.0</td>
<td>0.05</td>
<td>38×300s</td>
<td>7.80</td>
<td>25.4</td>
<td>25.2±0.2</td>
</tr>
<tr>
<td>April 1 1997</td>
<td>CFHT</td>
<td>1.0</td>
<td>0.12</td>
<td>16×480s</td>
<td>4.93h</td>
<td>24.6</td>
<td>24.6±0.2</td>
</tr>
<tr>
<td>April 2 1997</td>
<td>CFHT</td>
<td>1.2</td>
<td>0.12</td>
<td>18×480s</td>
<td>5.60h</td>
<td>24.6</td>
<td>backup April 1</td>
</tr>
<tr>
<td>Sept. 5 1997</td>
<td>Palomar</td>
<td>0.9</td>
<td>0.025</td>
<td>49×300s</td>
<td>5.78h</td>
<td>25.4</td>
<td>backup Sept 6</td>
</tr>
<tr>
<td>Sept. 6 1997</td>
<td>Palomar</td>
<td>0.9</td>
<td>0.025</td>
<td>27×480s</td>
<td>5.80h</td>
<td>25.4</td>
<td>25.6±0.2</td>
</tr>
<tr>
<td>Sept. 7 1997</td>
<td>Palomar</td>
<td>0.9</td>
<td>0.025</td>
<td>21×480s</td>
<td>3.68h</td>
<td>25.4</td>
<td>backup Sept 8</td>
</tr>
<tr>
<td>Sept. 8 1997</td>
<td>Palomar</td>
<td>0.9</td>
<td>0.025</td>
<td>24×480s</td>
<td>4.38h</td>
<td>25.4</td>
<td>25.6±0.2</td>
</tr>
<tr>
<td>Oct. 27 1997</td>
<td>Palomar</td>
<td>1.4</td>
<td>0.025</td>
<td>34×480s</td>
<td>5.80h</td>
<td>25.4</td>
<td>25.2±0.2</td>
</tr>
</tbody>
</table>

**Note.**—The typical seeing $\theta$ and area $A$ for each pencil-beam survey are given, along with the single-night integration time $T_{int}$, the elapsed time $\Delta T$ between the first and last exposures on that night's deep field, and instrumental zero-point. The 50% limit is the R magnitude at which that survey's detection efficiency falls to 50% (see Sec. 4); 'backup' indicates that the exposures were used to confirm candidates from the adjacent night (of superior quality), and did not have their limits directly measured.

2. **Observational procedures and data reduction**

This project was carried out using observations at the Palomar 5-meter and CFHT 3.5-meter telescopes. Since this program was driven by the requirements of observing very faint objects, we describe the instrumentation and data reduction methods before discussing the results obtained during the various observing runs of the program (Table 1).

2.1. **Instrumentation**

The 2048×2048 thinned Tektronix CCD COSMIC was used at prime focus of the 5-m Hale telescope. The Palomar chip has high quantum efficiency (85 - 90% from 550 - 750 nm), 0.28 arcsec pixels, and a square field of view 9.7' on a side. A Gunn $r$ filter was used for the majority of the Palomar observations to minimize sky brightness; transformation to Kron-Cousins $R$ magnitudes is straightforward. Following the results reported in Gladman and Kavelaars (1997), two additional Palomar observing runs occurred in the fall of 1997 (7 nights total, 5 usable).
The University of Hawaii 8k x 8k prime-focus CCD array on the Canada-France-Hawaii 3.6-m reflector was used with a conventional KC $R$ filter. The quantum efficiencies of this array’s 82048 x 4096 chips vary, but are considerably poorer (30–40%) than the Palomar chip, resulting in a search which was much less deep, but covered a larger field ($\approx 30 \times 30\arcmin$). All CFHT results reported here are from a 3-night observing run in early April 1997.

Since TNOs at opposition have retrograde motions slower than $5\arcsec$/hour, integration times of 480 sec limited trailing losses while still giving acceptable duty cycles. For $1\prime$ seeing, 480 sec exposures produced a SNR of about 6 for objects with $R \approx 24$ at Palomar, and with $R \approx 22.8$ at CFHT. Exposures at both telescopes were acquired while using an offset guide star. On each photometric night we observed photometric standard fields; NGC 7006 (Odehwan et al., 1992) from Palomar and various standard Landolt fields (Landolt 1992) from CFHT.

2.2. Analysis Method

All images involved in the deep searches were preprocessed to remove detector characteristics, including having bad columns fixed by averaging pixel values on either side. Cosmic rays were not removed, for fear of removing our faint moving sources, and because the subsequent data reduction eliminates them almost entirely. The data analysis software consists of an IRAF2 program which, given an angular rate and direction, recombines the images by shifting their pixels and then combining them. The offset for each frame is calculated based on an assumed drift rate and the time delay between the start of that exposure and the first frame in the sequence. Thus, all stationary objects will elongate, trailing in the direction of recombination; only objects moving near the specified angular rate will have their signal constructively add into a single seeing disk. Each angular rate $\dot{\theta}$ corresponds to first order to a different heliocentric distance. By recombining the frames at a variety of rates, we can search for objects from 10–100 AU. Since there appear to be many fewer Centaurs per square degree than TNOs (Jewitt and Luu 1996), the most likely discovery is of new TNOs in the range 30–50 AU. Our 480 sec integration time $T$ resulted in trailing losses only inside 18 AU, using the empirical trailing-onset criterion (Jewitt and Luu 1996) of $\dot{\theta}T/(FWHM) > 1$, in seeing with a FWHM of $0.9\arcsec$.

One would like to remove all fixed sources from these frames before beginning the processing. Experimentation with subtracting from each frame a median image created from all the un-shifted frames met with mixed success due to the problem of variable seeing (over 4–6 hours of integration), causing different point-spread functions for stellar images. Since the deep-search fields were selected to have few background stars (a few percent of the field area), this refinement produced negligible improvement, and thus was not used. Instead we settled on an algorithm which created two analysis images for each angular rate and direction considered.

- Each frame had its mean sky value subtracted and its flux equalized by scaling a bright (but non-saturated) star.
- The images were then shifted at the assumed rate and direction, creating a ‘stack’ of shifted images corresponding to those two parameters.
- A first analysis image was then created by simply summing the shifted set; this has the advantage of preserving all the signal but suffers because all cosmic rays appear in the ‘summed image’.
- The second analysis image (the ‘medianed image’) was created by rejecting the highest value for each pixel in the shifted ‘stack’, and then taking the median of the remaining pixels (e.g., Fig. 2); this eliminates effectively all cosmic rays, and most of the images of the faint stars (since non-bloomed stars contribute their PSF to only a small fraction of the images as they ‘pass by’ in the shifting sequence).

It was these ‘medianed images’ that were then searched for objects, although all candidates were examined in the ‘summed images’ to verify their reality. To search for moving objects, the analysis images were examined for point-like objects.

What resolution is necessary in parameter space in order to detect all objects? If the recombination rate (in pixels per hour) is too fast or too slow, the object’s signal will trail into many pixels and faint objects will not emerge above the noise. By experimenting with artificial objects implanted in the data, it was determined that a grid spacing of half the seeing per hour was sufficient to find all objects. To err on the safe side, a grid spacing of one-third the seeing per
hour was used, which resulted in all objects being visible on at least two of the medianed images (or more for brighter objects).

The orbital inclination of the objects will also produce motion in differing directions. However, our fields were within 10° of opposition, implying that the motion of all objects will be heavily dominated by their retrograde component (and means that mainbelt asteroids are far from their stationary points and cannot mimic the motion of outer solar system objects). Even orbital inclinations of 30° produce deviations of only 5° in the apparent direction of TNO motion across the sky (the dominant retrograde component being at roughly 23° to the equator for our spring and fall observations). The experimentally-determined grid separation required from our tests with implanted artificial objects was also 5°. Thus even though recombinations at solely the pure retrograde direction should suffice, to be cautious we searched all Palomar frames at apparent rates corresponding to motions of 18, 23, and 28° with respect to the equator. This should extend our sensitivities to all orbits with orbital inclinations < 45°. Since in all cases we found artificially implanted objects (see Sec. 4) that were moving at 18° or 28° we only searched the CFHT results at angles of 23°.

We found that the most effective way to search the combined frames was to work at constant recombination direction, and blink 4 adjacent rates in sequence. Real objects show a distinct pattern of their signal becoming stronger and then weaker as the correct rate is approached and passed. A best estimate of the rate and direction can then be established by producing a much finer resolution grid in rate and direction around the candidate object and determining which parameters maximize the signal. In practice we found that the rate and direction could be determined to similar precision by re-combining the first and second halves of the data at the candidate rate and direction, and then measuring the apparent motion on these two images; real objects of course have similar brightness on these two subsets, although the SNR is $\sqrt{2}$ smaller. The constant brightness (to the errors inherent in the photometry) in images created with half the data set is important, since if background noise were responsible for the appearance of a low-SNR object, it would have to be of constant amplitude along the direction of the trail in order to yield the same (spurious) amplitude and brightness profile in both halves of the integration time. All candidates had to show a pro-

![Image](image_url)

Fig. 2.— A medianed image of RR20 field, from data obtained Sept. 8/97 UT, showing 4 TNOs. From top to bottom they are 1997 RX9, 1997 RT5, 1996 RR20, and the $m_R = 25.6$ TNO pictured in Fig. 3. The recombination rate is 2.9°/hour retrograde at 23° from the equator, and the pictured field is $\approx 6.5' \times 8'$ in size. The image was constructed from twenty-five 480-sec images. The faint, bottom-left TNO, is shown better in Fig. 3.
file consistent with the oversampled PSF of the observing conditions, and exhibit the expected pattern of changing signal as the retrograde rate was tuned. We also examined the immediate neighbourhood of a candidate in all the images of the set to make sure no spurious event appeared (such as cosmic ray strikes) which might somehow produce a signal, even though the median process should eliminate such an eventuality. As a further check, each field was imaged on two adjacent nights; a faint object at low signal to noise can thus be verified by observing it on another night at the location where its measured motion should put it. We found that we could reliably work to a SNR of about 4 with almost 100% detection efficiency.

3. Results

The progress reported in this paper (subsequent to the previous null result reported in Gladman and Kavelaars 1997) comes from new data obtained during two observing runs at the Palomar 5-meter and one observing run at the CHFT 3.6-m reflector. Since the instruments used differed, resulting in very different limiting magnitudes and areal coverage, we will discuss the data obtained at the two observatories separately.

3.1. Palomar data

Data from two observing runs were available: 4 nights beginning Sept. 5 1997 UT, and from one night of a 3-night run beginning Oct. 26 1997 UT. Conditions during September were excellent with median seeing of about 0.9′′ on all 4 nights, allowing two deep fields to be obtained, on each of two nights under very stable conditions. The October conditions were much poorer (seeing ~ 1.3–1.7′′), allowing only one deep field to be obtained (with a shallower magnitude depth). Because of these differing depths, we shall discuss these two observing runs separately.

The single-frame limiting magnitude (SNR≈5) for the 8-minute September exposures was $R \simeq 24.1$. It was considered desirable if possible to have a known TNO in the frame as a ‘reference object’ to be recovered, and so before going to the telescope we examined all the fields that would contain known TNOs near opposition, and selected two which had low densities of other luminous objects in the APM catalogue. The relative motion of TNOs is large enough that such fields are just as likely to contain further new objects as are any randomly chosen field near the ecliptic. The previously known object is of course not counted in any estimate of the surface density coming from detections in that field.

The first field was based on the Sept. 7.0 1997 UT position of 1996 RR20, which was exhibiting retrograde motion near opposition at about 2.9′′/hr. Figure 2 shows the median recombination created by shifting the Sept. 8.0 images at this rate parallel to the ecliptic. 1996 RR20 is at right center with a SNR~ 40 in the recombined frame. In fact, this field contains four TNOs. Blinking the images at the telescope had immediately yielded a second bright TNO ($R = 23$, upper center in Fig. 2), subsequently designated 1997 RT5, which coincidentally was seen at a simultaneous observing run at La Palma by A. Fitzsimmons et al.; see Minor Planet Electron Circular (MPEC) 1997-R12. An orbit based on the discovery observations (and subsequent recovery in October) places it on a nearly circular orbit near 42 AU; the hypothesis of a plutino-type orbit generates much larger residuals (B. Marsden, private communication 1997). The next TNO was discovered immediately upon examining the first recombination of the frames at the RR20 rate, and subsequently designated 1997 RX9. This object had $R = 24.0 \pm 0.1$, and was clearly extended on the summed image at the RR20 rate in such a way as to imply both a different angular rate and direction of motion. The best recombination yielded an angular rate of 3.1′′/hr at an angle of 27.5°, indicating a heliocentric distance ~ 40 AU and large orbital inclination. The discovery observations implied $(a, e, i)$ of $ \simeq (42,0.30^\circ)$, which were only slightly modified to the elements of Table 2 after an October 1997 recovery by E. Helin and D. Rabinowitz at the 200-inch (see MPC 30791). 1997 RX9 is one of the faintest and most highly-inclined TNOs discovered to date. Both of these new objects were followed over the course of at least 4 hours on Sept. 7 and 8, yielding abundant high-precision astrometry.

The third and final new TNO in the RR20 field was much fainter, with $m_R \simeq 25.6$. However, the object is easily visible in the median frames despite the hinderance of a nearby star of moderate brightness, which it passed during its 4-hour track on Sept. 8 UT (Fig. 3). Unfortunately this object moved into the field on that night, and although we displaced the frames somewhat to compensate for night to night motion, this object was approximately 10′′ off the field on the previous night, meaning we were unable to obtain a 2-night arc on which to compute a preliminary
Table 2
Preliminary orbits for new TNOs

<table>
<thead>
<tr>
<th>Designation</th>
<th>$m_R$</th>
<th>$a$(AU)</th>
<th>$e$</th>
<th>$i$(°)</th>
<th>radius(km)</th>
<th>Orbit Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997 RT5</td>
<td>22.95±0.03</td>
<td>42.0</td>
<td>0.08</td>
<td>12.7</td>
<td>130</td>
<td>MPEC 1997-R12, 1998-L03</td>
</tr>
<tr>
<td>1997 GA45</td>
<td>23.7±0.5</td>
<td>43.9</td>
<td>0.09</td>
<td>8.3</td>
<td>80</td>
<td>MPEC 1998-G10</td>
</tr>
<tr>
<td>1997 RX9</td>
<td>24.0±0.1</td>
<td>42.1</td>
<td>0</td>
<td>50</td>
<td>80</td>
<td>MPEC 1997-S99, MPC 30791</td>
</tr>
<tr>
<td>faint TNO</td>
<td>25.6±0.3</td>
<td>44.3</td>
<td>0</td>
<td>&lt;10</td>
<td>40</td>
<td>1 night only, no designation</td>
</tr>
<tr>
<td>1997 RL13</td>
<td>25.8±0.3</td>
<td>44.5</td>
<td>0</td>
<td>&gt;6</td>
<td>40</td>
<td>MPEC 1998-E05</td>
</tr>
</tbody>
</table>

Note.—All orbits found are outside 40 AU. These objects have only provisional circular orbits; however, only 1997 RT5 and 1997 GA45 had sufficient observations to rule out a 2:3 resonant orbital solution. Radii are computed assuming geometric albedos of 4 percent.

orbit. However, we have no doubt as to the reality of this object, which appears in recombinations using only the first half, middle half, and second half of the frames (Fig. 3), moving at a constant rate and brightness. The frames contain no bright pixels from cosmic ray strikes along the track of motion. The PSF of the object, when combined at its correct rate of motion, was circular and of the stellar width, and the profile distorts exactly the correct pattern as the recombination rate is varied. The retrograde motion of 2.9 R°/hr indicates a heliocentric distance of 25 AU, with an orbital inclination of 16° (B. Marsden, private communication) relative to the ecliptic. Although a provisional circular orbit is reported in Table 2, eccentric orbits with $a = 35-55$ AU also satisfy the observations. This object will likely never be recovered, due to its faintness and the fact that our October nights were of insufficient quality to recover it in order to compute an orbit.

The field chosen for the Sept. 5 and 6 searches (observed both nights in good conditions) was that containing the TNO 1996 TR66, this 23rd magnitude object being easily recovered. This field yielded one new, very faint TNO ($R = 25.8 ± 0.3$), which was seen on both nights. The object’s motion (measured via either the recombination which maximized the signal or directly using the displacement from night to night) implied a heliocentric distance of 44.5 AU, and the object was subsequently designated 1997 RL13, with a provisional circular orbit. It should be noted that TR66, and hence this search field, was at an ecliptic latitude of $\simeq 6^\circ$; thus this is the object’s minimum orbital inclination. This is the faintest solar system object ever given a provisional designation, making it unlikely that it will be recovered. Assuming an albedo of 0.04, 1997 RL13 has a radius of only $\sim 40$ km, making it the smallest TNO given an orbit to date.

During the four September nights we also re-observed, the TNOs 1993 RO, 1995 QY9, 1995 QZ9, 1995 WY2, 1996 RR20, 1996 RQ20 (R-14), 1996 S74 (R-15), 1996 TK66 (S-10), 1996 TR66 (S-13), 1996 TO66, 1996 TP66, and 1996 TQ66; the (L-##) designation after an object supplies the 1997 Minor Planet Electronic Circular that reported the astrometry. The remaining observations can be found on the Minor Planet Circulars. These observations providing additional information for improved orbital solutions. The TNO 1997 SZ10 was discovered by D. Jewitt in the same field as 1996 RQ20 on the single night of Sept. 24; our Sept. 7 RQ20 recovery field ‘pre-covered’ SZ10, allowing a 3:2 resonant orbit to be established on a 3-week baseline in conjunction with a recovery by C. Hergenrother on Sept. 27 (see MPEC 1997-S16). We also conducted recovery attempts for the TNOs 1994 TG and 1995 YY3; despite a limiting magnitude much deeper than their estimated brightnesses, they were not within 5′ of their predicted locations. These two TNOs are probably now lost. This is somewhat puzzling for 1995 YY3, which was a multi-opposition object, but our second recovery attempt in October
was also unsuccessful. It is possible that stellar confusion obscured the objects in each case, but this is unlikely.

The October 1997 deep search was seriously hampered by clouds and poor seeing, resulting in only one deep field (0.025") being imaged in 1.4" median seeing, to a limiting magnitude of $R \approx 25.2$. In this case we chose not to use a field with a known object present, instead selecting a section of sky with a low background density of objects from the Palomar Sky Survey and APM catalogue, at $\alpha = 22^h 27^m 32^s$, $\delta = -6^\circ 47' 30''$ (J2000). No objects were discovered in this field, which is not surprising given the surface density estimates of $\sim 10/\text{arc}^2$ from previous work near this magnitude level (Jewitt et al. 1996) and our previous upper limit (PAL96 in Fig. 1).

3.2. CHFT data

The CHFT data obtained with the UH8k mosaic camera resulted in single-frame limiting magnitudes about one magnitude shallower than the Palomar data. The quantum efficiencies of two chips of the mosaic were so poor that we did not analyze those images, meaning that we had a reduced field of view of about $22 \times 30''$, which was later further reduced by trimming off the portions of the field not seen in all exposures caused by dithering. In our April 1997 3-night observing run we imaged two fields, one on two nights and the other on only a single night.

The first field was chosen so that one chip of the mosaic would be centered upon an elliptical galaxy in Virgo, allowing a simultaneous study of its globular cluster system (although half of that chip is useless for the TNO search due to crowding in the galaxy's halo). No new objects were found in the remaining 5.5 chips available to be searched on the two nights of April 1 and 2 1997 UT.

The second field, imaged on only the night of April 3 UT, was chosen to have the object 1994 GV9 ($R=23.1$) in it. Although too faint to see on individual exposures, this object was found within a few arcseconds of its predicted position after examining the recombined images. A second TNO was discovered roughly 3.5' northwest of GV9, which we followed for 7 hours, and was easily visible in the recombined frames. There was thus no doubt as to its reality. The new TNO was $\sim 0.6$ magnitudes fainter than GV9, and thus we estimate $R = 23.7 \pm 0.5$ for the new object, although the night was not photometric. This
TNO would not have been given a designation except for the happy coincidence that E. Fletcher et al. had obtained 9 images of 1994 GV9 on April 7/8 1997 UT at La Palma, and analysis of those frames allowed a recovery of the new object (see MPEC 1998-G10). Based on the five-night arc, the orbital elements in Table 2 were derived for 1997 GA45.

Few recovery attempts were made during this observing run. A previously un-numbered TNO was recovered, and subsequently designated 1997 CQ29 (see MPEC 1997-J02 for details). The mosaic's large field of view permitted the tracking of some main-belt asteroids over two nights, which resulted in a recovery of asteroid 2739 Tagua; successfully moving through the field, and the discovery and designation of a new \( R \simeq 18 \) asteroid 1997 GF38 (MPC 29736).

4. Limiting magnitude determination

After the images were aligned and the shifted images were trimmed of the small portions which move off the frame for the given shift rate, the Palomar frames covered an area of \( \simeq 0.025 \square^\circ \). Therefore, 0.05 \( \square^\circ \) of sky were searched in total in the deep fields of September. Since the single 8-minute frames had limiting magnitudes of \( R \simeq 24 \), a simple scaling indicates that the typically 3.5 hours of integration would be expected to yield a limiting magnitude of about 25.7 when recombined. Obviously our data reduction process might not be perfectly optimal. We established the limiting magnitude of the September data by writing a software algorithm that imprinted a random number of artificial objects in the data, at random places moving at random rates in random directions (although consistent with low eccentricity objects between 30 and 50 AU). These frames were then searched by eye in exactly the same fashion (or coincidentally with, in most cases) as for real objects. The discovery efficiency followed the normal limiting magnitude distribution behaviour (Harris 1990), being \( > 95\% \) until \( R \simeq 25 \), and then falling off to near zero over the next magnitude (Fig. 4). The 50\% efficiency level is at \( R \simeq 25.6 \pm 0.2 \), which we adopt as our limiting magnitude. This indicates that the shifting and recombination process has resulted in a loss of only \( \sim 0.1 \) magnitudes.

The detection efficiency function was determined for the October Palomar data in the same fashion, giving a limit of \( R \simeq 25.2 \pm 0.2 \) (since the best October night was not photometric), over 0.025 \( \square^\circ \). An

Fig. 4.— Discovery efficiency curves for the September Palomar data ( hollow squares ) and the 1994 GV9 field for the CFHT data ( filled circles ). The fraction \( f \) of artificial objects found in each magnitude bin is reported. The fits are smooth functions based on hyperbolic tangents which yield 50\% efficiency limits at \( R = 24.6 \) and \( R = 25.6 \) for the two surveys.
identical procedure was followed for the CFHT data, yielding a 50% limit at $R = 24.6 \pm 0.2$ (Fig. 4); although the two fields had different numbers of exposures, the seeing was sufficiently worse on April 3rd that the combined image limits for each night are the same to within 0.05 mags. The CFHT survey yielded 0.30$\oplus$ of searched area after trimming.

Since all of our surveys have had their detection efficiency functions evaluated (Fig. 4), we have combined all available data from all our surveys to create a cumulative surface density estimate (Table 3). For convenience, we have chosen bin boundaries so that bin centers lie at $R=24.6$ and 25.6, where two of our surveys have 50% completeness points. At each bin center in which we have detections, we have calculated the differential surface density by taking the number of objects and dividing by the 'effective area', the latter calculated by summing the product of the detection efficiency $f_i$ and searched area $A_i$ (Table 1) for each pencil-beam survey. The resulting cumulative surface density estimates are plotted in Fig. 5.

It should be noted that the surface density estimates are based on the combined surveys from CFHT and Palomar, which is in principle correct assuming no systematic errors at the level of several tenths of a magnitude are present in the magnitudes of detected objects or in the determination of the efficiency functions. If one considers only the September Palomar data set, which consisted of 4 photometric nights in almost identical stable conditions, then our sky density estimate becomes $\Sigma(< 25.9) = 120 \pm 60$ objects per square degrees, being somewhat larger than that listed in Table 3 since most of the detected objects are in this Palomar data set.

5. Comparison with other surveys

Our sky density estimates suffer the common problem of small-number statistics and a limited magnitude range within a single survey. In order to obtain a better estimate of the luminosity function, we shall combine our results with those from several other published surveys. Care is required while doing this, since not all surveys have been reported in the same way.

5.1. Surface density estimates

Fig. 5 plots estimates of $\Sigma(< R)$, along with our maximum likelihood fit (to be discussed below). It is important to note that since this is a cumulative plot of all objects brighter than a certain magnitude, the deepest edge of a bin boundary should be used; we have thus plotted our data (Table 3) and the JLT98 data in this fashion. All other estimates have been plotted at the stated $R$ magnitude corresponding to 50% completeness of the survey. In any case, it is not these sky density estimates that are used in the maximum likelihood fits.

Our sky density estimates (Fig. 5) are all about a factor of 3 above the extrapolated JLT98 luminosity function. There are several possible explanations for this (listed below).

1. We were lucky at the 1-$\sigma$ level and the average surface density is lower than our determination by a factor of 3. Note that we must remain systematically lucky at all of our magnitude levels.

2. Our real limiting magnitude is almost a magnitude fainter than our estimates. We doubt this because we believe that the experiments with artificial objects clearly indicate we cannot find objects fainter than $R = 26$ in the September data set.

3. A complex explanation could be that the surface density is not particularly uniform on the sky, and that the surveys all correctly report a local sky density. Each survey samples a different depth and location relative to opposition, so this explanation is not completely unreasonable. Our early September opposition survey was somewhat closer to Neptune than the 90° longitude separation which is usually searched in order to avoid the galactic plane. However, the surface density of objects might be expected to drop as one moves away from 90° separation from Neptune, where platinus are coming to perihelion and thus increase the local surface density (Malhotra 1995). Our greater surface density works in the opposite direction from this expectation.

4. The JLT98 sky density estimates, especially those for surveys fainter than $R = 24$, are in error. We in fact believe this possibility to be the most likely, which we now discuss. A definitive answer will require repeating the experiment, and the arrival of high-quality, large-format CCD mosaics will allow a pencil-beam survey to produce tens of detections rather than 5, giving sufficient detections in a single field to establish a high-quality luminosity function.

JLT98 report a luminosity function that rises by a factor of $\approx 4$ with each additional magnitude. As discussed above, with this estimate of the luminosity function the number of objects in the final magnitude,
over which the detection efficiency falls from 100% to zero (c.f. Fig. 4), should be roughly equal to the number found at all brighter magnitudes. This is not true of the objects reported in the surveys of Jewitt et al. (1996), and in particular for the $R = 24.8$ survey, which detected none of its 7 objects fainter than the reported $R = 24.5$ 100% completeness point. Thus, this survey is internally inconsistent with the derived luminosity function at more than the 2-sigma level. It could thus be the case that the first surveys reported in Jewitt et al. 1996 (from $R=23-24.8$) systematically underestimate $\Sigma$ and should thus be viewed as lower limits or, alternately, have stated limiting magnitudes that are too faint, as previously suggested by Weissman and Levison 1997. We show below that the recent JLT98 survey, which does have 60% of its objects at the faintest magnitudes, when analyzed by a maximum likelihood method yields a luminosity function slope that agrees with our results. The HST result also suffers from the inverse internal consistency problem, in that of order 40% of its objects should be brighter than its 100% completeness limit, in contrast to the zero found. Our estimate of the sky density estimates for the HST survey (discussed below) incorporate this effect.

5.2. Maximum likelihood fit to all surveys

We have analyzed the data from surveys available in the literature by adopting a simple model \( \log \Sigma = \alpha (R - R_0) \), and using Bayes’s theorem to infer the model parameters. The Bayesian approach is particularly suitable here because it allows us to readily combine information from disparate surveys simply by multiplying the likelihood functions for each survey. The likelihood functions can be derived using the Poisson distribution.

Most surveys report a detection efficiency (or detection limit) and the magnitudes of a finite number (possibly zero) of detected objects. For such surveys, the likelihood function for the parameters $P$ takes a form similar to that derived by Irwin et al. (1995). It can be written as

\[
\mathcal{L}_k(P) = \exp \left[ -\int dR \Omega_k \eta_k(R) \sigma(R) \right] \prod_i \int dR \xi_i(R) \sigma(R)
\]

where $k$ is an index denoting the survey, $\Omega_k(R)$ is the solid angle of survey $k$, $\eta_k(R)$ is the detection efficiency for a TNO of magnitude $R$ in that survey, $\sigma(R)$ is the TNO surface density per unit $R$ (i.e., the differential distribution, which depends on the parameters $P$), and the object likelihood function $\xi_i(R)$ describes the uncertainty for the magnitude of object $i$ in survey $k$. For $\eta_k(R)$, we use fits of smooth functions to reported detection efficiencies when the latter are reported, of the form

\[
\eta(R) = \frac{1}{2} \left[ 1 - \tanh \left( \frac{R - R_{50}}{W} \right) \right]
\]

### Table 3

<table>
<thead>
<tr>
<th>$m_R$</th>
<th>$N$</th>
<th>$\sum_i f_i A_i$</th>
<th>$\sum_i f_i A_i$</th>
<th>$\Sigma(&lt; R)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.1</td>
<td>1 ± 100%</td>
<td>0.37</td>
<td>2.7 ± 2.7</td>
<td>2.7 ± 2.7</td>
</tr>
<tr>
<td>23.6</td>
<td>1 ± 100%</td>
<td>0.37</td>
<td>2.7 ± 2.7</td>
<td>5.4 ± 3.8</td>
</tr>
<tr>
<td>24.1</td>
<td>1 ± 100%</td>
<td>0.35</td>
<td>2.9 ± 2.9</td>
<td>8.3 ± 4.8</td>
</tr>
<tr>
<td>25.6</td>
<td>2 ± 70%</td>
<td>0.025 ± 30%</td>
<td>80 ± 63</td>
<td>90 ± 60</td>
</tr>
</tbody>
</table>

Note.—Summary of cumulative sky densities for the combined CFHT and Palomar deep surveys. The uncertainty in the “effective area” $\Sigma_i f_i A_i$ is negligible except for the final row. Bins are 0.5 magnitude wide, with the given centers $m_R$. When plotted on the cumulative plot of Fig. 5, the faint edge $R$ of the bin is used. Note that these binned surface density estimates are not used by the maximum likelihood method to compute the luminosity function.
Fig. 5.— The trans-Neptunian luminosity function, plotted as the cumulative number of objects per square degree near the ecliptic brighter than a limiting R magnitude (for 100% completeness). Our new results (large circles) are listed in Table 3. The line is the result of the maximum likelihood fit to all CCD surveys with solid symbols and the LD90 and LJ88 CCD (R=24) upper limits. The PAL96 upper limit is not shown since those pencil-beam fields have been absorbed into our current estimates. The JLT98 data have been shifted to the faint end of the magnitude bins. The shown fit has $\alpha=0.76$ and $R_0=23.4$. See text for further discussion.

Fig. 6.— Best-fit parameters (cross) and credible regions (1, 2, and 3 sigma contours) from the maximum likelihood analysis of several CCD surveys with published efficiency functions. a The JLT98 survey, b JLT98, I05, and HST, c JLT98, I05, with our surveys, d all of the above with the LD90 and JL88 upper limits. The best fit parameters for d are $\alpha=0.76$ and $R_0=23.4$. 

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that is, the efficiency \( \eta \) falls to 50% at \( R = R_{50} \), over a characteristic half-width \( W \). For the upper limits we have taken the efficiency to fall from unity to zero across a range of 1 magnitude centered on the quoted 50% detection limit of the survey. For the object likelihood functions we use Gaussian functions with means equal to the best-fit values of \( R \) and standard deviations equal to the stated errors, or the root mean squares of the statistical and systematic errors when these are provided separately. This likelihood function improves on that of Irwin et al. (1995) in its incorporation of uncertainties and its use of the full detection efficiency function (Irwin et al. 1995) implicitly took \( \eta(R) \) to be Heaviside functions. Of these improvements, the latter is the most important. A full derivation of this type of likelihood function appears in Loredo and Wasserman (1995).

The HST survey (Cochran et al. 1995) did not report the magnitudes of individual detected objects; instead, the data consist of counts of detected objects in each of 20 magnitude bins, spanning a V magnitude range from 27.8 to 28.8. We assume a \( V - R \) color of 0.5; a \( \pm 0.5 \) magnitude error bar on the detection limit is given to account for this unknown color conversion. The 94 detected objects are individually of low SNR and thus to estimate the false detection rate, Cochran et al. (1995) analyzed the HST images by using retrograde candidate orbits, and object counts (presumably of false objects) were reported for each bin. A total of 65 retrograde objects were found. We built a likelihood function for these data by modelling the prograde and retrograde counts with Poisson counting distributions, the former with a mean equal to the sum of the model prediction and a background rate, and the latter with a mean equal to the background rate. The background rates for each bin are unknown, but the retrograde data provide us with estimates of them (with significant uncertainty). The likelihood function for each bin in such an “on-source/off-source” dataset is derived in Loredo (1992). In our analysis, we pooled the HST data into two bins (from 27.8 to 28.3, and 28.3 to 28.8). This was necessary because the reported counts are so low in each bin that, taken independently, they provide little constraint on the signal rate (i.e., the background and signal-plus-background rates are so poorly determined in the narrow bins that nearly every bin is consistent with zero signal when viewed independently of other bins). But the background rate is presumable similar in adjacent bins, and pooling adjacent data is a simple way to account for this.

For the differential magnitude distribution, we adopted the standard exponential model,

\[
\sigma(R) = \ln(10) \alpha 10^{\alpha(R-R_b)};
\]

that is, the cumulative distribution \( \Sigma(F) \) obeys \( \log \Sigma = \alpha(R - R_b) \), so that \( R_b \) is the magnitude where \( \Sigma = 1 \) TNO per square degree, and \( \alpha \) is the slope of the distribution when plotted with log-linear axes. Multiplying the likelihood functions of the various surveys produces a joint likelihood function for \( \alpha \) and \( R_b \). We adopted uniform priors, so the posterior distribution for \( \alpha \) and \( R_b \) is just the joint likelihood function, normalized, and the most probable parameter values are simply the maximum likelihood values. For our calculations, we normalized over the range \([0.05, 2.0]\) for \( \alpha \) and [19, 25] for \( R_b \) and found credible regions that enclose 68.3%, 95.4%, and 99.7% of the total probability. These values were chosen because of their familiarity from the Gaussian distribution, but the posteriors are not at all Gaussian and the likelihood values bounding the regions have to be found numerically; Irwin et al. (1995) presumed Gaussian statistics in finding their parameter regions.

The JL98 survey calculated a best-luminosity function via a least-squares fit to the cumulatively-binned surface density estimates. As mentioned above, because this is a cumulative distribution this should be considering the estimates to be at the faint end of the magnitude bins. More severely, using least-squares is incorrect for these data since (1) the errors are Poisson, not Gaussian, and (2) the errors in the points are highly correlated due to the cumulative distribution (i.e., the error for each fainter point contains the errors of all brighter ones). We have thus re-analysed the JL98 results using a maximum likelihood method based on their detailed, published efficiency function for the survey. We do not use the older Jewitt et al. data; the lack of objects at the faint end of those surveys is likely what forced the very shallow slope of the Irwin et al. (1995) maximum likelihood fits, a slope unsupported by the JL98 data.

A cost of using a formalism that allows combination of information from disparate surveys is that there is no simple graphical illustration of the fitting process that precisely displays the role of each survey in the fit. In Fig. 5 we follow the common practice of plotting separate estimates of the cumulative TNO surface density from each survey, along with the cumulative density distribution correspond-
ing to the best-fit model. Such a plot must be inter- 
reted with caution because it is not possible to 
construct model-independent estimates from the data 
due to the magnitude-dependent detection efficiencies 
of each survey.

Particularly troublesome is representation of the 
HST data on such a plot because the presence of 
many false detections complicates the estimation of 
the surface density and its uncertainty. For the pur- 
pose of plotting an estimate of the surface density of 
objects on our figures, we fixed $a$ at our best-fit value 
($a = 0.76$) and rewrote the $\sigma(R)$ model, replacing the 
$R_0$ parameter with $\Sigma_{\text{lim}}$, the cumulative TNO surface 
density for a limiting magnitude of $R = 28.1$. We 
then calculated the likelihood function for $\Sigma_{\text{lim}}$, and 
plopped a point with the maximum likelihood value of 
$\Sigma_{\text{lim}}$ at $R = 28.1$. The 2-bin likelihood function 
peaks at 20,000 per square degree. The endpoints of the 
vertical error bar indicate where the likelihood falls to $1/\sqrt{e}$ its maximum value (the range spanned 
by $\pm 1\sigma$ for a Gaussian likelihood). The location 
of this point is not very sensitive to $a$.

Fig. 6a shows the credible regions for only the 
JLT98 survey; note that very large luminosity func- 
tion slopes $a$ are permitted by this data set. We find a 
steep slope ($a = 0.73$) than the value of $a = 0.56 \pm 0.15$ 
given by JLT98, although that value is within the 
1-sigma confidence level of the maximum likelihood 
analysis. Both give $R_0 \approx 23.3$. Including the HST 
and Irwin et al. (1995) surveys (Fig. 6b) restricts this 
Corwedell, especially by the HST result (ironically) 
eliminating steep slopes. Fig. 6c shows how the best 
fit parameter change when our Palomar and CFHT 
survey results are included, and the HST data are not. 
Finally, Fig. 6d gives a combined fit that also includes 
all the previous surveys and the LD90 and LJ98 upper 
limits. The best-fit parameter values and uncertain- 
ties are $a = 0.76^{+0.13}_{-0.11}$ and $R_0 = 23.40^{+0.23}_{-0.18}$, where the errors 
indicate the range spanned by the joint 68.3% 
credible region in Fig. 6d). This implies that the 
cumulative sky density of TNOs increases by a factor 
of $10^a \approx 6$ per magnitude. This steeper luminosity 
function (Fig. 5) predicts more TNOs at faint magnitudes 
than extrapolation of the previous I95, Jew- 

witt et al. 1996, or JLT98 luminosity functions, and 
seems to nicely bring into accordance almost all previ- 
ous surveys; this includes the formerly problematic 
biigt photographic surveys and the HST result. It 
should be noted that the inclusion or removal of the 
HST result has very little influence on the location of 
the credible regions for the final fit (compare panel 
c & d). Clearly all of the results of these maximum 
likelihood fits overlap at the 1-sigma level, but the 
combined data set provides a much more well-defined 
best-fit region. There is still some uncertainty in the 
slope, especially interesting because the number of 
faint TNOs ($R > 30$) is a very strong function of $a$. 
Note that because the maximum likelihood method 
takes into account the 3 surveys providing upper lim- 
its, the best-fit luminosity function `appears' some- 
what low in Fig. 5 if one looks at only the positive 
detections; there are no lower limits to balance out 
the null results, and thus the sky density is pushed to 
lower values.

We have included on Fig. 5 three photographic sur- 
veys with limits $R \leq 20$ (Tombaugh 1961, Kowal 
1989, and Luu and Jewitt 1988; as reported in Ir- 
win et al. 1995 and JLT98). Although JLT98 ques- 
tion the validity of these surveys (as being difficult 
to quantify), our best fit luminosity function makes 
the non-detections by Kowal, and by Luu and Jewitt 
(1988), much less problematic than previous single 
power-law fits to the luminosity function. Plotting 
Tombaugh's Pluto detection on this figure may be 
questionable, since Pluto's albedo, and hence magni- 
tude, is probably enhanced by its active atmosphere, 
meaning it may not have been detected in his sur- 
vey if it had a dark surface. Nevertheless, the single 
detection is of course formally consistent with our 
luminosity function. The question of whether there 
exists a maximum-size cutoff in the size distribution 
(JLT98) is not directly addressed by our new results.

6. Inside and Outside the Belt

6.1. Centaurs

We also searched our two September Palomar deep 
fields for angular rates of up to $9^\circ$/hr, correspond- 
early circular orbits at about 13 AU. This 
data set is free from trailing losses outside of 18 AU, 
and the trailing loss mounts to 0.3 mags at 13 AU. 
No new Centaurs were found in 0.05$\Delta^d$ to magnitude 
R=25.6. Given the Jewitt et al. (1996) estimate of 
$\sim 0.5$ Centaurs/$\Delta^d$ brighter than $R=24.2$, this null 
result is not surprising. Even if the sky density increases 
by a factor of 6 per magnitude, at $R=25.6$ we expect 
only $\sim 6$ per square degree, meaning our 0.05$\Delta^d$ sur- 
vey had only a 25% chance of finding one. Because 
of this null result, we did not search the larger but 
shallower ($R \approx 24.6$) CFHT data set for Centaurs,
since it involves looking at a large number of chips at a large number of rates, and had only a small chance of finding any objects.

6.2. The Belt Outside 50 AU

We also searched the September Palomar data set for rates down to 1.4°/hr, corresponding to heliocentric distances of nearly 100 AU. No TNOs were found at rates lower than 2.6°/hr, meaning we did not observe objects at heliocentric distances greater than 50 AU, where other surveys have also as yet failed to find any objects. While the existence of Pluto and 1996 TL66, which journey outside of 50 AU during their orbits, clearly implies that there are objects in this region, we as yet have no direct observational evidence for a `dynamically cold disk' in this region; that is, no objects on nearly circular orbits have been found outside of 50 AU. Is this a surprise? Dones (1997) has discussed this issue.

Imagine looking in a square ecliptic field of linear size $\xi$ radians, and corresponding linear dimension $\xi r$, where $r$ is the heliocentric distance. Let us assume a single power-law cumulative size distribution (independent of heliocentric distance) of the form $N(diam \geq D) \propto D^{-Q}$, and a volume number density proportional to $r^{-\beta}$. Note that $Q=q-1$, where $q$ is the differential slope. If the surface mass density of the primordial nebula dropped as $r^{-2}$ then we expect $\beta \sim 2-3$, consistent with constraints derived from Monte Carlo modelling of the known TNO distribution (JIT98). Assume that the Kuiper belt proper ends at some inner edge $r_{\text{min}}$; at this distance there is a minimum diameter object which can be seen (we will be assuming constant albedos). As we move to shells of greater heliocentric distance, the flux from a particle of the same size drops as $1/r^4$, and so the minimum visible size increases as $r^2$, and thus the number of visible objects drops as $r^{-2Q}$ due to the size variation. The number of objects visible in the shell therefore obeys

$$dN \propto \xi^2 r^2 dr \sim r^{-\beta + 2Q}$$

and the cumulative surface density in this field from heliocentric distance $r_1$ to $r_2$ is

$$\Sigma(r_1, r_2) = \int_{r_1}^{r_2} dN/\xi^2.$$ \hfill (3)

We can thus derive that the fraction of objects that should be further out than some distance $r_s$ is

$$\frac{\Sigma(r_s, \infty)}{\Sigma(r_{\text{min}}, r_s)} = \left[ \frac{r_{\text{min}}}{r_s} \right]^\gamma - 1.$$ \hfill (4)

where $\gamma = 5 - 2q - \beta$.

If we assume that there is no maximum diameter for TNOs, and that the inner edge is at 30 AU, then the fraction of the objects that should be outside $r_s=50$ AU depends heavily on the size index $q$. Irwin et al. (1995) show that $\alpha = (q - 1)/5$ if the radial distribution is smooth; we will take $\beta = 2$ although $\beta=3$ is more appropriate for a primordial disk with constant inclination. Dones (1997) used the shallower size distribution $q = 3$, which predicts that more than 1 quarter of all TNOs should have been discovered outside 50 AU. For JIT98's result of $q = 4$, one finds that 8% of TNOs should be outside 50 AU; for a somewhat steeper $q \approx 4.8$ (from our best-fit $\alpha$) this drops to 4%. Thus, the lack of detections outside 50 AU in our pencil-beam surveys is not a surprise. For the entire ensemble of $\sim 55$ TNOs, one should expect several or $\sim 1$ such object(s) depending on the size distributions, and on the complications introduced by a more realistic model. For example, including a maximum diameter (meaning that large objects do not exist to be seen) or including a 'plutino' component trapped in resonance with Neptune will both drop the expected fraction of objects outside of 50 AU. We also do not know if all previous surveys were uniformly sensitive to objects moving as slowly as 2°/hour or less. We conclude that as yet there is not a convincing problem, and that a doubling or tripling of the TNO population will be needed before one should begin to worry about the lack of distant objects.

7. Discussion

We discovered 5 TNOs in our combined CFHT and Palomar pencil-beam surveys. We used a maximum likelihood analysis to combine our results with 4 other published TNO surveys. Including our deep pencil-beam work, we conclude that the luminosity function of TNOs is steeper than previous estimates, with the number of TNOs increasing by a factor of $10^{q-6}$ per magnitude. This rapid increase implies that a deep survey $R \sim 26$ with a sensitive, large field of view CCD mosaic (say 0.25 square degrees), should discover tens of TNOs in a single field; we have been allocated observing time on the CFHT to attempt
this project. Our best-fit luminosity function apparently brings into accordance almost all published faint-object surveys. Our results neither directly confirm nor deny the validity of the HST detections; our best estimate of the luminosity function, if extrapolated to $R \simeq 28$, predicts a sky density of $\sim 4,000/\Delta^2$, about 1.5-sigma below the HST estimate.

Although our original intent was to work to $R \sim 26$ regardless of the sky density, it is interesting to note that our pencil-beam method will actually find more TNOs for a fixed telescope time than the 'classical' method of looking for moving objects in 3 exposures separated by $\sim 1$ hour, due to the steep luminosity function. In a background limited environment, 6 exposures are required to work one magnitude fainter than a single exposure. A classical search acquires one exposure of 6 different fields; however, it must repeat the 6 fields 3 times. Thus, after this has been completed a deep search that concentrated on a single field has 18 times the flux and thus goes 1.6 magnitudes deeper than any one of the single images from the classical method. Thus, the ratio, $N_d/N_c$, of the number of objects discovered by the deep survey to the classical survey is

$$\frac{N_d}{N_c} = 6 \times 10^{1.6 \alpha}$$

where the factor of 6 appears due to the greater areal coverage of the classical method. The methods thus discover equal numbers of objects for $\alpha \approx 0.5$, and the deep search method discovers more objects for all steeper slopes. The point here is that the pencil-beam method uses all the available photons to contribute to the depth of the survey, whereas the classical method uses only one-third, since it is the exposure limit of a single image that determines the depth of the classical survey. For $\alpha \approx 0.7$ our pencil-beam method discovers more than twice as many objects per night. Of course, most of these objects are near the magnitude limit of the survey and are thus not easily recoverable in order to monitor and improve their orbits. The deep method is thus better suited to study the large-scale structure of the belt; for example, by compiling better statistics on the number of objects as a function of ecliptic latitude.

Our best estimate for the luminosity function implies a surface density at magnitude $R \sim 29$ (radius $\sim 10$ km at 45 AU) of $\sim 4 \times 10^4$ TNOs per square degree which, assuming a belt of latitudinal extent $\pm 15^\circ$ implies $4 \times 10^5$ Kuiper Belt objects from 30 to 50 AU, in rough agreement with previous estimates based on the number of short-period comets (Levison and Duncan 1997). However, the uncertainty of the radii of a typical short-period comets and the steepness of the luminosity function results in it being very easy to tune this number by small variations of the magnitude of a 'comet' in the Kuiper Belt. Nevertheless, this steeper luminosity function implies that the Kuiper Belt could be the current source of the Jupiter-family comets, although a component of objects coming from the 'scattered disk' cannot be ruled out (see Duncan and Levison 1997).

The lack of detections of Centaurs or objects beyond 50 AU in our Palomar data is consistent with a simple extrapolation of the size and radial distributions. A doubling or tripling of the number of TNOs needs to occur without discoveries outside 50 AU before there is a convincing problem with the lack of detections here. A single large ($30' \times 30'$) pencil-beam survey in the ecliptic to $R > 26$ should answer the question.

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