HALF-LIVES AND PRE-SUPERNova WEAK INTERACTION
RATES FOR NUCLEI AWAY FROM THE STABILITY LINE

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Abstract: A detailed model for the calculation of beta decay rates of the fp shell nuclei for situations prevailing in pre-supernova and collapse phases of evolution of the core of massive stars leading to supernova explosion has been extended for electron-capture rates. It can also be used to determine the half-lives of neutron-rich nuclei in the fp/fpg shell. The model uses an averaged Gamow-Teller (GT) strength function. But it can also use the experimental log ft values and GT strength function from (n, p) reaction studies wherever available. The calculated rate includes contributions from each of the low-lying excited states of the mother including some specific resonant states (”back resonance”) having large GT matrix elements.

1 Introduction

Nuclei away from the line of stability play an important role in a number of problems in nuclear astrophysics. The wealth of experimental information on such nuclei right
up to the drip line ones obtained from experiments with radioactive nuclear beams (RNB) in recent years has focussed many issues in the r-process nucleosynthesis, stellar evolution, etc. In this article we discuss why the weak interaction rates of these nuclei in stellar conditions are important for the pre-supernova (preSN) and supernova (SN) evolution of massive stars and describe our work in constructing a model for calculating these rates for nuclei important at this stage with $50 < A < 80$. This model also predicts the half-lives of neutron/proton-rich nuclei, an important ingredient in the structure studies of the RNB nuclei. The beta-decay of the n-rich nuclei plays a significant role at the silicon burning stage of the preSN, and the rates for some nuclei considered important at typical temperatures and densities have been reported [1]. The electron capture (EC) rates on nuclei with neutrons filling the fp-shell orbits are also needed for the collapse state of the SN. We report here on the model for the calculation of the EC rates for them.

2 Pre-Supernova and Supernova Evolution

The last stage of stellar burning involves successive addition of \( \alpha \)-particles on silicon and other intermediate mass nuclei at temperatures typically at $4 \times 10^9$ K. In the time scale of a few days the final end product $^{56}$Fe is reached and a degenerate core is created where no further energy generation through nuclear reactions is possible. If the core mass exceeds the Chadrasekhar mass then the gravitational collapse of the core starts with the matter in the envelope having a quasi-free fall [2]. During the further evolution of the core one of the most important qualities is the lepton fraction $Y_l$. This is because at densities a few times the nuclear matter density the collapse ultimately stops with the stiffer baryonic matter at the central region giving rise to a shock wave whose energy is strongly dependent on $Y_l$, being proportional to $Y_{lf}^{10/3}(Y_{lf} - Y_{li})$, where $Y_{lf}$ and $Y_{li}$ are the final and initial lepton fractions. Recent findings indicate that even at the Si-burning stage beta decay
and electron captures on the relevant nuclei are important in order to determine the $Y_e$ (the electron fraction) at the beginning of the collapse and the network of nuclei involved in the weak interaction processes needs to be enlarged with the inclusion of some neutron-rich nuclei [3,4]. For this purpose beta decay rates on some nuclei in the fp shell have recently been calculated [1,5] and the possible set of important nuclei at different densities and temperatures have been listed with the simplified assumption of nuclear statistical equilibrium. Here one finds that some neutron-rich nuclei contribute significantly to leptonisation in spite of their small abundances since they have large beta decay rates because of large Q values. Electron capture rates on some of these fp shell nuclei again become very important during the collapse phase. During the collapse deleptonisation takes place through rapid electron capture on nuclei with $A > 56$ until the neutron number exceeds 40 when the allowed electron capture gets blocked.

### 3 Half-Lives and Beta-decay Rates of Neutron-rich Nuclei

We first briefly describe the physics input in the model to calculate the presupernova beta-decay rates [1]. It is based on a statistically averaged allowed beta-decay, in particular the Gamow-Teller(GT) strength function. For the GT sum rule strength for $\beta^-$ decay we use an expression involving the occupancies of neutron particles and proton holes which is seen to be a good approximation for the fp shell nuclei using the framework of spectral distribution theory [6,7,8]. One observes that for $\beta^-$ decay with $N > Z$, one can reach only the tail of the GT giant resonance due to energy conservation. This giant resonance is assumed to be distributed as a Gaussian in energy, a feature seen in studies in large full shell-model spaces in the absence of strong collectivity. The centroid of this Gaussian is fixed from the results of (p,n) reaction studies and a phenomenological formula for its position involving
\(N - Z\) and \(A^{1/3}\) is employed \[9\]. Recently, Sutaria and Ray \[10\] improved this by using a fit to new data with mostly fp shell nuclei and a calculation of rates based on their centroids is being pursued. The strength width coming from the nuclear Hamiltonian is left as a parameter and is determined globally by a best fit to the observed half-lives of a number of n-rich fp shell nuclei with \(A > 60\). It is seen that large shell configuration mixing and possible coupling to \(\Delta\) excitation gives rise to a long tail in the GT strength distribution at higher energies causing quenching of the observed strength. This is taken into account by an overall quenching factor of 0.6 in the sum rule following Aufderheide et al. \[4\]. In this work we report the calculated half-lives of some more fp shell nuclei with neutrons filling the \(g_{9/2}\) orbit.

For the latter case we put the neutrons in excess of 40 in the \(g_{9/2}\) orbit and add three times the number of neutrons in the \(g_{9/2}\) orbit to the GT sum rule. For example for \(^{68}\)Fe, we put 2 neutrons in \(g_{9/2}\) and add 6 to the GT sum rule calculated for \(^{66}\)Fe. In Table 1 we present our predictions compared with experimental values as well as with predictions by microscopic theories \[11\] and gross theory or its modified form \[12\].

In Table 2, column A is with a Gaussian GT distribution with the width parameter set at 6.3 MeV and column B is with the same width at 7.5 MeV and a skewed distribution with skewness -0.3 to bring more strength in the ground state domain.

In the calculation of rates in stellar conditions the partial blocking of the electrons produced by the Fermi sea outside is important and is taken into account using the electron chemical potential obtained by solving an integral equation numerically. This chemical potential has built into it both finite temperature correction and corrections coming from the production of electron-positron pairs. The model also includes contributions from excited states which, though reduced by the factor \(exp(-E_i/kT)\), are appreciable because of larger Q-values than the ground state. For their sum rule strength we use the same expressions where the occupancies are calculated by spectral distributions at the respective excitation energies and the
strength distribution is assumed to have the same form. For some nuclei at high densities ($\rho > 10^9 g cm^{-3}$) these excited state contributions are an order of magnitude more than the ground state contributions.

In Table 2 we present the $\beta^-$ decay rates for typical densities and temperatures for four isotopes of cobalt along with their Q-values. We find that the rates are the highest for the largest Q-value for $^{64}$Co. Our rates for some nuclei include experimental log ft's to the ground state/lowlying excited state where the predicted half-lives without them are in disagreement. For $^{62}$Co and $^{62}$Fe one finds that this inclusion of one or two lowlying log ft's brings the predicted half-lives very close to experimental values. The rates also include contributions from a particular resonant state of the mother nucleus which is in equilibrium with the electron capture rate on the daughter. This resonant state has a large overlap with the daughter ground state or low excited state and can get connected to them by proton-neutron single particle allowed GT transition. These so called ‘back resonance’ rates are calculated by following the prescription of Fuller, Fowler and Newman II [13] and is described in Kar, Ray and Sarkar [1].

4 Model for Electron Capture Rates

We briefly describe the major features of the calculation for the electron capture rates. They are:

1) The GT sum-rule for electron capture uses the same expressions as in the case of beta-decay, replacing the neutron occupancy by the proton occupancy and the proton hole occupancy by the neutron hole occupancy. The quenching factor used is 0.5, a typical value from (n,p) data. Thus the GT sum for EC ($S_{\beta^+}$) is much smaller than the corresponding $\beta^-$ sum-rule ($S_{\beta^-}$) on the same nucleus and the difference between the two is always $3(N - Z)$, a model independent result. For example, for $^{62}$Fe the unquenched $S_{\beta^-}$ and $S_{\beta^+}$ are 36.7 and 6.7 whereas the corresponding
quenched values are 21.5 and 3.4 respectively.

2) For the strength distribution one uses the Gaussian form in the case of EC rates also. For the centroid of the Gaussian one uses the empirical form obtained by Sutaria and Ray [10] through fits to (n,p) data and for the width one uses an average value of 2 MeV.

3) For nuclei with \( N > Z \), EC connects the ground state of the mother nucleus with isospin \( T_0 \) to states of the daughter nucleus with isospin \( (T_0 + 1) \) only. As a result, Fermi transitions are ruled out by isospin conservation.

4) The phase space factor for EC is of course different from the \( \beta^- \) decay with 2 particles in the exit channel instead of 3.

The code used for this calculation has two sectors with electron capture Q-values both positive and negative. For the positive Q-value case, for the ground state one has the options of using (i) experimental log ft’s to low-lying states of the daughter, (ii) shell model log ft’s, and (iii) GT strength function of our model. For the negative Q-value case the code can calculate the ground state rates from (i) \( B_{GT}(E) \) distribution of (n,p) reactions given as strength/MeV; (ii) shell model log ft’s and (iii) GT strength-function of our model. For the low-lying excited states in both the cases the only options available are using (i) shell model log ft’s, if available, and (ii) the GT strength function of our model. For electron capture a special excited state of the mother is considered whose shell model configuration differs from the ground state of the daughter by a single proton and a single neutron so that it has a large overlap with the daughter state and the electron capture rate is in equilibrium with the back reaction rate of the daughter. In Table 3 we give EC rates on \( ^{56}\text{Fe} \) for typical preSN temperatures and densities. One sees that the contributions from the excited states are sometimes an order of magnitude larger than the rate from the ground state of the nucleus. For \( ^{56}\text{Fe} \) the values with 28 excited states are compared to the contribution coming from the lowest 4 excited
states.

5 Conclusion

In conclusion, we stress that we have built models for calculating the beta decay and electron capture rates for fp shell nuclei for use in problems of preSN and SN evolution. This model can be used to predict half-lives of beta decay of nuclei away from the line of stability. We plan to extend this to predict half-lives for very n-rich nuclei near the drip line as well as to electron capture and positron decay for the p-rich nuclei. This also should be extended to lower densities and temperatures so that it can be used to supply the weak interaction rates in the r- and s-process nucleosynthesis calculations.

References


Table 1

Comparison of the $\beta^-$ decay rates from the ground state of the mother nucleus for the four different isotopes of Cobalt. Values are given for $Y_e = 0.47$.

<table>
<thead>
<tr>
<th>Temperature $\log_{10} \rho$ (K)</th>
<th>Nucleus &amp; Q-value (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$^{62}\text{Co}$</td>
</tr>
<tr>
<td></td>
<td>5.322</td>
</tr>
<tr>
<td></td>
<td>$3 \times 10^9$</td>
</tr>
<tr>
<td>-2.0</td>
<td>$4 \times 10^9$</td>
</tr>
<tr>
<td></td>
<td>$5 \times 10^9$</td>
</tr>
</tbody>
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Table 2

Comparison of calculated and experimental Half-Lives.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Experimental</th>
<th>This work</th>
<th>Gross Theory</th>
<th>QRPA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>$^{68}\text{Co}$</td>
<td>0.18±0.10</td>
<td>0.54</td>
<td>0.10</td>
<td>0.81</td>
</tr>
<tr>
<td>$^{68}\text{Fe}$</td>
<td>0.10±0.06</td>
<td>1.05</td>
<td>0.36</td>
<td>0.42</td>
</tr>
<tr>
<td>$^{69}\text{Ni}$</td>
<td>11.4</td>
<td>6.53</td>
<td>2.46</td>
<td>20</td>
</tr>
<tr>
<td>$^{71}\text{Cu}$</td>
<td>19.53</td>
<td>13.12</td>
<td>5.25</td>
<td>7.6</td>
</tr>
<tr>
<td>$^{72}\text{Cu}$</td>
<td>6.6</td>
<td>1.47</td>
<td>0.38</td>
<td>2.7</td>
</tr>
<tr>
<td>$^{73}\text{Cu}$</td>
<td>3.9</td>
<td>2.75</td>
<td>0.96</td>
<td>1.7</td>
</tr>
<tr>
<td>$^{69}\text{Co}$</td>
<td>0.27</td>
<td>0.72</td>
<td>0.19</td>
<td>0.68</td>
</tr>
<tr>
<td>$^{71}\text{Ni}$</td>
<td>1.86</td>
<td>1.85</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>$^{72}\text{Ni}$</td>
<td>2.10</td>
<td>4.25</td>
<td>1.87</td>
<td></td>
</tr>
<tr>
<td>$^{58}\text{Cr}$</td>
<td>7.0</td>
<td>22.0</td>
<td>10.0</td>
<td></td>
</tr>
<tr>
<td>$^{63}\text{Mn}$</td>
<td>0.25</td>
<td>0.94</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>$^{59}\text{Cr}$</td>
<td>0.74</td>
<td>1.81</td>
<td>0.54</td>
<td></td>
</tr>
</tbody>
</table>
Table 3

Electron-capture rates for $^{56}\text{Fe}$. Values are given for $Y_e = 0.47$ and $T = 6 \times 10^9\text{K}$.

<table>
<thead>
<tr>
<th>log $\rho_{10}$</th>
<th>$\mu_e$ in MeV</th>
<th>Rate (g.s.)</th>
<th>Rate</th>
<th>Rate</th>
<th>Total Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(log ft=2.48) with 4 states with 28 states</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2.0</td>
<td>1.00</td>
<td>$2.4 \times 10^{-4}$</td>
<td>$1.46 \times 10^{-3}$</td>
<td>$4.39 \times 10^{-3}$</td>
<td>$4.39 \times 10^{-3}$</td>
</tr>
<tr>
<td>-1.0</td>
<td>3.40</td>
<td>0.008</td>
<td>0.224</td>
<td>0.396</td>
<td>0.396</td>
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</tbody>
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