

3.1. An Initial Guess Function

II. THE PARAMETERIZATION

The parameterization of the parameter $\alpha$ is achieved through the following equation:

$$\alpha = \frac{m}{\mu} \left( 1 + \frac{n}{\mu} \right)$$

where $m$ and $n$ are parameters that can be determined from data.

In the parameterization of the parameter $\beta$, we have the following equation:

$$\beta = \frac{m}{\mu} + \frac{n}{\mu}$$

The parameter $\gamma$ is determined through the relationship:

$$\gamma = \frac{m}{\mu} \left( 1 + \frac{n}{\mu} \right) + \frac{m}{\mu}$$

We can define the effective mass fraction of a parameter $\delta$ as:

$$\delta = \frac{m}{\mu} \left( 1 + \frac{n}{\mu} \right)$$

The effective mass fraction of a parameter $\epsilon$ is given by:

$$\epsilon = \frac{m}{\mu} \left( 1 + \frac{n}{\mu} \right) + \frac{m}{\mu}$$

IV. CONCLUSION

The results presented in this paper show that the parameterization of the parameters $\alpha$, $\beta$, and $\gamma$ allows for a more accurate determination of the parameter $\delta$.

ACKNOWLEDGMENTS

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References


DARK MATTER AND THE CHEMICAL EVOLUTION OF IRREGULAR GALAXIES

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Figure 1: A schematic diagram of the chemical evolution of irregular galaxies.
Table 1

$r$ Values

<table>
<thead>
<tr>
<th>Object</th>
<th>Ref</th>
<th>$0.01-85 , M_\odot$</th>
<th>$0.01-120 , M_\odot$</th>
<th>$0.08-85 , M_\odot$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar vicinity</td>
<td>KTG</td>
<td>1.30</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Solar vicinity</td>
<td>KTG</td>
<td>1.85</td>
<td>1.512</td>
<td>1.510</td>
</tr>
<tr>
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<td>0.70</td>
<td>0.820</td>
<td>0.820</td>
</tr>
<tr>
<td>NGC 6752</td>
<td>FCBRO</td>
<td>1.90, 2.33</td>
<td>1.879</td>
<td>1.876</td>
</tr>
<tr>
<td>NGC 7099</td>
<td>PCK</td>
<td>2.00</td>
<td>1.812</td>
<td>1.809</td>
</tr>
<tr>
<td>NGC 6397</td>
<td>PCK</td>
<td>1.60</td>
<td>1.202</td>
<td>1.201</td>
</tr>
</tbody>
</table>

$^a$ IMF $\propto \begin{cases} 
\frac{m^{-\alpha_1}}{m} & \text{if } m_1 \leq m < 0.5 , \\
\frac{m^{-2.2}}{m} & \text{if } 0.5 \leq m < 1.0 , \\
\frac{m^{-2.7}}{m} & \text{if } 1.0 \leq m \leq m_u. 
\end{cases}$

$^b$ $m^{-1.3}$ if $m_1 \leq m < 0.27$, $m^{-2.33}$ if $0.27 \leq m < 0.5$.

The IMF for the solar neighborhood adopted in this paper is what was called KTG IMF (Kroupa et al. 1993) in CCPS, except that here the upper limit is taken to be 85 $M_\odot$, and it is given by

$$\xi(m) = \begin{cases} 
0.506 \, m^{-1.3} & \text{if } 0.01 \leq m < 0.5 , \\
0.271 \, m^{-2.2} & \text{if } 0.5 \leq m < 1.0 , \\
0.271 \, m^{-2.7} & \text{if } 1.0 \leq m \leq m_u. 
\end{cases} \tag{5}$$

where $\xi(m) \, dm$ is the number of stars in the mass interval from $m$ to $m + dm$. This function is extended to a minimum mass of 0.01 to take into account the fraction of dark matter hidden in objects of substellar mass. This function is normalized to one; that is,

$$\int_{m_1}^{m_u} \xi(m) \, dm = 1, \tag{6}$$

where $m_1$ and $m_u$ are the lower and upper limits, are taken to be 0.01 and 85, respectively, unless otherwise stated.

3.2. Chemical Evolution Models and the Value of $r$

An $r$ value was defined in CCPS as follows

$$r(m_l, m_u) = \frac{\int_{m_l}^{m_u} \xi(m) \, dm}{\int_{m_l}^{m_u} \xi'(m) \, dm}, \tag{7}$$

where $m_l$ and $m_u$ are the low-mass and high-mass end of the IMF respectively, $\xi(m)$ is the KTG IMF, and $\xi'(m)$ is an IMF with different slope in $m_l \leq m < 0.5$ range (see equation 5 and Table 1); $r$ depends on $m_l$ because $\xi(m)$ and $\xi'(m)$ are normalized to one (see equation 6).

The chemical evolution models depend on $r$ because the net yields of the heavy elements, and in particular that of O, decrease when $r$ increases. (See the definition of net yield in Peimbert, Colin & Sarmiento 1994, and models for different $r$ values in CCPS).

The closed-box model based on the yields by Maeder (1992) is unable to reproduce simultaneously the $\mu$, O, C/O, Z/O, and $\Delta[Y] / \Delta O$ values of the typical irregular galaxy studied by CCPS. CCPS were able to reproduce the observed constraints based on an O-rich outflow model with $\gamma = 0.23$ and $r(0.01, 120) = 2.66$, where $\gamma$ is the fraction of O produced by SNe that is ejected to the intergalactic medium without mixing with the interstellar gas.

We want to study the effect of using different yields in the models. We will produce closed-box models for different $r$ values to fit the observational constraints. We will discuss if these $r$ values are in agreement with the observed IMFs of globular clusters and of the solar vicinity.

3.3. Solar Vicinity

We will study two problems: the $r$ values determined for different IMFs and the effect of the yields by Maeder and WIW & WW on the $M_{sub}$ value for chemical evolution models of the solar vicinity.

In Table 1 we present $r$ values for different mass ranges and for different values of the slope for the low-mass range, $\alpha_1$, given by IMF $\propto m^{-\alpha_1}$ for $m_1 \leq m < 0.5$; we have adopted the IMF slopes given by KTG for $m > 0.5$. In Table 2 we present the average mass of the objects in the IMF from $m_\tau$ to $m_u$, when $m_u = m_\tau = 0.01$ we include all objects in the IMF and when $m_\tau = 0.01$ we include only stellar objects in the IMF.

The KTG simulations of star-count data reach the maximum confidence when the scale height for their model reaches 270 pc; in this case $\alpha_1 = 1.3$. Based on their model KTG suggest for the solar vicinity that $0.70 < \alpha_1 < 1.85$, for the $0.08 \leq m < 0.5$ range. We adopted $\alpha_1 = 1.3$ as the preferred value, and for this value we define $r = 1$ for the three mass ranges presented in Table 1 (see the first line of this table).

We present three mass ranges in Table 1 for the following reasons: Maeder (1992), and WIW (1993) use $m_u$ equal to 120 and 85 respectively, while CCPS and in this paper we have adopted $m_u = 0.01$, in the last column we present the lower mass included by KTG, $m_\tau = 0.08$.

Carigi (1996) computed a chemical evolution model of the solar neighborhood with $M_{sub} = 0$ based on the yields by Maeder (1992) adopting the KTG IMF for the $0.01 \leq m \leq 120$ range. For this model the mass fraction in
substellar objects amounts to 15.1% and the present day
gas fraction is 0.15 in agreement with the data \((0.05-0.20,\)
Tosi 1996).\)

We have computed a model for the solar neighborhood
under the same assumptions as those adopted by Carigi
(1996), but based on the yields by LWL and WW, and
adopted a 0.01 ≤ m ≤ 85 mass range \((r = 1).\) We obtain
for this model a mass fraction in substellar objects of 14% \(\text{and}
the present day gas fraction is 0.07, also in agreement
with the observations (Tosi 1996).

Both models of the solar vicinity could accommodate
an additional modest amount of substellar objects or non
baryonic dark matter and still reproduce the observed
abundances and the \(\sigma_{\text{gal}}/\sigma_{\text{tot}}\) value.

If there is a significant fraction of non baryonic dark
matter that does not participate in the chemical evolution
process, it has to be subtracted from \(M_{\text{tot}}\) increasing the
value of \(\mu_{IMF}.\) Therefore to reach the observed abundances
with lower gas consumption the IMF needs to have a larger
fraction of massive stars, and if we keep the KTG slope for massive stars constant we need to reduce the
fraction of low-mass stars in the model, and consequently
\(r\) values smaller than one.

On the other hand, if \(M_{\text{tot}}\) is larger than the observed value adopted by us, and if this difference is due to a
greater mass fraction in substellar objects in the solar vicinity, then the \(\mu_{IMF}\) would become smaller and the model
would need a higher gas consumption and \(r\) values
higher than one to be able to reproduce the observed abundances.

3.4. Globular Clusters

Based on HST observations Ferraro et al. (1997, hereinafter FCBR0) determined the mass function for the
lower main sequence of the globular cluster NGC 6752
\((Z = 0.03 Z_\odot).\) They found \(x\) values of 0.90 and 1.33
for the mass ranges 0.15 - 0.30 \(M_\odot\) and 0.25 - 0.55 \(M_\odot\),
respectively, where \(\xi(x)\) is proportional to \(m^{-1.4}\).
Therefore we have adopted for NGC 6752:

\[\xi_{\text{6752}}(m) \propto \left\{ \begin{array}{ll}
m^{-1.9} & \text{if } m \leq m_\odot, \\
m^{-2.3} & \text{if } 0.27 \leq m \leq 0.5, \\
m^{-2.2} & \text{if } 0.5 \leq m \leq 1.0, \\
m^{-2.7} & \text{if } 1.0 \leq m \leq m_u. 
\end{array} \right. \]

Note that to derive an \(r\) value we need to define the IMF
in the \(m_\odot-m_u\) range and that for \(m \geq 0.5\) we are adopting KTG for all globular clusters.

Also based on HST observations Piotto, Cool, & King
(1997, hereinafter PCK) have determined the mass function
for four globular clusters. Three of them have very similar
mass functions: NGC 6341, NGC 7078 and NGC 7099.
Based on the mass luminosity relation by D'Antona and
Mazzitelli (1995) PCK derive for NGC 7099 that \(x = 1.0\)
for masses below 0.4 \(M_\odot\). Consequently, we have adopted
for NGC 7099:

\[\xi_{\text{7099}}(m) \propto \left\{ \begin{array}{ll}
m^{-2.0} & \text{if } m \leq m_\odot, \\
m^{-2.2} & \text{if } 0.5 \leq m \leq 1.0, \\
m^{-2.7} & \text{if } 1.0 \leq m \leq m_u. 
\end{array} \right. \]

For the same mass range, and the same mass luminosity
relation PCK find \(x = 0.6\) for NGC 6397. They suggest
that the lower value of \(x\) derived for NGC 6397, relative
to the other three clusters, could be due to selective loss
of low-mass stars by evaporation and tidal shocks.

In Table 1 we present the NGC 6752, NGC 7099 and
NGC 6397 \(r\) values for the different mass ranges considered.
By comparing the \(r\) values for different mass ranges it is
found that the effect on \(r\) introduced by changing the
upper mass end from 120 to 85 \(M_\odot\) is negligible. Moreover,
it is also found that the \(r\) values for NGC 6752 and
NGC 7099 are significantly higher than for the solar vicinity.
This result implies that the fraction of low-mass stars is
higher for globular clusters than for the solar vicinity,
and might imply that the fraction of substellar objects is
higher also.

4. CHEMICAL EVOLUTION MODELS

All the models in this paper reproduce at least two observational constraints: the O abundance by mass in the
interstellar medium (ISM), and \(\mu.\) In addition each model predicts different element abundance ratios by mass
that can be compared with other observational constraints.

We computed three types of models: closed-box with
continuous SFRs, closed-box with bursting SFRs, and
O-rich outflow with continuous SFRs. The assumptions adopted in our models are:

a) The initial composition of the gas is primordial:
\(Y_\odot = 0.23, Z_\odot = 0.00.\)

b) We have computed models for three galaxy ages, \(t_g:\)
0.1, 1.0, and 10.0 Gyr

c) The star formation rate is proportional to the gaseous
mass, \(\text{SFR} = \nu M_{\text{gas}.}\) The efficiency, \(\nu,\) is mainly determined
by the need to reach \(\mu_{IMF}\) at the age of the model.
For a continuous SFR \(\nu\) is constant in time. On the other
hand, when we consider a bursting SFR

\[\nu = \left\{ \begin{array}{ll}
\text{constant} & \text{if } t_g \leq t < t_g + 40 \text{ Myr}, \\
0 & \text{if } t_g + 40 \text{ Myr} \leq t \leq t_g + 1, 
\end{array} \right. \]

where \(t_g = (t_0 - 0.2) \left(\frac{\nu}{\text{const}}\right)^{1/\nu}\) Gyr is the burst starting time,
\(n\) is the total number of bursts, and \(1 \leq \nu \leq n.\)

d) We have adopted several IMFs (see §3). For \(m \geq 0.5\) all of them have the same slopes as those given by KTG.
The \(r(0.01, 85)\) value is varied until the desired oxygen abundance is obtained. This \(r(0.01, 85)\) value corresponds to
a unique IMF with a slope for stars with \(m < 0.5\) denoted by \(a_1.\) In what follows \(r\) corresponds to the 0.01-85 mass
interval unless otherwise noted.

e) We drop the instantaneous recycling approximation,
IRA, and assume that the stars eject their envelopes after leaving the main sequence. The main sequence lifetimes
are taken from Schaller et al. (1992). The possible reduction
of the O yields of massive stars due to the production of
black holes as suggested by Maeder (1992) has not been
considered.

f) We have used the stellar yields and remnant masses
due to: i) Renzini & Voli (1981) for \(1.0 \leq m \leq 8.0\)
(\(\alpha = 1.5, \eta = 1/3); ii) WW for \(11 \leq m \leq 40\) (models
"B" for \(30, 35\) and \(40 \text{ M}_\odot); iii) LWL for \(m = 60\) and
\(85.\) We also consider the changes in the stellar yields
due to the stellar initial metallicity. Only massive stars,
those with \(m > 8,\) enrich the ISM with oxygen.

g) For SNIA we have taken into account the yields by
Nomoto, Thielemann, & Yokoi (1984, model W7). Only a fraction of binary stars, in the \(3 \leq m_1 + m_2 \leq 16\) range,
Table 2
<m> Values 

<table>
<thead>
<tr>
<th>( \alpha )^a</th>
<th>0.1-85 M(_{\odot} )</th>
<th>0.1-120 M(_{\odot} )</th>
<th>0.01-85 M(_{\odot} )</th>
<th>0.01-120 M(_{\odot} )</th>
<th>0.08-85 M(_{\odot} )</th>
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<tbody>
<tr>
<td>1.30</td>
<td>0.501</td>
<td>0.503</td>
<td>0.194</td>
<td>0.195</td>
<td>0.452</td>
</tr>
<tr>
<td>1.85</td>
<td>0.391</td>
<td>0.393</td>
<td>0.074</td>
<td>0.074</td>
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<tr>
<td>0.70</td>
<td>0.626</td>
<td>0.629</td>
<td>0.429</td>
<td>0.431</td>
<td>0.503</td>
</tr>
<tr>
<td>1.90, 2.33</td>
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<td>0.346</td>
<td>0.062</td>
<td>0.062</td>
<td>0.293</td>
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<td>2.00</td>
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<td>1.60</td>
<td>0.440</td>
<td>0.441</td>
<td>0.116</td>
<td>0.116</td>
<td>0.386</td>
</tr>
</tbody>
</table>

^a As in Table 1.

become SNIa; where \( 2.2 \leq m_1 \leq 8.0 \) and \( 0.8 \leq m_2 \leq 8.0 \). We have determined such fraction by fitting the observed solar Fe abundance.

In models with outflow of O-rich material we assume that a fraction, \( \gamma \), of the mass expelled by type II SNe is ejected to the intergalactic medium without mixing with the ISM. The WW yields have been computed without considering stellar winds, therefore the ejected mass during a SN explosion is equal to the initial mass minus the stellar remnant.

5. NGC 1560 AND II Zw 33

There are two irregular galaxies with good O/H values for which \( \mu_{IMF} \) can be determined: NGC 1560 and II Zw 33, also known as Markarian 1094. For these two galaxies we can compute closed-box models for different ages, each model characterized by an \( r(0.01, 85) \) value, which corresponds to a specific IMF.

5.1. \( \mu_{IMF} \) and O/H

From the studies of the rotation curves of a few dwarf irregular galaxies it is found that they are dominated by non baryonic dark matter (e.g., Bural 1996, Salucci & Penci 1997), some of them well within the core of the mass distribution (e.g., Moore 1994). In general most of the \( M_{\odot} \) is present outside the Holmsberg radius. Unfortunately, for most of these non baryonic dominated galaxies, for which a rotation curve is available, chemical abundance determinations do not exist. From the very reduced group for which rotation curves as well as oxygen abundances are available we have extracted NGC 1560 and II Zw 33 to build chemical evolution models. For these galaxies \( M_{H2}, M_b, \) and consequently \( \mu_{IMF} \) are known and are presented in Table 3 (see Walter et al. 1997 and Broeils 1992).

The O/H gasvaces value for NGC 1560 comes from Richer & McCall (1995) and for II Zw 33 comes from Esteban & Peimbert (1995). To derive the O abundances by mass we have considered the contribution of O expected to be in dust grains (0.04 dex) and the effect of temperature variations over the observed volume (0.16 dex); consequently we have added 0.2 dex to the gasvaces values derived under the assumption of a constant temperature distribution inside the II H regions (see CCPS and references therein).

5.2. Closed-box Models with Continuous SFRs for NGC 1560 and II Zw 33

Closed-box models for NGC 1560 and II Zw 33 have been computed under the assumptions presented in §4. The models reproduce \( \mu_{IMF} \) and the O abundance by mass shown in Table 3.

In Table 4 we present three models computed for NGC 1560 with continuous SFRs. Each line of this Table represents a different model. For each age we find a unique \( r(0.01, 85) \) value. Columns 3 to 6 show the mass fractions defined in equations (1) and (2). The C/O, \( \Delta Y/\Delta O \), and \( \Delta Z/\Delta O \) ratios are presented in columns 7 to 9; in all tables C, O, Y and Z are given by mass.

The models with \( \gamma = 0.00 \) and \( M_{\odot} = 0.0 \) for II Zw 33 are presented in Tables 5 and 6, for these models \( \mu_{IMF} \approx 0.29 \). From these models we note that: a) \( r \) increases with model age because a larger fraction of stars have enriched the ISM with heavy elements and the model needs to reduce \( M_{\odot} \) (columns 5 and 6 of Tables 4 and 5, respectively); b) if \( r \) increases, \( \alpha_1 \) increases and \( M_{\odot} \) becomes higher; c) \( M_\odot \) changes little with age; d) despite the fact that \( M_{\odot} \) decreases with age \( M_{\odot} \) grows with age because the fraction of stars that have had time to end their evolution is higher; e) for NGC 1560 \( M_{\odot} + M_{\odot} \approx 61 \% \) and \( r \approx 1.73 \) while for II Zw 33, \( M_{\odot} + M_{\odot} \approx 43 \% \) and \( r \approx 2.75, \) at 10 Gyr; f) the 1.0-Gyr and 10-Gyr models predict C/O and Z/O ratios higher than those determined observationally for our typical irregular galaxy (see Tables 9); g) the 0.1-Gyr models reproduce well the observed C/O and Z/O ratios but predict lower \( \Delta Y/\Delta O \) values than observed.

5.3. Additional Models for II Zw 33

There is no compelling observational evidence for large systematical IMF variations in galaxies for \( m > 1.0 \) (e.g., Kennicutt 1998 and references therein). Therefore it is possible that the IMF could be the same everywhere for \( m < 1.0 \). This would imply a unique \( r \) value for all objects. Therefore we will explore if it is possible to produce chemical evolution models for II Zw 33 with \( r = 1.8 \), the average value for NGC 6752, NGC 7099, and NGC 1560 (\( t_\odot = 10 \) Gyr). There are two groups of models that satisfy the \( r = 1.8 \) requirement; closed-box models with \( M_{\odot} \neq 0.0 \) and O-rich outflow models with \( M_{\odot} = 0.0 \). In what
CHEMICAL EVOLUTION OF IRREGULAR GALAXIES

Table 3
Properties of NGC 1560 and II Zw 33

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>$\log(M_{\text{total}}/M_\odot)$</th>
<th>$\log(M_{\text{gas}}/M_\odot)$</th>
<th>$\log\mu$</th>
<th>$\log\mu_{\text{IMF}}$</th>
<th>$10^3\Omega$</th>
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</thead>
<tbody>
<tr>
<td>NGC 1560</td>
<td>9.83</td>
<td>9.20</td>
<td>-0.63</td>
<td>-0.34</td>
<td>2.03</td>
</tr>
<tr>
<td>II Zw 33</td>
<td>9.71</td>
<td>9.17</td>
<td>-0.54</td>
<td>-0.54</td>
<td>1.93</td>
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</table>

Table 4
Models for NGC 1560$^a$

<table>
<thead>
<tr>
<th>$t_g$(Gyr)</th>
<th>$r$</th>
<th>$M_{\text{Sub}}$(%)</th>
<th>$M_{\text{d}}$(%)</th>
<th>$M_{\text{rest}}$(%)</th>
<th>$M_{\text{rem}}$(%)</th>
<th>C/O</th>
<th>$\Delta Y/\Delta O$</th>
<th>Z/O</th>
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<tr>
<td>0.1</td>
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<td>8.7</td>
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<td>8.8</td>
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<td>0.8</td>
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<td>3.678</td>
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<td>10.0</td>
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<td>9.0</td>
<td>4.5</td>
<td>2.2</td>
<td>0.324</td>
<td>4.167</td>
<td>2.429</td>
</tr>
</tbody>
</table>

$^a M_{\text{sub}} + M_{\text{d}} + M_{\text{rest}} + M_{\text{rem}} + M_{\text{gas}} + M_{\text{ neb}} = 100.0\%$, with $M_{\text{gas}} = 23.4\%$, $M_{\text{ neb}} = 48.9\%$, and $\mu_{\text{IMF}} = 0.46$

Table 5
Models for II Zw 33$^a$

<table>
<thead>
<tr>
<th>$t_g$(Gyr)</th>
<th>$r$</th>
<th>$\gamma$</th>
<th>$M_{\text{sub}}$(%)</th>
<th>$M_{\text{d}}$(%)</th>
<th>$M_{\text{rest}}$(%)</th>
<th>$M_{\text{rem}}$(%)</th>
<th>$M_{\text{ neb}}$(%)</th>
<th>$\mu_{\text{IMF}}$</th>
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<tr>
<td>0.1</td>
<td>2.502</td>
<td>0.00</td>
<td>38.6</td>
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<td>1.0</td>
<td>2.632</td>
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<td>41.0</td>
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<td>10.0</td>
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<td>0.00</td>
<td>43.3</td>
<td>17.8</td>
<td>6.6</td>
<td>3.5</td>
<td>0.0</td>
<td>0.29</td>
</tr>
</tbody>
</table>

$^a M_{\text{sub}} + M_{\text{d}} + M_{\text{rest}} + M_{\text{rem}} + M_{\text{gas}} + M_{\text{ neb}} = 100.0\%$, with $M_{\text{gas}} = 28.8\%$
follows we will explore these two possibilities further.

5.3.1. Closed-box Models with $M_{rb} \neq 0$

In Tables 5 and 6 we present the main characteristics of models with $r = 1.8$ and $M_{rb} \neq 0$ for II Zw 33. The two main differences between the models with $r = 1.8$ and those with $r > 2.5$ are that, as expected, $M_{rb}$ decreases and $M_{gas}$ increases with decreasing $r$. The increase in $M_{rb}$ implies an increase in $\mu_{MP}$. Alternatively the changes in the C/O, $\Delta Y/\Delta O$, and Z/O values are negligible. These models would imply that the $M_{rb}$ determination by Walter et al. (1997) is not correct.

5.3.2. O-rich Outflow Models with $M_{rb} = 0$

We have produced O-rich outflow models ($r \neq 0.0$) for II Zw 33 with $r = 1.8$ and $M_{rb} = 0$ (see Tables 5 and 6). The O-rich outflow models predict C/O, $\Delta Y/\Delta O$, and Z/O values higher than the closed-box models (see Table 6). The C/O and Z/O values are not known for II Zw 33, our models predict that if O-rich outflows have been important during the evolution of this object its C/O and Z/O values should be considerably higher than those observed in other irregular galaxies (see §7.2).

6. A TYPICAL IRRGULAR GALAXY

For NGC 1560 and II Zw 33 we have only $\mu_{MP}$, $M_{rb}$, and O as observational constraints; these constraints are not enough to decide if O-rich outflows have been present in these objects. We have decided to model a typical irregular galaxy because for it we can use average observational values for $\mu$, O, C/O, $\Delta Y/\Delta O$, and Z/O, derived from a set of well observed irregular galaxies; these observational constraints will permit us to address the issue of the importance of O-rich outflows for the evolution of the typical dwarf irregular galaxy.

In what follows we will estimate the general properties of a typical irregular galaxy. We will use the same set of galaxies that was used by CCPS. These galaxies were chosen because their properties are well known, in particular the chemical composition of their gaseous content.

For the irregular galaxies in the CCPS sample we do not have a $\mu_{MP}$ value because we do not know which is the contribution of the halo dark matter, $M_{rb}$ to $M_{t, gal}$. Therefore the observed $\mu$ value is a lower limit to $\mu_{MP}$ (see equation 4).

6.1. The $\mu$ Value

The $\mu$ value depends on the $M_{gas}$ to $M_{t, gal}$ ratio. We will revise the $M_{gas}$ and $M_{t, gal}$ values adopted by CCPS for each galaxy and the new adopted values will be presented in Table 7.

CCPS neglected the contribution of H$_2$ to $M_{gas}$ due to the low CO content of the irregular galaxies. Nevertheless, there are observational and theoretical considerations that favor a CO-to-H$_2$ conversion factor, $X_{CO-H_2}$, that increases for systems of lower metallicity (Maloney & Black 1988; Wilson 1995; Arimoto, Sofue, & Tsujimoto 1996). Therefore we can not exclude the possibility of having low-metallicity irregular galaxies with a relatively high molecular hydrogen content.

The H$_2$ mass estimated for II Zw 40 by Tacconi & Young (1987), for NGC 6822 by Israel (1997), and adopted for II Zw 33 by Walter et al. (1997) is about 10% of H I mass. Consequently, we will multiply the H I + He I gaseous mass of irregular galaxies by a factor of 1.1 to take into account the contribution due to H$_2$, with the exception of I Zw 18 and UGC 4483, where no such correction will be applied (Carigi & Peimbert 1998).

Madden et al. (1997) recently estimated an unusually high H$_2$ column density (a factor of five that of H I) in three regions of IC 10, their result is based on [C II] 158 micron observations and an argument of thermal balance. If this result is applied to the whole galaxy one finds that $\mu = 1.2$ which is impossible since by definition $\mu$ has to be smaller than 1. We consider that there is no room for such high amounts of H$_2$ in dwarf irregular galaxies (see also Lequeux 1996).

To produce a homogeneous set of $\mu$ values we need to estimate $M_{t, gal}$ at a given distance from the galactic center. Wherever possible we have chosen the Holmberg radius, $R_H$, because most of the gas and stars are inside it, for larger radii $M_{rb}$ becomes larger and $\mu$ becomes smaller deviating more from $\mu_{MP}$.

Based in part on the previous discussion we have introduced the following changes to Table 1 of CCPS and have generated the $M_{t, gal}$ and $M_{gas}$ values of Table 7: a) we have added a 10% to the H I + He I gaseous mass to consider the H$_2$ contribution for all galaxies except for II Zw 40, where H$_2$ is explicitly taken by Tacconi & Young (1987), and I Zw 18 and UGC 4483, where we are assuming a null H$_2$; b) we have adopted a Hubble constant of $H_0 = 100 h$ kms Mpc$^{-1}$ with $h = 0.65$, while CCPS adopted $h = 0.70$, errors in the distance, d, will alter $\mu_{MP}$ because $M_{gas} \propto d^2$ while $M_{t, gal} \propto d$; c) we have revised for each galaxy the determination of $M_{t, gal}$. The discussion on the $M_{t, gal}$ and $\mu$ determinations for each galaxy follow.

I Zw 18 — The total mass within $R_H$ (Staveley-Smith, Davies, & Kimman 1992, hereinafter SDK) give a ratio of $M_{gas}$ to $M_{t, gal}$ greater than one. On the other hand, assuming that the high radial velocity gradient is due to rotation, Petrosian et al. (1997) derive a total mass (within a radius of 0.48 kpc) higher than that derived by SDK. The difference is due to the higher rotational velocity estimated by Petrosian et al. Our $M_{t, gal}$ and $M_{gas}$ values are those adopted by Carigi & Peimbert (1998), which are a compromise between the values given by SDK and Petrosian et al. Incidentally, the $\mu$ value adopted here is close to that adopted by CCPS.

UGC 4483.— There are two determinations of $M_{t, gal}$, one by SDK and another by Lo, Sargent, & Young (1993), which together with the $M_{gas}$ derived by SDK yield an average $\mu$ value of 0.76. The radius at which $M_{t, gal}$ is derived, 1.42 kpc (putting UGC 4483 at the distance given by SDK but with $h = 0.65$), is close to its $R_H = 1.28$ kpc (SDK).

Mk 600.— We adopted as the total mass within $R_H$ that derived by SDK but assuming an $h = 0.65$.

SMC, LMC, & II Zw 40.— These three galaxies have a total mass derived within a radius that is lower than $R_H$, being II Zw 40 the extreme case. This is not a problem as long as $\mu$ does not suffer a significant change when going from this radius to $R_H$. The radius at which the total mass of SMC is derived (Hindman 1967) is close to its $R_H$ (Balkowski, Chamarua, & Welicich 1978), therefore we
### Table 6

**Abundance Ratios for II Zw 33**

<table>
<thead>
<tr>
<th>$t_2$ (Gyr)</th>
<th>$r$</th>
<th>$\gamma$</th>
<th>C/O</th>
<th>$\Delta Y/\Delta O$</th>
<th>Z/O</th>
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<td>2.502</td>
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### Table 7

**Properties of Selected Galaxies**

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<tr>
<th>Galaxy</th>
<th>$\log(M_{\text{total}}/M_\odot)$</th>
<th>$\log(M_{\text{gas}}/M_\odot)$</th>
<th>$\log \mu$</th>
<th>$Y$</th>
<th>$10^3O^a$</th>
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</thead>
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<tr>
<td>I Zw 18</td>
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<td>8.21</td>
<td>$-0.05$</td>
<td>0.230</td>
<td>0.317</td>
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<td>UGC 4483</td>
<td>8.07</td>
<td>7.95</td>
<td>$-0.12$</td>
<td>0.239</td>
<td>0.639</td>
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<tr>
<td>Mrk 600</td>
<td>8.88</td>
<td>8.76</td>
<td>$-0.12$</td>
<td>0.240</td>
<td>1.967</td>
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<tr>
<td>SMC</td>
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<td>8.71</td>
<td>$-0.41$</td>
<td>0.237</td>
<td>2.268</td>
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<tr>
<td>II Zw 40$^b$</td>
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<td>8.62</td>
<td>$-0.84$</td>
<td>0.251</td>
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<tr>
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<td>9.74</td>
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<td>II Zw 70</td>
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<td>8.67</td>
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</tr>
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<td>NGC 4449</td>
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<td>9.63</td>
<td>$-1.28$</td>
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<td>4.526</td>
</tr>
<tr>
<td>average</td>
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<td>8.75</td>
<td>$-0.54$</td>
<td>0.243</td>
<td>2.589</td>
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</table>

$^a$The O gaseous values have been multiplied by 1.58 (0.2 dex, see text).

$^b$Values quoted for the northern cloud. Molecular and ionized hydrogen is added.
would not have expected a very different $\mu$ value if $M_{\text{tot},\text{H}}$ had been determined inside $R_H$ (we are also assuming that the contribution from the gas beyond $R_H$ is negligible). As the distance determined by Welch et al. (1987) of 61 kpc has now been used, as opposed to 70 kpc, its $M_{\text{tot},\text{H}}$ and $M_{\text{gas}}$ values have changed accordingly. On the other hand, according to Kunkel, Demers, & Irwin (1996) most of the mass of the LMC is located within the inner 5 deg, we are thus not making a big mistake by using the $\mu$ associated to the inner region of 4.2 deg (Lequeux et al. 1979); apparently there is no massive dark halo in LMC (Kunkel et al. 1997). The H I core of II ZW 40 was studied by Gottlieb & Weliachew (1972) and its corresponding $\mu$ was adopted by CCPs from Lequeux et al. From the $M_{\text{tot},\text{H}}$ values derived by Brinks & Klein (1988), we find $\mu$ value of 0.14 for the northern cloud of II ZW 40 (see Table 7). This low value of $\mu$ might imply the presence of a significant amount of dark matter.

IC 10. — The Keplerian estimate of $M_{\text{tot},\text{H}}$ (Shostak 1974) was derived within a radius which is very close to $R_H$ ($R_H$=4.0 kpc, if the distance to the galaxy is taken to be 3 Mpc). Recent determinations of the distance to IC 10 put it close to 1 Mpc (e.g., Wilson et al. 1996). This value contrasts with the 3 Mpc adopted by Lequeux et al., from Sandage & Tammann (1975). We have adopted 1.5 Mpc and at this distance $\mu \approx 0.51$.

II ZW 70 & NGC 6822. — The total masses of these two galaxies are derived within a radius which is greater than their $R_H$ values by about a factor of two (Balkowski et al. 1987; Gottlieb & Weliachew 1977). By using these $M_{\text{tot},\text{H}}$ values, we may be underestimating the value of $\mu$ as compared with the values derived for other galaxies in the sample. A very detailed modeling of the rotation curve of these two galaxies is needed to know the contribution of a dark halo to the total mass.

NGC 4449. — The total mass of this galaxy, within a radius of 37 kpc (by far greater than its optical radius) has been estimated recently by Bajaja, Huchtmeier, & Klein (1994). Its $\mu = 0.052$ is considerably lower than those usually estimated for dwarf irregular galaxies, in particular, lower than the other galaxies of our sample. The rotation curve indicates the presence of dark matter in its extended H I halo but, as in the case of NGC 6822 and II ZW 70, a very detailed modeling of its rotation curve is still needed.

It is interesting to note that the mean $\mu$ value presented in Table 7 differs only by 2% from the mean $\mu$ value obtained by CCPs.

6.2. O, C/O, $\Delta Y/\Delta O$, and Z/O

Columns 5 and 6 of Table 7 present the helium and the oxygen abundances by mass, the data are the same as those presented by CCPs. In the last two lines of Tables 9 and 11 we present the C/O, $\Delta Y/\Delta O$, and Z/O values derived by CCPs from their sample of irregular galaxies.

More recent determinations of He abundances permit to derive other $\Delta Y/\Delta O$ values. From the data of Ito, Thuan, & Lipovetsky (1997) on extragalactic H II regions it is obtained that $\Delta Y/\Delta O = 3.1 \pm 1.4$. Based on observations by many authors Olive, Steigman, & Skillman (1997) obtain a pregalactic helium abundance, $Y_p$, of $0.234 \pm 0.002$, alternatively Ito et al. find $Y_p = 0.243 \pm 0.003$. By adopting $Y_p = 0.240 \pm 0.006$ and combining this value with the $Y$ and $O$ abundances of the galactic H II region M17, that amount to $0.280 \pm 0.006$ and $(8.69 \pm 1.3) \times 10^{-3}$, respectively (Peimbert, Torres-Peimbert, & Ruiz 1992), it follows that $\Delta Y/\Delta O = 4.6 \pm 1.1$. We have added 0.08 dex to the gaseous O abundance to consider the fraction of O atoms embedded in dust (Ettel et al. 1998). Finally, from fine structure in the main sequence based on Hipparcos parallaxes Pagel & Portinari (1998) obtain that $\Delta Y/\Delta O = 5.6 \pm 3.6$. These three $\Delta Y/\Delta O$ values are in good agreement with the value presented in Tables 9 and 11.

![Fig. 1 — Mass fractions of (a) substellar objects, $M_{\text{sub}}$, and of (b) non baryonic dark matter, $M_{\text{db}}$, for different $r(0.01, 85)$ values. The data correspond to closed-box models with continuous SFRs for different ages of the typical irregular galaxy (see Table 8).](image)

6.3. Closed-box Models with Continuous SFRs

The properties of the typical irregular galaxy were obtained from the galaxies presented in Table 7 and are given in the last row of this table.

We have computed closed-box models with continuous SFRs and different ages for the typical irregular galaxy. All the models reproduce the observational constraints, $\mu = 0.288$ and $O = 2.589 \times 10^{-2}$. For the typical irregular galaxy we do not know the amount of $M_{\text{sub}}$, therefore the observed $\mu$ value is a lower limit to $\mu_{\text{HMF}}$. Consequently, we have computed models for a range of $r(0.01, 85)$ values; we think it is unlikely that the $r$ value is smaller than one (see Table 1), and $r_{\text{max}}$ is the value for $M_{\text{sub}} = 0.0$, $r$ cannot be higher than $r_{\text{max}}$ because $M_{\text{sub}}$ would become negative. In Table 8 we present the model results for the different mass fractions defined in this paper.

The distributions of $M_{\text{sub}}$ and $M_{\text{db}}$ for different $r(0.01, 85)$ values are plotted in Figure 1. From this fig-
ure it can be seen that $M_{\text{sub}}$ increases with $r$ and $M_{\text{hb}}$ decreases with $r$. The increase of $M_{\text{sub}}$ with $r$ is due to an increase of the slope of the low-mass end of the IMF with $r$. The decrease of $M_{\text{hb}}$ with $r$ is due to the lower efficiency in the O production and therefore to the decrease of $\mu_{\text{MF}}$ (see Table 8 and equation 4). Furthermore for a given $r$ value $M_{\text{sub}}$ decreases with the age of the model while $M_{\text{hb}}$ increases with it.

In Table 9 we present the predicted abundance ratios by our closed-box models with continuous SFRs, the predicted abundance ratios by CCPS, and the observed ratios. From this table it can be seen that: a) the predicted abundance ratios for a given model age are almost independent of $r$; b) the predicted ratios increase with model age due to the production of Y and C by intermediate mass stars, while O is produced only by massive stars; c) the C/O and Z/O values point to models with ages not older than one Gyr, while the $\Delta Y/\Delta O$ value indicates older ages; d) our models predict considerably larger values for C/O, $\Delta Y/\Delta O$, and Z/O than the models by CCPS, this result is due to the difference between the yields by WIW & WW and those by Maeder (1992).

Our sample in Table 7 was chosen to have a large spread in $Y$ and O to derive a meaningful $\Delta Y/\Delta O$ value, but our sample is not completely homogeneous because the O abundances for I Zw 18 and UGC 4483 are very small and the $M_{\text{obs}}$ for NGC 4449 was estimated from observations at distances far away from $R_H$. To have a more homogeneous sample we can eliminate the three objects in Table 7 that deviate most from the average $Y$ and O values: I Zw 18, UGC 4483, and NGC 4449. For this reduced sample we obtain the following average quantities: $\log Y = -0.56$ and $10^8 O = 2.915$, in very good agreement with the values adopted for the typical irregular galaxy. Moreover, the chemical evolution models computed to adjust the average values of the reduced sample are very similar to those computed for the typical irregular galaxy.

6.4. Closed-box Models with Bursting SFRs

The presence of old populations in many of the dwarf irregular galaxies in the local group indicates that star formation started ~10 Gyr ago (e.g., Mateo 1998; Pagel & Tautvaisienė 1998). The nature of the star formation histories of the galaxies is diverse (eg., Mateo 1998). The most massive dwarf irregular galaxies appear to have had a continuous SFR, while ordinary dwarf irregular galaxies appear to have undergone a gasping SFRs (eg., Tosi 1998). It is interesting then to see if by changing the continuous nature of the SFR to a discontinuous one (bursting), our results presented in §6.3 change drastically.

Figure 2 shows in six panels the evolution of the SFR, $\mu_{\text{MF}}$, O, C/O, $\Delta Y/\Delta O$, and Z/O. The model is for $t_2 = 1.0$ Gyr and has three bursts each lasting 40 Myr. The behavior of O, C/O, $\Delta Y/\Delta O$, and Z/O are as expected. The oxygen abundance increases suddenly during each burst and then stays constant in the quiescent phase. On the other hand, C, Y and Z increase during the quiescent phase because they are also produced by intermediate mass stars. There is a period of time after the second or third burst in which these ratios decrease because the growth rate of the O abundance is higher than those of C, Y, and Z. The spike in the C/O, $\Delta Y/\Delta O$, and Z/O values at the beginning of the model is due to the C and Y production in very massive stars relative to that of O.

Table 10 is similar to Table 8 but with an extra column that gives the number of bursts, $n$, of each model. We did not compute a bursting SFR model for 0.1 Gyr, because the 0.1-Gyr model with continuous SFR presented in Tables 8 and 9 can be considered as a bursting model with single burst lasting 100 Myr. It can be seen from Table 10 that the $r$ parameter is not very sensitive to $n$. The $r$ parameter increases slightly as we reduce the number of bursts.

The abundance ratios for the typical irregular galaxy with bursting SFRs are given in Table 11. This Table is similar to Table 9 except for the inclusion of a new column representing the number of bursts of the model and the absence of the 0.1-Gyr model. All of the remarks drawn from Table 9 can also be drawn from Table 11; in particular, the 1.0-Gyr and 10-Gyr models reproduce well the observed $\Delta Y/\Delta O$ value but underestimate the observed C/O and Z/O values. A slight improvement in the comparison of the predicted versus observed C/O and Z/O values is introduced by bursting SFRs (specially when $n = 2$), but high C/O and Z/O values continue to be predicted. Furthermore, from Figure 2 and the observed values in Table 11, it can be seen that after a burst the predicted versus the observed C/O and Z/O values become closer, but the $\Delta Y/\Delta O$ values become farther apart.

7. Discussion

7.1. The $r$ Value

The $r$ value is almost independent of moderate changes in the high-mass limit of the IMF, alternatively it depends strongly on the low-mass limit and on the slope of the IMF at the low-mass end (see Table 1).

The average $r(0.01, 85)$ value for the globular clusters NGC 6752 and NGC 7099 amounts to 1.85, while for $r(0.08, 85)$ amounts to 1.32 (see Table 1). These $r$ values are higher than those for the solar vicinity and might mean two different things: a) that the $r$ value for the solar vicinity is not well known and that the $r$ value for globular clusters is representative of the solar vicinity, or b) that systems with lower metallicity have higher $r$ values. In this discussion we have not considered NGC 6397 due to the possible selective loss of low-mass stars by evaporation and tidal encounters.

A system with $r > 1$ has a larger mass fraction of objects with $m < 0.5$ than a system with $r = 1$, this is true for any value of $m_I < 0.5$, including the case when $M_{\text{sub}} = 0.0$.

7.2. Model Results

We have computed closed-box models for NGC 1560 and I Zw 33 that match the observed O and $\mu_{\text{MF}}$ values. For the 10 Gyr models the $r$ values are equal to 1.73 and 2.75, respectively.

If the IMF is universal it follows that the same $r$ should apply to all galaxies. From globular clusters it is obtained that $r \sim 1.8$, this is the best $r$ determination available and it might apply to all galaxies.

Since closed-box models with continuous SFRs and $M_{\text{sub}} = 0.0$ yield $2.5 < r < 2.75$ for I Zw 33, we decided to compute other types of models with $r = 1.8$ for this galaxy. The closed-box continuous SFR models with $r = 1.8$ require that 28% $< M_{\text{hb}} < 37\%$, in contradiction with the
Table 8

Continuous SFR Models for the Typical Galaxy

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<thead>
<tr>
<th>$t_\odot$ (Gyr)</th>
<th>$r$</th>
<th>$M_{\text{sub}}$ (%)</th>
<th>$M_{e,\ell}$ (%)</th>
<th>$M_{\text{rest}}$ (%)</th>
<th>$M_{\text{rem}}$ (%)</th>
<th>$M_{\text{nb}}$ (%)</th>
<th>$\mu$ IMF</th>
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<tr>
<td>0.1</td>
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<th>$M_{\text{rem}}$ (%)</th>
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<td>16.5</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>2.000</td>
<td>32.7</td>
<td>19.9</td>
<td>8.4</td>
<td>4.5</td>
<td>4.8</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>2.075</td>
<td>36.1</td>
<td>20.9</td>
<td>8.6</td>
<td>4.7</td>
<td>0.0</td>
<td>0.30</td>
</tr>
</tbody>
</table>

$^a$ $M_{\text{sub}} + M_{e,\ell} + M_{\text{rest}} + M_{\text{rem}} + M_{\text{gas}} + M_{\text{nb}} = 100.0\%$, with $M_{\text{gas}} = 29.7\%$

Table 9

Abundance Ratios for the Typical Galaxy (Continuous SFRs)

<table>
<thead>
<tr>
<th>$t_\odot$ (Gyr)</th>
<th>$r$</th>
<th>C/O</th>
<th>$\Delta Y/\Delta O$</th>
<th>Z/O</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.000</td>
<td>0.165</td>
<td>2.788</td>
<td>1.832</td>
</tr>
<tr>
<td></td>
<td>1.400</td>
<td>0.166</td>
<td>2.873</td>
<td>1.855</td>
</tr>
<tr>
<td></td>
<td>1.800</td>
<td>0.167</td>
<td>2.965</td>
<td>1.876</td>
</tr>
<tr>
<td></td>
<td>1.855</td>
<td>0.167</td>
<td>2.978</td>
<td>1.883</td>
</tr>
<tr>
<td></td>
<td>3.240</td>
<td>0.135</td>
<td>1.783</td>
<td>1.736</td>
</tr>
<tr>
<td>1.0</td>
<td>1.000</td>
<td>0.283</td>
<td>3.608</td>
<td>2.247</td>
</tr>
<tr>
<td></td>
<td>1.400</td>
<td>0.292</td>
<td>3.663</td>
<td>2.273</td>
</tr>
<tr>
<td></td>
<td>1.800</td>
<td>0.303</td>
<td>3.721</td>
<td>2.299</td>
</tr>
<tr>
<td></td>
<td>1.967</td>
<td>0.307</td>
<td>3.747</td>
<td>2.315</td>
</tr>
<tr>
<td></td>
<td>3.290</td>
<td>0.225</td>
<td>2.597</td>
<td>1.927</td>
</tr>
<tr>
<td>10.0</td>
<td>1.000</td>
<td>0.320</td>
<td>4.093</td>
<td>2.415</td>
</tr>
<tr>
<td></td>
<td>1.400</td>
<td>0.322</td>
<td>4.130</td>
<td>2.427</td>
</tr>
<tr>
<td></td>
<td>1.800</td>
<td>0.324</td>
<td>4.170</td>
<td>2.439</td>
</tr>
<tr>
<td></td>
<td>2.000</td>
<td>0.324</td>
<td>4.191</td>
<td>2.439</td>
</tr>
<tr>
<td></td>
<td>2.075</td>
<td>0.325</td>
<td>4.199</td>
<td>2.445</td>
</tr>
<tr>
<td></td>
<td>3.390</td>
<td>0.240</td>
<td>2.946</td>
<td>1.938</td>
</tr>
<tr>
<td>Obs$^b$</td>
<td></td>
<td>0.212</td>
<td>4.48</td>
<td>1.85</td>
</tr>
<tr>
<td>errors(±)$^b$</td>
<td></td>
<td>0.071</td>
<td>1.02</td>
<td>0.20</td>
</tr>
</tbody>
</table>

$^a$ These lines corresponds to closed-box models from CCPS

$^b$ Taken from CCPS
### Table 10

**Bursting SFR Models for the Typical Galaxy**

<table>
<thead>
<tr>
<th>( t_g ) (Gyr)</th>
<th>( n )</th>
<th>( r )</th>
<th>( M_{\text{sub}} ) (%)</th>
<th>( M_{\text{st}} ) (%)</th>
<th>( M_{\text{rem}} ) (%)</th>
<th>( M_{\text{nb}} ) (%)</th>
<th>( \mu_{\text{IMF}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>2</td>
<td>1.930</td>
<td>27.5</td>
<td>17.6</td>
<td>13.2</td>
<td>1.6</td>
<td>10.4</td>
</tr>
<tr>
<td>3</td>
<td>1.894</td>
<td>27.2</td>
<td>17.8</td>
<td>13.2</td>
<td>1.7</td>
<td>10.4</td>
<td>0.33</td>
</tr>
<tr>
<td>5</td>
<td>1.860</td>
<td>26.8</td>
<td>18.0</td>
<td>13.4</td>
<td>1.7</td>
<td>10.4</td>
<td>0.33</td>
</tr>
<tr>
<td>( \infty )^b</td>
<td>1.800</td>
<td>25.9</td>
<td>18.3</td>
<td>14.1</td>
<td>1.6</td>
<td>10.4</td>
<td>0.33</td>
</tr>
<tr>
<td>10.0</td>
<td>2</td>
<td>1.841</td>
<td>24.0</td>
<td>16.4</td>
<td>9.0</td>
<td>4.4</td>
<td>16.5</td>
</tr>
<tr>
<td>3</td>
<td>1.832</td>
<td>24.2</td>
<td>16.7</td>
<td>8.4</td>
<td>4.5</td>
<td>16.5</td>
<td>0.36</td>
</tr>
<tr>
<td>5</td>
<td>1.818</td>
<td>24.1</td>
<td>16.7</td>
<td>8.8</td>
<td>4.2</td>
<td>16.5</td>
<td>0.36</td>
</tr>
<tr>
<td>( \infty )^b</td>
<td>1.800</td>
<td>24.5</td>
<td>17.3</td>
<td>7.8</td>
<td>4.2</td>
<td>16.5</td>
<td>0.36</td>
</tr>
</tbody>
</table>

\(^a\) \( M_{\text{sub}} + M_{\text{st}} + M_{\text{rem}} + M_{\text{gas}} + M_{\text{nb}} = 100.0\% \), with \( M_{\text{gas}} = 29.7\% \)

\(^b\) These lines correspond to continuous SFRs

### Table 11

**Abundance Ratios for the Typical Galaxy (Bursting SFRs)**

<table>
<thead>
<tr>
<th>( t_g ) (Gyr)</th>
<th>( n )</th>
<th>( r )</th>
<th>( C/O )</th>
<th>( \Delta Y/\Delta O )</th>
<th>( Z/O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>2</td>
<td>1.930</td>
<td>0.289</td>
<td>4.024</td>
<td>2.349</td>
</tr>
<tr>
<td>3</td>
<td>1.894</td>
<td>0.304</td>
<td>3.943</td>
<td>2.349</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.860</td>
<td>0.319</td>
<td>3.894</td>
<td>2.358</td>
<td></td>
</tr>
<tr>
<td>( \infty )^b</td>
<td>1.800</td>
<td>0.303</td>
<td>3.721</td>
<td>2.299</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>2</td>
<td>1.841</td>
<td>0.291</td>
<td>4.355</td>
<td>2.389</td>
</tr>
<tr>
<td>3</td>
<td>1.832</td>
<td>0.303</td>
<td>4.288</td>
<td>2.406</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.818</td>
<td>0.309</td>
<td>4.171</td>
<td>2.408</td>
<td></td>
</tr>
<tr>
<td>( \infty )^b</td>
<td>1.800</td>
<td>0.324</td>
<td>4.170</td>
<td>2.439</td>
<td></td>
</tr>
<tr>
<td>Obs^b</td>
<td></td>
<td></td>
<td>0.212</td>
<td>4.48</td>
<td>1.85</td>
</tr>
<tr>
<td>errors(±)^b</td>
<td></td>
<td></td>
<td>0.071</td>
<td>1.02</td>
<td>0.20</td>
</tr>
</tbody>
</table>

\(^a\) These lines corresponds to continuous SFRs

\(^b\) Taken from CCPS
results of $M_{\text{vir}} = 0$ derived by Walter et al. (1997). O-
rich continuous SFR models with $r = 1.8$ and $M_{\text{vir}} = 0.0$
implies C/O $> 0.33$ for models older than 1.0 Gyr. This
C/O ratio is higher than the highest values detected for an
irregular galaxy (0.248 for 30 Doradus in LMC) and
considerably higher than 0.158, the average C/O value for
the eight irregular galaxies for which C/O is known (Gar-
values for H II Zw 33 is not known and conceivable could be
considerably higher than those of other irregular galaxies
but it seems unlikely.

We have also computed closed-box models with contin-
uous SFRs for the typical irregular galaxy that match the
observed $\mu_0$, O, and $\Delta Y/\Delta O$ values for different $r$
values (see Tables 8 and 9). To choose one of these models
we need to know the $r$ or the $M_{\text{vir}}$ value. Based on the models
for NGC 1560 and on the $r$ values for globular clusters we
consider that the best models should be around $r = 1.8$
(A maximum value of $r = 2.1$ is obtained when $M_{\text{vir}} \rightarrow 0.0$
for the typical irregular galaxy (see Table 8). We do not know which is the behavior of the IMF for
$m < 0.1$, nor the value of $r$, but by assuming that $\alpha_1$ is
the same down to $m = 0.01$, the solution for the typical
irregular galaxy with $r(0.01, 0.15) = 1.80$ at 10 Gyr implies
that $M_{\text{vir}} = 24.5 \%$ and $M_{\text{vir}} = 16.5 \%$. If $M_{\text{vir}}$ is smaller
than that obtained from a given $\alpha_1$, it is possible to ob-
tain a closed-box model that fits the same observational
constraints by increasing $M_{\text{vir}}$ (see Table 8).

We consider that O-rich outflows are not very impor-
tant for the typical irregular galaxy because O-rich outflow
models predict higher C/O and Z/O values than those ob-
served. The changes in the final abundance ratios between
the 0.1-Gyr and the 1.0-Gyr models (see Table 9) are more im-
portant than the changes between different SFR histories,
for a given $t_2$ (see Table 11). For 1.0 Gyr $\leq t_2 \leq 10$ Gyr
the changes between bursting and continuous SFR mod-
els are very small (see Table 11). For $t_2 = 0.1$ Gyr, the con-
tinuous SFR model can be considered as a single-burst
model and the differences between this model and those
for 1.0 Gyr and 10 Gyr are very significant (see Table 9).

We also consider that outflow of well-mixed material is
not important. Outflow models of well-mixed material
will have lower $r$ and $M_{\text{vir}}$ values than those presented in Ta-
bles 4, 5, 8, and 10; but to produce drastic reductions in
$r$ these models require the ejection of large amounts of
gas to the intergalactic medium, that have not yet been
observed around irregular galaxies. The previous discus-
sion is based on CCPS models for outflow of well-mixed
material; their models were made for $r = 1$ while their
closed-box model for 10 Gyr gives $r = 3.39$, and the ratio
of the ejected mass to the mass left in the galaxy is 7.95.

Furthermore, in general infall of material with pregalac-
tic abundance, $Y = Z = 0$, is not important because
these models are not able to match low O values with moder-
ately low $\mu_{\text{MPH}}$ values (Peimbert et al. 1994).

### 7.3. CCPS O-rich Outflow Models

Are the closed-box models presented in Tables 8-11 the
only possibility to adjust the observational constraints of
the typical irregular galaxy? The answer is no. By us-
ing the yields by Maeder (1992) CCPS have shown that it
is also possible to adjust the observational constraints by
means of O-rich outflow models.

The two reasons given by CCPS to support O-rich out-
flow models were the high $r$ and the low $\Delta Y/\Delta O$
values predicted by the closed-box models; for the 10 Gyr
model these values amount to 3.390 and 2.946, re-
spectively (see Table 9). This CCPS model was made un-
der the $M_{\text{vir}} = 0.0$ assumption. Our 10-Gyr closed-box
model for the typical irregular galaxy with $M_{\text{vir}} = 0.0$
yields $r = 2.075$, and $\Delta Y/\Delta O = 4.190$. The differences
between our model and the CCPS model are only due to
differences in the adopted yields.

By introducing $M_{\text{vir}}$ values different from zero in the
CCPS models it is possible to reduce $r$ to a reasonable
value for a closed-box model; but $\Delta Y/\Delta O$ would still be
lower than observed because it is almost independent of $r$.

### 8. Conclusions

The IMF from globular clusters has a larger fraction of
low-mass stars, and consequently a larger value of $r$, than
the KTG IMF. This result is based on the assumption that
the IMF slopes for $m > 0.5$ are the same for all objects.
The difference in the $r$ value, if real, could be due to the
lower metallicity of the globular clusters relative to the so-
lar vicinity. Alternatively, considering that the IMF seems
to be metallicity independent at higher masses, the dif-
ference in the $r$ value could be due to observational errors;
since the accuracy of the IMF determination for globu-
lar clusters is higher than for the solar vicinity maybe the
lower end of the KTG IMF should be modified to agree
with that derived from globular clusters.

The $r$ values required by the closed-box models based
on the yields by WIW & WW for NGC 1560 and H II Zw 33,
where $M_{\text{vir}}$ has been determined, are considerably higher
than one. Moreover, the $r$ value for NGC 1560 is very
similar to those derived from globular clusters.

The models based on the yields by WIW & WW predict
lower $r$ values than those based on the yields by Maeder
(1992). For a given model that fits $\mu_{\text{MPH}}$ and O, the yields
by WIW & WW predict higher C/O, $\Delta Y/\Delta O$, and Z/O
values than the yields by Maeder. The closed-box mod-
els with continuous SFRs based on the yields by WIW
& WW can fit the observational constraints provided by
the well observed irregular galaxies. In other words, the
O-rich outflows that are required by the yields of Maeder
for the typical irregular galaxy are not required by the
yields of WIW & WW.

For models with the same age the C/O, $\Delta Y/\Delta O$, and
Z/O ratios are almost independent of the $r$ value.

The fit between the C/O and Z/O ratios predicted by
the closed-box models with continuous SFRs and the ob-
servational constraints is only fair.

It is possible to obtain lower $r$ values by adopting: a) O-
rich outflow models or b) closed-box models with higher
$M_{\text{vir}}$ values and lower $M_{\text{vir}}$ values (see Table 5). Never-
theless, O-rich outflow models can be disregarded for the
typical irregular galaxy because they predict higher C/O
and Z/O values than closed-box models and consequently
larger differences with the observed values.

A C/O determination and a better determination of
$\mu_{\text{MPH}}$ is needed for H II Zw 33 to be able to discriminate
among the different models that we have computed for
this object.

The dark matter mass fraction for the models com-
Fig. 2— Bursting SFR model for the typical irregular galaxy with \( t_o = 1.0 \) Gyr, \( n = 3 \), and \( r (0.01, 85) = 1.894 \) (see Table 11). C, O, Y, and Z are given by mass.

By comparing bursting SFR models with continuous SFR models of the same age the differences in the final abundance ratios are very small. Consequently, it can be said that the shape of the SFR does not affect the results considerably. The largest differences occur just after a burst: the C/O, \( \Delta Y/\Delta O \), and Z/O values decrease, diminishing the differences of C/O and Z/O with the observed values but increasing the differences of the \( \Delta Y/\Delta O \).
with observed value.

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