Parity Violation in Effective Field Theory and the Deuteron Anapole Moment

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Abstract

Effective field theory, including pions, provides a consistent and systematic description of nucleon-nucleon strong interactions up to center-of-mass momentum $p \sim 300$ MeV per nucleon. We describe the inclusion of hadronic parity violation into this effective field theory and find an analytic form for the deuteron anapole moment at leading order in the expansion.

I. INTRODUCTION

An increasing amount of both experimental and theoretical interest is being focused on nuclear parity violation. The problems in reproducing parity violating observables in light nuclei (such as $^{18}$F [1]) and in the nuclear anapole measurement of cesium [2] by parameters in single boson exchange potentials [3,4] suggest that we re-examine the theoretical framework with which hadronic parity violation is studied. The low momentum transfers involved in these parity violating processes makes them amenable to an effective field theory treatment. After much effort [5–23], a consistent power counting and calculational scheme has been developed which provides a systematic treatment of the low-momentum nucleon-nucleon strong interactions [23], up to a center-of-mass momentum for each nucleon of $p \sim 300$ MeV. Analytic expressions for $NN$ scattering phase shifts and deuteron observables [24,25] have been obtained which describe the data well. Using this technique we construct an effective field theory describing weak interactions in the two-nucleon sector, and as an example obtain an analytic expression for the deuteron anapole moment at leading order in the expansion.

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II. STRONG INTERACTION AMPLITUDES

The strong interactions of the pions are described by the Lagrange density

\[ L_0 = \frac{f^2}{8} TrD_\mu \Sigma D^\mu \Sigma^\dagger + \frac{f^2}{4} \lambda Trm_q(\Sigma + \Sigma^\dagger) + ... \]  

(2.1)

where the pion fields are incorporated in a special unitary matrix,

\[ \Sigma = \exp \frac{2i\Pi}{f}, \quad \Pi = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix}, \]  

(2.2)

with \( f = 132 \text{ MeV} \). The strong interactions of the nucleons are described by a Lagrange density that we write as

\[ L = L_0 + L_1 + L_2 + \ldots, \]  

(2.3)

where \( L_n \) contains \( n \)-body nucleon operators.

The one-body terms in the Lagrange density are

\[ L_1 = N^\dagger \left( iD_0 + \frac{D^2}{2M} \right) N + \frac{i g_A}{2} N^\dagger \sigma \cdot (\xi D \xi^\dagger - \xi^\dagger D \xi) N + ... \]  

(2.4)

where \( g_A = +1.25 \) and the ellipses denote operators with more insertions of the light quark mass matrix, meson fields, and spatial derivatives. The two-body Lagrange density contributing to S-wave interactions may be written as

\[ L_2 = - \left( C_0^{(3S_1)} + D_2^{(3S_1)} \lambda \text{Tr} m_q \right) (N^T P_{i,0} N)^\dagger (N^T P_{i,0} N) \\
+ \frac{C_0^{(3S_1)}}{8} \left[ (N^T P_{i,0} N)^\dagger \left( N^T \left[ P_{i,0} \overline{D}^2 + \overline{D}^2 P_{i,0} - 2\overline{D} P_{i,0} \overline{D} \right] N \right) + h.c. \right] \\
- \left( C_0^{(1S_0)} + D_2^{(1S_0)} \lambda \text{Tr} m_q \right) (N^T P_{0,a} N)^\dagger (N^T P_{0,a} N) \\
+ \frac{C_0^{(1S_0)}}{8} \left[ (N^T P_{0,a} N)^\dagger \left( N^T \left[ P_{0,a} \overline{D}^2 + \overline{D}^2 P_{0,a} - 2\overline{D} P_{0,a} \overline{D} \right] N \right) + h.c. \right] \\
+ ... \]  

(2.5)

where the ellipses denote operators involving more insertions of the light quark mass matrix, meson fields, and spatial derivatives. The \( P_{i,0} \) and \( P_{0,a} \) are spin-isospin projectors defined by

\[ P_{i,0} \equiv \frac{1}{\sqrt{8}} \sigma_2 \sigma_1 \tau_2, \quad \text{Tr} P_{i,0}^\dagger P_{j,0} = \frac{1}{2} \delta_{ij} \]  

\[ P_{0,a} \equiv \frac{1}{\sqrt{8}} \sigma_2 \tau_2 \tau_a, \quad \text{Tr} P_{0,a}^\dagger P_{0,b} = \frac{1}{2} \delta_{ab} \]  

(2.6)

The coefficients have been determined from fits to the S-wave phase shifts in both the \(^1S_0\) and \(^3S_1\) channels [23]. Renormalized at \( \mu = m_\pi \) in the power divergence subtraction scheme (PDS) [23], it is found that
\[ C_0^{(3S_0)} = -3.34 \text{ fm}^2, \quad D_2^{(1S_0)} = -0.42 \text{ fm}^4, \quad C_2^{(1S_0)} = 3.24 \text{ fm}^4, \]
\[ C_0^{(3S_1)} = -5.51 \text{ fm}^2, \quad D_2^{(3S_1)} = 1.32 \text{ fm}^4, \quad C_2^{(3S_1)} = 9.91 \text{ fm}^4. \]  

The relative importance of an operator to a particular observable is given by the running of the operator in the infrared. Dimensional regularization with the PDS scheme provides a simple and consistent determination of this running. Consider a general operator arising from strong interactions that connects states with angular momentum \( L \) and \( L' \). If this operator has \( m \) insertions of the light quark mass matrix \( m_q \) and \( 2d = L + L' + 2n \) spatial gradients, we will denote its coefficient by \( C_{m,n}^{(L,L')} \). It was shown in [23,24] that such a coefficient scales like \( \mu^{-(n+m+1)} \) for the \( 1S_0 \) and \( 3S_1 - 3D_1 \) partial waves, and like \( \mu^0 \) for all other partial waves. For nucleon nucleon scattering in the \( 1S_0 \) channel the momentum independent contact term with no insertions of \( m_q \) has coefficient \( C_0^{(1S_0)} = C_{0,0}^{(0,0)} \sim \mu^{-1} \), and is the leading contribution. Each loop graph in the time-ordered product of two insertions of this operator scales like \( \mu C_{0,0}^{(0,0)} \sim 1 \). Therefore the bubble chain of such insertions constitutes the leading amplitude. At subleading order two additional operators appear. There is a contribution from a single insertion of an operator with \( d = 1, m = 0 \) (coefficient \( C_2^{(1S_0)} \)), and another from a single insertion of an operator with \( d = 0, m = 1 \) (coefficient \( D_2^{(1S_0)} \)). At the same order, there are contributions from the exchange of a single potential pion, which scales like \( \mu^0 \). For the remaining discussion we will represent the small expansion parameters, the external momentum \( p \), the quark masses \( m_q \), and the renormalization scale \( \mu \), by the generic quantity \( Q \).

III. PARITY VIOLATING INTERACTIONS

The standard model of electroweak interactions gives rise to parity violating four-quark operators that exhibit enhanced symmetry in particular limits. For instance, in the limit that the angles of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the weak mixing angle vanish, and the mass of the two quarks in each generation are degenerate, the standard model Lagrange density has a global \( SU(2)_L \otimes SU(2)_R \) chiral symmetry [32]. All parity violation in this limit must be \( \Delta I = 0 \). The large difference between the mass of the charm and strange quarks breaks this symmetry. For hadronic processes the strange quarks can be considered dynamical while the charm quarks are “integrated out” of the theory. Therefore, in the limit of vanishing mixing angles in the CKM matrix and in the neutral gauge boson sector, all \( \Delta I = 1 \) parity violation comes from strange quarks. In the real world with non-zero mixing angles, the up and down quarks also contribute, but with coefficients suppressed by \( \sin^2 \theta_w \), and by the Cabibbo angle.

Excellent reviews of parity violating observables in the context of meson exchange models of weak nucleon-nucleon scattering appear in [26–30]. In this work we do not reproduce the results obtained in potential models for various parity violating observables, but instead focus on the effective field theory description of parity violation. This will facilitate the systematic inclusion of higher order effects such as relativistic corrections and dynamical meson exchange. A first step was undertaken in [31] where a Lagrange density consistent with chiral symmetry was constructed for the single nucleon sector. As with any effective field theory constructed to reproduce the low-energy behavior of some unknown or unsolved
theory, there are couplings in the theory that must be determined by experiment in order for the theory to be predictive.

The parity violating four-quark operators generated by the standard model of electroweak interactions can be classified according to how they transform under $SU(2)_L \otimes SU(2)_R$ chiral transformations [31]. By explicit construction they have decompositions $(1,1), (3,1) \oplus (1,3)$, and $(5,1) \oplus (1,5)$, and so can be unambiguously classified by their transformation under the isospin subgroup. Operators transforming as $(3,3)$ also occur but do not violate parity. The following operators [31]

\begin{align}
X^a_L &= \xi^a \tau^a \xi, \\
X^a_R &= \xi^a \tau^a \xi, \\
X^a_L - X^a_R &= -2\sqrt{2} f^a_{\alpha\beta} \pi^\alpha \tau^\beta + \ldots, \\
X^a_L + X^a_R &= 2\tau^a + \ldots,
\end{align}

(3.1)

transform as $X \rightarrow UXU^\dagger$ under chiral transformations. Rewriting the Lagrange density of [31] in terms of two-component non-relativistic nucleon fields, so that they are expressed in terms of the same field operators as the strong $NN$ Lagrange density in eqs.(2.4) and (2.5), we have\(^1\)

\begin{align}
\mathcal{L}_{\Delta I=0}^{PV} &= h^{(0)}_{\pi NN} N^a A_0 N + \ldots \\
\mathcal{L}_{\Delta I=1}^{PV} &= -h^{(1)}_{\pi NN} \frac{1}{4} f_\pi N^a (X^a_L - X^a_R) N \\
&+ h^{(1)}_{\pi NN} \frac{1}{2} N^a T_{ab} \left[ A_0 (X^a_L + X^a_R) \right] - h^{(1)}_{A} \frac{1}{2} N^a \sigma^a N Tr \left[ A_a (X^a_L - X^a_R) \right] + \ldots \\
\mathcal{L}_{\Delta I=2}^{PV} &= h^{(2)}_{\pi NN} N^a \left( X^a_R A_0 X^b_R + X^a_L A_0 X^b_L \right) N \\
&- h^{(2)}_{A} T_{ab} N^a \left( X^a_R \sigma \cdot A X^b_R - X^a_L \sigma \cdot A X^b_L \right) N + \ldots,
\end{align}

(3.2)

where the ellipses represent higher order terms in the chiral and momentum expansion. $A_\mu$ is the axial vector meson field $A_\mu = \frac{i}{2} \left( \xi^\dagger D_\mu \xi - \xi D_\mu \xi^\dagger \right)$, with $D_\mu = \partial_\mu + ieA_\mu$, and $T_{ab}$ is defined in [31] to be

\begin{equation}
T_{ab} = \frac{1}{3} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -2
\end{pmatrix}.
\end{equation}

(3.3)

In the power counting we employ, the size of the coefficients in the Lagrange density of eq. (3.2) ($h^{(1)}_{\pi NN}, h^{(0,1,2)}_{\pi NN}$, and $h^{(1,2)}_{A}$) is set by naive dimensional analysis (NDA). Were the coefficients to dramatically deviate from their NDA size the power counting must be modified.

Expanding the interactions in eq.(3.2) to $\mathcal{O}(\pi)$ gives

\begin{equation}
\mathcal{L}_{\pi NN}^{PV} = -i h^{(1)}_{\pi NN} \pi^+ p^1 n + \text{h.c.},
\end{equation}

(3.4)

where the $h_V$-type interactions involving a single $\pi$ do not contribute due to the conservation of the vector current. The strong interaction between two nucleons induced by the exchange

\(^1\)Note that the sign of $h^{(1)}_{\pi NN}$ is opposite that used in [31].

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of a single potential pion scales like $Q^0$ in the effective field theory; two factors of momentum from the derivative coupling and two powers of $q$ and $m_\pi$ in the pion propagator give

$$V(q) \sim \frac{(\sigma \cdot q)(\sigma \cdot q)}{q^2 + m_\pi^2}.$$  (3.5)

The weak interaction induced between two nucleons from the single insertion of the operator with coefficient $h^{(1)}_{\pi NN}$ scales like $Q^{-1}$, since the weak $\pi NN$ vertex is independent of momentum,

$$V(q) \sim \frac{\sigma \cdot q}{q^2 + m_\pi^2}.$$  (3.6)

The meson-nucleon vertices induced by the other operators in eq.(3.2) involve more derivatives so their contribution to the weak nucleon nucleon interaction is suppressed by additional powers of the expansion parameter $Q$. If for some reason $h^{(1)}_{\pi NN}$ is much smaller than indicated by NDA, as suggested by the $^{18}\text{F}$ experiments [1], while the others coefficients remain their NDA size, then this weak one pion exchange interaction will no longer be the leading order.

In addition to the contribution from the exchange of one or more mesons, there are contributions to the weak interaction between nucleons coming from weak four-nucleon contact operators. The leading order weak interaction (CP invariant) between nucleons in an $S$-wave in both initial and final states is described by the Lagrange density

$$\mathcal{L} = \eta^{(1)}_{S_0} \frac{f}{2} \left( N^T P_{0,a} N \right) \dagger N^T \{ P_{0,a}, X^3_L - X^3_R \} \dagger N + \text{h.c.}$$

$$= -i \eta^{(1)}_{S_0} \{ p^T \sigma_2 n + n^T \sigma_2 p \} \dagger \left( p^T \sigma_2 p \pi^- - n^T \sigma_2 n \pi^+ \right) + \text{h.c.} + \mathcal{O}(\pi^3).$$  (3.7)

Notice that this does not generate an interaction involving only two nucleons, but does give rise to an interaction between two nucleons and pions. We have defined the modified anti-commutator by

$$\{ A, B \}_T = AB + B^T A.$$  (3.8)

Coefficients of these operators are defined to have the form $\eta^{(\Delta I)}_{(2s+1) L_I J_I (2s'+1) L_{I'} J_{I'}} n^*$, where the nucleon quantum numbers are defined to be those in the absence of pions, and $n$ is the number of $\nabla^2$ and $m_\pi$ insertions. The $\beta$-function for these operators shows that that coefficient $\eta^{(1)}_{S_0}$ scales like $\mu^{-2}$. In general, operators in this channel with $n$ insertions of $\nabla^2$ and $m$ insertions of the light quark mass matrix scales like $\mu^{-2(n+m+1)}$. At next order there are contributions from operators involving more insertions of the light quark mass matrix, insertions of the axial vector meson field $A_{\mu}$ and more derivatives acting on the nucleon fields. This is similar to those found in eq.(2.5). There is no analogous interaction involving nucleons in the $^3S_1$ channel that is CP invariant.

The dominant contribution to low-energy parity violation in $pp$ scattering is from the $^1S_0 - ^3P_0$ weak amplitude interfering with the strong scattering amplitude. The four nucleon operators for such weak scattering are, for $\Delta I = 0$
\[ \mathcal{L} = \eta^{(0)}_{S_{00}P_{00}} \left( N^T P_{0,0} N \right)^\dagger N^T \left[ P_{i,a} \overrightarrow{D}^i - \overrightarrow{D}^i P_{i,a} \right] N + \text{h.c.} \quad , \]

and for \( \Delta I = 1 \)

\[ \mathcal{L} = \eta^{(1A)}_{S_{00}P_{00}} \left( N^T \{ P_{0,a}, X^3_L + X^3_R \} T N \right)^\dagger N^T \left[ P_{i,a} \overrightarrow{D}^i - \overrightarrow{D}^i P_{i,a} \right] N + \text{h.c.} + \eta^{(1B)}_{S_{00}P_{00}} \left( N^T P_{0,a} N \right)^\dagger N^T \left[ (P_{i,a}, X^3_L + X^3_R) T \overrightarrow{D}^i - \overrightarrow{D}^i (P_{i,a}, X^3_L + X^3_R) T \right] N + \text{h.c.} \quad . \]

We have not shown the \( \Delta I = 2 \) Lagrange density. The coefficients of these operators, \( \eta^{(0)}_{S_{00}P_{00}} \), \( \eta^{(1A)}_{S_{00}P_{00}} \), and \( \eta^{(1B)}_{S_{00}P_{00}} \) scale like \( \mu^{-1} \) and hence contribute to the \( pp \) scattering amplitude at order \( Q^0 \). In general, operators in this channel with \( n \) insertions of \( \nabla^2 \) and \( m \) insertions of the light quark mass matrix scales like \( \mu^{-(n+m+1)} \). At next order in the expansion terms involving more insertions of spatial gradients and the light quark mass matrix give a contribution at order \( Q^1 \), suppressed by only one power of \( Q \) in the expansion. Therefore, at next-to-leading order (NLO), there are contributions from both subleading terms in the strong interaction and from subleading terms in the weak interaction. The construction of the four nucleon operators contributing to weak processes between higher partial waves follows straightforwardly from the above discussions, and we will not detail them in this work.

**IV. THE ANAPOLE MOMENT OF THE DEUTERON**

As an example of a parity violating quantity that can be described using this effective theory, we will calculate the anapole moment of the deuteron at leading order in the expansion. This is of interest because of the possibility of measuring or constraining the coefficient \( h^{(1)}_{\pi NN} \). Attempts to determine \( h^{(1)}_{\pi NN} \) through parity violating observables in nuclei have been frustrated by the uncertainties inherent in complicated nuclear systems. Parity violating observables in the interaction between leptons and single nucleons also receive contributions from \( h^{(1)}_{\pi NN} \) but there are contributions from both isoscalar and isovector \( Z^0 \) mediated interactions as well. In contrast, the deuteron has only isoscalar interactions, dominated by the strangeness content of the deuteron. Fortunately, there are currently two experiments under consideration [33,34] which will look at the theoretically clean deuteron system. The \( \vec{n}p \rightarrow d\gamma \) process [33] has been treated in this effective field theory in ref. [35] and parity violation in electron-deuteron scattering [34] is the subject of this section.

The anapole moment of a particle is its spin dependent coupling to the electromagnetic field. For the nucleon

\[ \mathcal{L} = A_N \frac{1}{M^2_N} N^\dagger \sigma^k N \partial_{\mu} F^{\mu k} \quad . \]

The equations of motion for the electromagnetic field, \( \partial_{\mu} F^{\mu \nu} = e \overrightarrow{\gamma}^{\nu} \Psi \), allow this operator to be written entirely in terms of local four-Fermi contact terms. It is convenient to keep the operator in the form of eq. (4.1) for calculational purposes. Parity violating lepton hadron scattering that depends upon the spin of the hadronic target receives contributions from both the anapole moment of the object and from the exchange of \( Z^0 \) gauge bosons between the lepton current and the hadronic target. The anapole moment of elementary particles
FIG. 1. Leading order diagrams contributing to the deuteron anapole moment. The crossed circles denote operators that create or annihilate two nucleons with the quantum numbers of the deuteron. The solid square denotes the weak operator with coefficient $h_{\pi NN}^{(1)}$ and the solid circle denotes minimal coupling to the electromagnetic field. Wavy lines are photons, solid lines are nucleons, and dashed lines are mesons.

depends upon the choice of $R_{\xi}$ gauge [36], since its contribution cannot be separated from the contributions of $W$-box and $Z$-loop graphs, which all occur at order $g^4$ in the electroweak coupling. However, since the anapole moment of the hadrons (composite objects) is dominated by the long distance behavior of QCD, and the weak vertices at the hadronic level are independent of the choice of $R_{\xi}$ gauge, these are well-defined contributions to the spin dependent weak scattering amplitude.

The anapole moment of the deuteron is $A_D$, the coefficient in

$$L = iA_D \frac{1}{M_N^2} \epsilon_{abc} D^{\dagger a} D^b \partial_\mu F^{\mu c},$$

where $i\epsilon_{abc} D^{\dagger a} D^b$ is the deuteron spin operator. The leading contribution to $A_D$ comes from weak meson exchange starting at order $Q^{-1}$, dominated by the operator with coefficient $h_{\pi NN}^{(1)}$ (see fig. (1)). The weak four nucleon operators start contributing at order $Q^0$ and we will neglect them in this work.

Explicit computation of the graphs shown in fig. (1) leads to a deuteron anapole moment of

$$A_D = \frac{egA h_{\pi NN}^{(1)} M_N^2}{12\pi f m_\pi} \left[ 1 + \frac{m_\pi^2 + 3\gamma m_\pi + 12\gamma^2}{6(m_\pi + 2\gamma)^2} \right],$$

where $\gamma = \sqrt{M_N B}$ is the binding momentum of the deuteron with $B$ the deuteron binding energy. The first term in eq. (4.3) is from the graphs that give the dominant contribution to the nucleon anapole moment [36] (the last four graphs in fig. (1)), and the second term is the contribution from potential pion exchanges between the two nucleons in the deuteron.
(the first six graphs in fig. (1)). These two terms are the same order in the expansion, but the contribution from the individual nucleon anapole moments is numerically larger. Higher order corrections to this result that are parametrically suppressed by the expansion parameter(s) arise from higher dimension operators in both the weak sector and in the strong sector. These corrections are typically on the order of 30%. In addition to the contributions from the higher dimension weak meson operators in (3.2) and the contributions from the weak four nucleon operators there is a contribution from four nucleon anapole operators, e.g.

\[ \mathcal{L} = A_{4N} i\epsilon_{ijk}(N^TP_{i,0}N)^\dagger(N^TP_{j,0}N) \partial_\mu F^{\mu k} + \ldots, \]  

(4.4)

that contributes at next-to-next-to-leading order in the \( Q \) expansion. The coefficient \( A_{4N} \) scales like \( \mu^{-2} \) in the PDS scheme.

The deuteron spin dependent amplitude for electron deuteron scattering receives contributions from both the deuteron anapole moment and from the direct exchange of a \( Z^0 \) (and the associated radiative corrections). The spin-dependent matrix element for \( eD \) scattering, at tree-level in the standard model is

\[ \mathcal{A}^{(pv)} = 10^{-7} \left( 5.4 g_{\Delta S} - 0.17 \left( \frac{h_{\pi NN}^{(1)}}{10^{-7}} \right) \right) \frac{1}{M_N^2} e^{\nu} e^\ast a e_b, \]  

(4.5)

where \( \varepsilon^*_a \) and \( \varepsilon_b \) are the polarization vectors of the final and initial deuterons and \( g_{\Delta S} \) is the strange axial matrix element of the nucleon. Loop corrections in the standard model are fractionally larger than one would naively expect largely because of the suppression of the tree-level amplitude by a factor of \( (1 - 4 \sin^2 \theta_w) \) [37]. The estimated value of \( h_{\pi NN}^{(1)} \) is in the range \( 0 \to 11 \times 10^{-7} \) from the work of DDH [27], while naive dimensional analysis estimates give \( h_{\pi NN}^{(1)} \sim 5 \times 10^{-7} [31,32] \). Present determinations of \( g_{\Delta S} \) from lepton scattering [38,39], including SU(3) breaking effects, are \( \sim -0.15 \pm 0.15 \) (e.g. [40]). This suggests that the deuteron anapole moment contribution to \( \mathcal{A}^{(pv)} \) is comparable to the contribution from \( Z^0 \) exchange. However, for values of \( h_{\pi NN}^{(1)} \) suggested in [3] or larger, the deuteron anapole moment makes the larger contribution to \( \mathcal{A}^{(pv)} \). While it is clear that this will not provide the cleanest extraction of \( h_{\pi NN}^{(1)} \), we encourage the experimental community to continue their investigation of the parity violating spin dependent interactions of the deuteron (e.g., [34]) in order to constrain \( h_{\pi NN}^{(1)} \) and help resolve the current controversy surrounding this parameter. (Of course, if \( h_{\pi NN}^{(1)} \) turns out to be much smaller than NDA suggests, then the hadronic contribution we have computed here will not be the dominant one.)

V. CONCLUSIONS

We have shown how to systematically include hadronic parity violation from the standard model of electroweak interactions into an effective field theory description of nucleon nucleon interactions, including photons and pions. Exchange of a single potential pion with coupling \( h_{\pi NN}^{(1)} \) is the leading contribution to the weak scattering of two nucleons in the effective field theory expansion. The most general parity violating four nucleon operators categorized by partial wave and isospin change were discussed and the lowest dimension operators presented.
The leading four nucleon operator that violates parity has at least one pion in addition to nucleons. As an example we computed the leading contributions to the deuteron anapole moment, which could be used to place a constraint on \( \rho_{\pi NN}^{(1)} \).

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VI. ERRATUM AND ADDENDUM

Recent work by Khriplovich and Korkin (KK) [41] has demonstrated that a leading order (LO) contribution to the deuteron anapole moment was omitted in our paper [42]. In addition to this omission there is a factor of $-2$ error in one of the contributions. In this “Erratum and Addendum” we correct the errors, and discuss the chiral limit of this observable in more detail.

There is a contribution to the deuteron anapole moment arising from the isovector magnetic moment of the nucleon which was omitted in [42], as detailed in [41]. This interaction is described by the lagrange density

$$\mathcal{L}_{1,B} = \frac{e}{2M_N} N^\dagger (\kappa_0 + \kappa_1 \tau_3) \sigma \cdot B N \quad ,\quad (6.1)$$

where $\kappa_0 = \frac{1}{2}(\kappa_p + \kappa_n)$ and $\kappa_1 = \frac{1}{2}(\kappa_p - \kappa_n)$ are the isoscalar and isovector nucleon magnetic moments in nuclear magnetons, with

$$\kappa_p = 2.79285 \quad , \quad \kappa_n = -1.91304 \quad .\quad (6.2)$$

The magnetic field is conventionally defined $B = \nabla \times A$. The experimentally determined values of $\kappa_0$ and $\kappa_1$ include the pion loop corrections that we need to this order.

Using the same definition of the deuteron anapole moment as given in [42] of

$$\mathcal{L} = iA_D \frac{1}{M_N^2} \epsilon_{abc} D^\mu D^\nu \partial_\mu F^{\nu c} \quad ,\quad (6.3)$$

our corrected result becomes

$$A_D = -\frac{eg_A h^{(1)}_{\pi NN} M_N^2}{12\pi f} \left[ \kappa_1 \frac{m_\pi + \gamma}{(m_\pi + 2\gamma)^2} + \frac{1}{2m_\pi} - \frac{m_\pi^2 + 3m_\pi \gamma + 12\gamma^2}{6m_\pi(m_\pi + 2\gamma)^2} \right] \quad ,\quad (6.4)$$

where $\gamma = \sqrt{M_N B}$ with $B$ the deuteron binding energy. The first term in eq. (6.4) is generated by the nucleon isovector magnetic moment. The second term is the leading contribution from the nucleon anapole moment, which is purely isoscalar. The third term in eq. (6.4) arises from the pion exchange interaction between the nucleons. Combining the last two terms gives the somewhat more compact expression,

$$A_D = -\frac{eg_A h^{(1)}_{\pi NN} M_N^2}{12\pi f} \left[ \kappa_1 \frac{m_\pi + \gamma}{(m_\pi + 2\gamma)^2} + \frac{2m_\pi + 9\gamma}{6(m_\pi + 2\gamma)^2} \right] \quad .\quad (6.5)$$

Inserting the appropriate values for constants into eq. (6.5), the tree-level parity violating matrix element for electron-deuteron scattering that depends upon the deuteron spin becomes

$$\mathcal{A}^{(p\nu)} = 10^{-7} \left( 5.4 g_{\Delta S} - 0.21 \left( \frac{h^{(1)}_{\pi NN}}{10^{-7}} \right) \right) \frac{1}{M_N^2} \epsilon^{abi} \epsilon^{c} \epsilon^{\ast \ast}_{a} \epsilon_{b} \quad .\quad (6.6)$$

This expression replaces eq. (4.5) in [42]. The coefficient of $h^{(1)}_{\pi NN}$ has been changed from 0.17 to 0.21. The individual contributions to the numerical value of 0.21 that appears
in eq. (6.6) are 0.17 from the nucleon isovector magnetic moment, 0.07 from the anapole moment of the nucleon, and −0.03 from minimal coupling interactions between the nucleons. The amplitude is dominated by the interaction with the nucleon magnetic moment. It is important to note that the expression given in eq. (6.6) is valid only for momentum transfer $|k| \ll \gamma, m_\pi$. For momentum transfers larger than this, there will be an additional form factor that multiplies the coefficient of $h^{(1)}_{\pi NN}$.

In the limit of the deuteron having zero binding energy, $\gamma \to 0$, $A_D$ becomes

$$A_D = -\frac{e g_A h^{(1)}_{\pi NN} M_N^2}{12\pi f m_\pi} \left[ \kappa_1 + \frac{1}{3} \right] , \quad (6.7)$$

which agrees with the analogous limit of the expression given in [41]. The expressions given in [41] are only valid in the limit that the deuteron $\gamma \ll m_\pi$, as is stated below eq. (12) in [41]. The approximation they make is justified for the actual numerical values of $\gamma$ and $m_\pi$.

A nontrivial check of our calculation can be made by considering the chiral limit $m_\pi \ll \gamma \ll \Lambda_\chi$. Since the deuteron is an isoscalar object, with a vanishing one-pion strong coupling, there cannot be a $1/m_\pi$ divergence in this limit. Here the charged pions cannot resolve the deuteron as a bound state of two nucleons and a logarithmic dependence on $m_\pi$ is the most divergent behavior that can appear. For $m_\pi \ll \gamma$ eq. (6.5) becomes

$$A_D = -\frac{e g_A h^{(1)}_{\pi NN} M_N^2}{48\pi f \gamma} \left[ \kappa_1 + \frac{3}{2} \right] , \quad (6.8)$$

which has the expected form. The behavior shown in eq. (6.8) does not agree with the results obtained in [41], who find

$$A_D \sim \left( \kappa_1 - \frac{7}{24} \right) \frac{1}{\gamma} + \frac{1}{m_\pi} . \quad (6.9)$$

In fact, since it was assumed that $\gamma \ll m_\pi$ in deriving the results of [41], the $m_\pi \ll \gamma$ limit cannot be taken, and the dependence on $\gamma/m_\pi$ is correct only in the limit of small $\gamma/m_\pi$.

Our calculation does not include contributions beyond leading order in $1/M_N$ or $1/\Lambda_\chi$ in the expressions for either the nucleon or deuteron anapole moment. Beyond the terms proportional to $h^{(1)}_{\pi NN}$ that contribute at higher orders, as discussed in [41], there are also terms generated from additional weak interaction vertices involving pions and nucleons. A detailed discussion of such interactions and their contribution to the nucleon anapole moment is given in [31]. Therefore, the higher order correction included in [41] is only one of several contributions, and only provides an indication of the size of higher order terms.
REFERENCES


[34] The SAMPLE Collaboration, M. Pitt and E. Beise (co-spokespeople), MIT Bates proposal #94 – 11.