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N. TOOMI, J. FUJIMOTO, S. KAWABATA, Y. KURIHARA
and
T. WATANABE

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Luminosity spectrum measurement in future e^+e^- linear colliders using large-angle Bhabha events

N. Toomi, J. Fujimoto, S. Kawabata, Y. Kurihara ^a and T. Watanabe ^b

^a High Energy Accelerator Research Organization (KEK) Tsukuba, Ibaraki 305, Japan

^b Department of physics, Kogakuin University Shinjuku, Tokyo 160, Japan

In future e^+e^- linear colliders, the colliding energy cannot be monochromatic due to the large effect of beam-beam interactions before e^+e^- collisions. The luminosity must have a continuous spectrum which distributes around the nominal energy and has a long tail toward the low-energy region. An accurate measurement of the luminosity spectrum is indispensable for a precise experiment at high-luminosity e^+e^- linear colliders.

It has been found that the luminosity spectrum can be determined by measuring the acollinearity angle of Bhabha events by a tracking device with good angular resolution. The beam parameters, *i.e.* the beam-energy spread, the topological beam size and the number of particles in a beam bunch, can be also extracted from the measured luminosity spectrum by a likelihood fitting.

1. Introduction

In current e^+e^- colliding experiments, the luminosity is measured using small angle Bhabha scattering under the assumption that the colliding energy is monochromatic, except for the effect of the initial-state radiation. In future e^+e^- linear colliders, the topological beam size at the interaction point should be kept on the order of nano-meters in order to achieve sufficient luminosity to satisfy the experimental requirements. As a result of such a small beam size, electrons and positrons may lose their energies due to the beam-beam effect before they collide. Consequently, the colliding energy, $\sqrt{s}$, is no more monochromatic, but shows a continuous spectrum. Moreover, the beam-energy spread, around 1%, makes the luminosity measurement more complex than in the current e^+e^- colliders [1]. A precise determination of the luminosity spectrum (the center-of-mass energy distribution of e^+e^- collisions) is indispensable for experimental studies, especially at the particle threshold, such as in t -quark physics [2].

Miller et al. have proposed to measure the luminosity spectrum using the acollinearity angle of large-angle Bhabha events [3]. The energy difference between the colliding electron and positron results in the acollinearity angle of the Bhabha scattering. The potentiality of this idea has been

studied by means of our luminosity spectrum generator, *Luminous*, and a Bhabha-event generator produced by the GRACE system [4].

2. Event generation

Yokoya et al. have developed simulation codes, ABLE [5] and CAIN [6], to calculate a detailed beam-beam interaction, and have shown a precise distribution of the colliding energy of the e^+e^- colliders as well as those of the $e\gamma$ and $\gamma\gamma$ colliders. They also proposed an empirical function expressing the energy distribution of colliding electrons or positrons based on a detailed simulation in a simple form [7]. In the experimentally interesting energy region of more than $\sqrt{s}=450\text{GeV}$, the function can reproduce CAIN's result.

E_{beam}/GeV	250	σ_x/nm	260.0
$N_{particles}/10^{10}$	0.7	σ_y/nm	3.04
$\epsilon_x/10^{-6}\text{mrad}$	3.3	$\sigma_z/\mu\text{m}$	90
$\epsilon_y/10^{-6}\text{mrad}$	0.048	f_{rep}	150
β_x^*/mm	10.0	n_{bunch}	85
β_y^*/mm	0.1		

Table 1
Accelerator parameters for JLC.

We have developed *Luminous* [8], which gener-

ates the energy of colliding electrons or positrons efficiently using an empirical function as the probability density function. Through this work the JLC parameters listed in Table 1 are used for the beam parameters.

Besides the beam-beam interactions, the beam-energy spread also distorts the luminosity spectrum. Since the exact shape of the beam-energy spread is not known well, a flat distribution around the nominal energy ($\pm 1\%$) is assumed in the calculations. This effect is visualized in Fig.1(A).

For Bhabha-event generation, a program package for an automatic Feynman-amplitude calculation system, GRACE, is used. Numerical integration and unweighted event generation have been carried out using BASES/SPRING [9]. In this study only the lowest order diagrams of Bhabha scattering were taken into account.

3. Luminosity spectrum measurement

The momentum difference, ΔP , between the initial electron and positron can be approximately described by the acollinearity angle, θ_A , of the Bhabha event as

$$\Delta P \simeq \frac{\theta_A P_b}{\sin \bar{\theta}}, \quad (1)$$

where P_b stands for the nominal beam momentum, and $\bar{\theta}$ is an average polar angle of the final electron and positron [3]. The nominal beam momentum is assumed to be measured independently. The CM energy, $\sqrt{s_{meas.}}$, can be approximately obtained by

$$\sqrt{s_{meas.}} = \sqrt{s_0} - \Delta P, \quad (2)$$

where $\sqrt{s_0}$ is the nominal CM energy obtained from an independent energy measurement. This was the reason why Miller et al. proposed to use the acollinearity angle, since the energies of the final electron and positron could not be measured accurately enough. Since, however, the acollinearity angle is positive definite, $\sqrt{s_{meas.}}$ is always less than (or equal to) $\sqrt{s_0}$. Although the beam-energy spread may produce those events with the effective CM energy above the nominal value, as shown in Fig.1(A), the measured

CM energy distributes only below that value, as shown in Fig.1(B). This is one of the limitations of this method. When the beam has a finite-energy spread, a direct measurement of the luminosity spectrum requires other information in addition to the acollinearity angle and the nominal beam energy.

Figure 1. (A) Luminosity spectrum weighted by the Bhabha cross section without (with) the beam-energy spread, shown by the solid (dashed) histogram. (B) The same distribution calculated based on the acollinearity angle using Bhabha events without (with) any beam-energy spread, shown by the solid (dashed) histogram.

In order to clearly see the effect of the detector resolution the beam-energy spread is assumed to be zero in this section. The measured spectrum by such an ideal detector, which determines the production angle of a charged particle without error, is compared with that by a detector with angular resolutions of 3mrad and 5mrad, as shown in Fig.2. For finite resolution cases, the production angle of charged particles was smeared with a Gaussian distribution independently. The Bhabha events corresponding to $10fb^{-1}$ (about one-year's accumulation) with a

polar angle between 45° and 135° were used as a standard experimental data sample throughout this work. The result with a resolution of 1mrad is hardly distinguished from the ideal detector case. The measured spectra with a worse resolution than 3mrad show visible deviations from that of the ideal case.

Figure 2. Luminosity spectrum calculated from the measured acollinearity angle of Bhabha events by a tracker with an angular resolution of perfect (solid histogram), 3mrad (dashed histogram) and 5mrad (dotted histogram).

The luminosity spectrum around nominal energy is essentially important because almost of all events observed in the experiment come from this energy region as shown in Fig.1 and Fig.2. Consider a quantity (call the effective luminosity hereafter) defined by

$$I = \int_{495}^{\infty} \mathcal{L}(\sqrt{s}) d\sqrt{s}, \quad (3)$$

where $\mathcal{L}(\sqrt{s})$ is either the measured luminosity spectrum or the generated one. Fig.3 shows how the ratio of the measured effective luminosity, $I_{meas.}$, to the generated one, $I_{gen.}$, depends on the angular resolution of the detector.

Even with the ideal detector, however, the measured CM energy distribution can NOT perfectly reproduce the generated one. When, for example, the electron and positron lose the same amount of energy by the beam-beam effect, the shape of this event is collinear in spite of the fact that its

Figure 3. Measured cumulative luminosity above a CM energy of 495.0 GeV $I_{meas.}$ normalized by the generated one $I_{gen.}$ v.s. the angular resolution. Results with two different minimum acceptance-angles are shown.

CM energy is not equal to the nominal one. This is another limit of this method. It namely works well only when the probability for initial particles to lose a sizable amount of energy is negligibly small. A deviation of 2.5% with the ideal detector shown in Fig.3 is due to this effect.

For the small-angle region the resolution of ΔP in equation(1) is deteriorated by the projection factor on the beam axis ($1/\sin\theta$). When only those events, detected around the central part of the detector, are used, they give a good ΔP resolution. On the other hand, the statistics of the Bhabha events decreases very rapidly along with an increasing minimum polar angle, θ_{min} , of the detector acceptance ($\propto \sin^{-3}(\theta_{min})$). If θ_{min} of the Bhabha events is set equal to 20° , instead of 45° , the statistics increases by about one order of magnitude. Even if the angular resolution can be kept constant down to 20° , and if it is larger than 1mrad, the measured spectrum is distorted according to the decrease in the minimum acceptance-angle, as shown in Fig.3. When the resolution can be kept at 1mrad, the acceptance can be set lower to 20° .

4. Beam parameter fitting

The beam-energy distribution just before collisions is described by the beam parameters, such as: the horizontal, vertical and longitudinal beam

sizes $(\sigma_x, \sigma_y, \sigma_z)$, and the number of particles in a beam bunch, N . In the empirical function, there are two independent variables, $N/(\sigma_x + \sigma_y)$ and σ_z . Including the beam-energy spread, ΔE , we have three independent parameters[†] for each beam. Among three combinations of them, simultaneous determinations of $N/(\sigma_x + \sigma_y)$ and σ_z and that of $N/(\sigma_x + \sigma_y)$ and ΔE were studied, while another remaining parameter was assumed to be known for each case. For this study, a *standard* experimental data sample (called 'e-sample' hereafter) with an angular resolution of 1 mrad was prepared with the beam parameters given in Table 1 and a beam-energy spread, ΔE , of 1%.

The effective CM energy, $\sqrt{s}$, of each event is calculated from the measured acollinearity angle of the final electron and positron by equations (1) and (2), of which the distribution corresponds to the measured luminosity spectrum. Strictly speaking, this spectrum is not of the luminosity itself, but is weighted by the cross section of Bhabha scattering. Although the luminosity spectrum should be calculated by taking the Bhabha cross section into account, it is not necessary in our method. Thus, this spectrum is called the luminosity spectrum in this paper for simplicity.

The estimates of two unknown parameters, ξ_1 and ξ_2 , are searched for by the simplest method (grid search), where each parameter varies between $\pm 3\%$ of its nominal value. At each point in the two-parameter space a simulating the data sample (called 'l-sample' hereafter) is generated with 10-times more statistics than the e-sample. For each l-sample, after smearing the production angles of the electron and positron, a histogram of the effective CM energy, $\sqrt{s}$, for all events is made, where the energy region between 375 and 500 GeV is divided into 50 bins. Only for this energy region is our likelihood fitting method applied, since it covers more than 85% of the whole

region and the empirical function is considered to be a good approximation in this region. The obtained histogram is further normalized to unity. This resultant histogram is called the probability histogram, and is used as a probability-density function, $\mathcal{P}$, which can be written as

$$\mathcal{P}_i(\xi_1, \xi_2) \equiv N_i/N, \quad (4)$$

where i indicates the bin number, N_i is the number of l-sample events in the i -th bin, and N is the total number of events with $\sqrt{s} \geq 375$ GeV in the l-sample. For each l-sample the following log-likelihood is calculated:

$$\ln \mathcal{L}(\xi_1, \xi_2) \equiv \sum_{j=1}^n \ln \mathcal{P}^j, \quad (5)$$

where n is the total number of e-sample events and $\mathcal{P}^j$ is the value of the probability density function of the j -th event. If the effective CM energy, $\sqrt{s}$, of the j -th event falls in the k -th bin of the probability histogram, the value of the probability density function, $\mathcal{P}^j$, is given by

$$\mathcal{P}^j = \mathcal{P}_k(\xi_1, \xi_2). \quad (6)$$

The estimates of two parameters are given by those values where the log-likelihood has its maximum. The 68.3% joint likelihood region in two-parameter space, spanned by ξ_1 and ξ_2 , is inside the contour defined by the equation

$$\ln \mathcal{L}(\xi_1, \xi_2) = \ln \mathcal{L}(\max) - 1.15, \quad (7)$$

where $\ln \mathcal{L}(\max)$ is the maximum value of log-likelihood. This region has comparable meaning to the 1σ interval in the one-parameter case.

Fig.4(A) shows the log-likelihood distribution in the two-parameter space of $N/(\sigma_x + \sigma_y)$ and σ_z , where the innermost contour represents the boundary of the 68.3 % joint likelihood region and for each step of $\Delta \ln \mathcal{L} = 1.15$ a contour is drawn. The parameter $N/(\sigma_x + \sigma_y)$ is quite sensitive to the shape of the luminosity spectrum, but σ_z is not, and no correlation between them is found. Since there are two maxima in the distribution due to small statistics of the e-sample, it is not an appropriate situation to say anything about the estimate and error. For the e-sample of 10-times

[†]Since the electron and positron beams have mutually independent values of the beam parameters, six parameters are to be estimated from the experimental data. As the first step of a feasibility study of the luminosity measurement, it is assumed throughout this paper for simplicity that both the electron and positron beams have completely identical beam parameters.

more statistics than the standard, we can see only one maximum in the distribution, as shown in Fig.4(B).

The determination of σ_z by this method requires at least a few years' data accumulation, and is not practicable. This is due to a weak sensitivity of σ_z to the shape of the luminosity spectrum. In other words, an inaccurate knowledge about σ_z does not affect the determination of the other parameters. Furthermore, since the bunch length, σ_z , is a parameter of the bunch compressor, and is known with around ten percent accuracy, we can take this value as a known parameter, σ_z .

Figure 4. Log-likelihood distributions for the parameters of $N/(\sigma_x + \sigma_y)$ and σ_z . Each parameter is varied between $\pm 4\%$ of the nominal value. The distribution with 10-times more statistics of the e-sample than usual (B). The innermost contour in these distributions shows a 1σ deviation.

The likelihood distribution as a function of $N/(\sigma_x + \sigma_y)$ and ΔE is shown in Fig.5(A), where the innermost contour shows the boundary of the 68.3% joint likelihood region. The estimates of $N/(\sigma_x + \sigma_y)$ and ΔE are slightly different from the nominal ones. This uncertainty of the central value is due to the small statistics of the e-sample. In order to confirm the validity of this method, the same fitting procedure was performed for the e-sample with 100-times more statistics than that of the standard. In this case, the estimated beam parameters are much closer to the given values, as shown Fig.5(B).

Figure 5. Log-likelihood distributions for the parameters of $N/(\sigma_x + \sigma_y)$ and the beam-energy spread for an e-sample. Each parameter is varied between $\pm 3\%$ of the nominal value. The innermost contour in these distributions shows the 1σ deviation (A). The distributions show 100-times more statistics of the e-sample than usual (B).

The fitting accuracy of ΔE as a function of the angular resolution of the detector is shown in Fig.6, where the central value is estimated by fitting the likelihood distribution with a Gaussian form when the parameter $N/(\sigma_x + \sigma_y)$ is assumed to be its given value.

Figure 6. Fitting accuracy of the beam-energy spread as a function of the angular resolution of the detector. The mean value is calculated by fitting the likelihood distribution to a Gaussian form when the parameter $N/(\sigma_x + \sigma_y)$ is nominal.

5. Luminosity measurement

For measurements of the absolute values of various cross sections in a real experiment we need to know the precise value of the integrated luminosity as a function of $\sqrt{s}$. For this purpose, we tried to calculate the integrated luminosity assuming the empirical function. Fig.7 shows the luminosity spectrum calculated by using the beam parameters determined by the likelihood fitting. The integrated luminosity is obtained by the formula

$$\mathcal{L} = N_{events} \sum_{i=1} \mathcal{P}_i(B^{meas.}, \Delta E^{meas.})/\sigma_i, \quad (8)$$

where N_{events} is the observed number of events and $\mathcal{P}_i$ is a probability density function defined in equation(4). $B^{meas.}$ is an estimate of the nominal beam parameter, $N/(\sigma_x + \sigma_y)$, and δ_B is its fitting error by the likelihood fitting. $\Delta E^{meas.}$ is an estimate of the beam-energy spread and $\delta_{\Delta E}$ is its fitting error. σ_i is the cross section of Bhabha events for i -th bin. The luminosity is calculated as a function of $\sqrt{s}$, and the integrated luminosity is estimated by integrating the luminosity over $\sqrt{s}$. With the given values of the beam parameters, the integrated luminosity of $10fb^{-1}$ for the standard data sample is calculated to be $9.990 \pm 0.070fb^{-1}$ when using the beam-parameter values obtained by the likelihood fitting. The error is estimated by

$$\sigma_{\mathcal{L}} = \left\{ \sum_{i=1} |\mathcal{P}_i(B^{meas.}) - \mathcal{P}_i(B^{meas.} \pm \delta_B)|^2 + |\mathcal{P}_i(\Delta E^{meas.}) - \mathcal{P}_i(\Delta E^{meas.} \pm \delta_{\Delta E})|^2 \right\}^{1/2}. \quad (9)$$

The calculated value of the integrated luminosity shows good agreement with the assumed value.

6. Conclusion and Discussion

The potentiality of measuring the luminosity spectrum using the acollinearity angle of the large-angle Bhabha events was studied by means of the luminosity spectrum generator *Luminous* and a Bhabha-event generator produced by the GRACE system. For the energy distribution of the e^+/e^- beam before collisions, an empirical function is assumed, which has two independent

Figure 7. Luminosity spectrum calculated by the empirical function using the given value (solid histogram) and the parameters determined by a likelihood fitting (closed circles).

parameters, $B \equiv N/(\sigma_x + \sigma_y)$ and σ_z , besides the nominal beam energy. The nominal beam energy is assumed to be known. For the beam-energy spread, ΔE , a flat distribution around the nominal energy ($\pm 1\%$) is assumed in the calculations.

Among three combinations of three beam parameters, the simultaneous determination of parameters $N/(\sigma_x + \sigma_y)$ and σ_z , and that of $N/(\sigma_x + \sigma_y)$ and the beam-energy spread ΔE were studied. The experimental data sample of an integrated luminosity of $10fb^{-1}$ with a minimum acceptance angle of 45° is used as a standard data sample. As a result of the former combination, the parameter $N/(\sigma_x + \sigma_y)$ is quite sensitive to the shape of the luminosity spectrum, but σ_z is not, and no correlation between them has been found. It was found that the determination of σ_z by this method requires at least a few years' data accumulation and is not practicable. This is due to a weak sensitivity of σ_z to the shape of the luminosity spectrum. Since an inaccurate knowledge about the parameter, σ_z , does not affect the determination of other parameters, we can take the σ_z value expected from the bunch compressor as being a known parameter. For the latter combination, $N/(\sigma_x + \sigma_y)$ and ΔE can be determined with accuracies of 0.5 and 1.0 percent, respectively. When the minimum acceptance angle, θ_{min} , is lowered to 20° by keeping an intrinsic angular resolution of 1mrad, comparable accuracies

of the beam parameters to those for the standard data sample may be achieved, even with the experimental data of $1fb^{-1}$ integrated luminosity.

In this study the empirical function proposed by Yokoya et.al was assumed. The real energy distribution of initial particles may not always be described exactly by this function, but may be a good approximation. A great advantage of our method is that since the function is expressed by the basic beam parameters in a relatively simple form, the fitting result of the parameters does not only allow us to estimate the luminosity, but is also easily compared them with the actual beam parameters. There may be such an alternative function with several parameters that reproduce the measured energy distribution quite well as a result. If the parameters of this function could have some relation with the beam parameters, it may be a very acceptable one. However, if not, there is no way to confirm whether the resultant parameters are acceptable, except that they reproduce the measured distribution quite well.

In this work the initial-state radiation was not studied. Since the effect of the initial-state radiation on Bhabha scattering has been studied very precisely, it may be possible to subtract this effect from the observed distribution. For a precision experiment, however, the luminosity spectrum measurement including the effect of the initial-state radiation should be studied in detail.

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