No-Lose Theorem for Higgs Boson Searches at a Future Linear Collider

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Assuming perturbativity up to a high energy scale $\sim 10^{16}$ GeV, we demonstrate that a future e^+e^- linear collider operating at $\sqrt{s} = 500$ GeV with $\int \mathcal{L} = 500 \text{ fb}^{-1}$ per year (such as the recently proposed TESLA facility) will detect a Higgs boson signal regardless of the complexity of the Higgs sector and of how the Higgs bosons decay. [S0031-9007(99)08409-4]

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In this paper, we consider a model with spread out, but unresolved Higgs boson [1] resonances and show that, no matter how they decay, an e^+e^- collider of sufficient luminosity (L) is guaranteed to find a signal. The L needed is that to detect a broad enhancement in the M_X spectrum in $e^+e^- \rightarrow ZX \rightarrow e^+e^-X + \mu^+\mu^-X$ over the M_X region allowed if the model remains perturbative up to energy scales of the order of 10^{16} GeV.

For simplicity, we will adopt a notation that assumes CP conservation in the Higgs sector; generalization is straightforward. A general neutral Higgs field Φ_α in an arbitrary (t_α, Y_α) representation is written in terms of its real (CP -even) and imaginary (CP -odd) components as $\Phi_\alpha^0 = \frac{1}{\sqrt{2}}(h_\alpha^{0r} + v_\alpha + ih_\alpha^{0i})$. (v_α is the vacuum expectation value of the field as a result of electroweak symmetry breaking.)

The crucial ingredient for the no-lose theorem is that there must be Higgs bosons with mass below and in the vicinity of a perturbatively limited mass scale m_B (of the order of the Fermi scale) which, in aggregate, have net coupling squared to ZZ that is at least as large as the standard model (SM) value. The proof begins with two crucial inequalities. Defining

$$\phi_Z^0 = \frac{1}{N_Z} \sum_\alpha t_{3\alpha}^2 h_\alpha^{0r} v_\alpha, \quad (1)$$

where N_Z is a normalization constant, the first inequality is [2,3]

$$\langle \phi_Z^0 | M^2 | \phi_Z^0 \rangle \leq \lambda v^2 \equiv m_B^2, \quad (2)$$

where M^2 is the mass-square operator for the neutral Higgs boson, λ is some adimensional quartic Higgs coupling, and $v^2 = 1/(\sqrt{2} G_F) \sim (246 \text{ GeV})^2$. The coupling λ is not asymptotically free and can be bounded above by triviality arguments assuming it remains perturbative up to a high-energy scale Λ . The model-dependence of m_B will be discussed below. The second inequality is

$$g_{ZZ\phi_Z^0}^2 \geq g_{ZZh_{SM}}^2 = g_Z^2 m_Z^2 / \cos^2 \theta_W. \quad (3)$$

(The equality holds only for $t_{3\alpha} = 0, \pm 1/2$.) We now define $K_i = |\langle \phi_Z^0 | h_i^0 \rangle|^2$, where $\sum_i K_i = 1$, and insert

complete sets of h_i^0 states in (2) to obtain

$$\langle \phi_Z^0 | M^2 | \phi_Z^0 \rangle = \sum_i K_i m_{h_i^0}^2 \leq m_B^2, \quad (4)$$

Since ϕ_Z^0 defines the combination of states that couples to ZZ , we have $g_{ZZh_i^0}^2 = K_i g_{ZZ\phi_Z^0}^2$. Similar results apply in the WW sector.

In the SM with $\Lambda \sim M_{Pl}$, $m_B \sim 190$ GeV [4], and modest luminosity at $\sqrt{s} = 500$ GeV would be sufficient to detect the unique Higgs boson [5]. In the minimal supersymmetric standard model (MSSM), there are two CP -even Higgs bosons, and, for $\Lambda_{SUSY} \leq 1$ TeV and $\Lambda = \Lambda_{GUT} \sim 10^{16}$ GeV, $m_B \leq 125$ GeV [6]. Equation (4) implies that, when $K_1 \rightarrow 0$, $m_{h_2} \leq m_B$ and $K_2 \rightarrow 1$. For $\sqrt{s}$ well above m_B , this is sufficient to guarantee Higgs detection in the $h_{1,2}Z$ channels alone [5]. (At LEP2, the complementary channel h_1A must be used to cover the $K_1 \rightarrow 0$ case although large radiative corrections render that channel inoperative in some small region of parameter space [7].) In the NMSSM (the MSSM extended by a singlet chiral multiplet), $m_B \sim 140$ GeV [8] for the same Λ_{GUT} , and there are three CP -even Higgs bosons. Using the fact that $\sigma_{Zh}(m_h)$ is a decreasing function of m_h (for constant g_{ZZh}^2) and using our previous results, the minimum cross section for any one of the Higgs bosons,

$$\sigma_{\min} \equiv \min[\{\max\}_i \sigma_{Zh}(h_i^0)], \quad (5)$$

[where $\sigma_{Zh}(h_i^0) = K_i \sigma_{SM}(m_{h_i^0})$] must be bigger than [8,9] $\sigma_{SM}(m_B)/3$, as achieved for $m_{h_i^0} = m_B$ and $K_i = 1/3$ for $i = 1, 2, 3$. For $\sqrt{s} = 300$ GeV (and $m_B = 140$ GeV), $\sigma_{\min} \sim 61 \text{ fb}$. (This improves over Ref. [9] by a different use of the mass inequalities.) For this $\sigma_{\min}$, an integrated luminosity of $L = 10 \text{ fb}^{-1}$ would be more than adequate to detect at least one of the three CP -even scalars. In contrast, there are substantial regions of NMSSM parameter space for which none of the Higgs bosons will be discovered at the LHC [10].

In general supersymmetric (SUSY) models with arbitrary particle content, m_B can be as high as 200 GeV for $\Lambda \sim 10^{17}$ GeV [11]. As fundamental bosons at the electroweak scale are natural only in SUSY, we will take

$m_B \lesssim 200$ GeV for the general case (our results are, however, independent of whether the model is supersymmetric or not). The success of fits to precision electroweak data using one SM-like Higgs boson with mass $\lesssim 200$ GeV also suggests that, in a multi-Higgs model, the Higgs bosons with large ZZ coupling must have average mass $\lesssim 200$ GeV.

The minimal cross section (5) gets smaller if more singlets are added to the model [9] (a large number of singlets is predicted in some string models) or if there is CP violation with mixing of scalar and pseudoscalar Higgses. Roughly, $\sigma_{\min} \sim \sigma_{\text{SM}}(m_B)/N$ where N counts the number of neutral mass eigenstates that can mix with ϕ_Z^0 . However, since $\sigma_{\min}$ is achieved when all N Higgs bosons have mass $\sim m_B$, one can simply add their signals together (for small widths, interference effects can be neglected) inside a mass bin of size determined by the experimental resolution, Δm (defined as full width at half maximum) yielding an easily detected SM-strength signal. The actual worst-case situation in the limit of large N arises when all of the Higgs bosons that can couple to ϕ_Z^0 have masses spread out below and somewhat above m_B , but are spaced more closely than Δm , and, assuming large $\sqrt{s}$, have coupling K_i such that $K_i \sigma_{\text{SM}}(m_{h_i})$ closely mimics the background of relevance. Then, the only observable is the excess of overall rate as compared to background over some mass interval. The resonance peaks of the Higgs bosons will not be observable.

To analyze this type of situation more quantitatively, we note that for spacing smaller than Δm the number of Higgs bosons is irrelevant. Thus, we convert to an integral notation. Our crucial sum rules become

$$\int dm_h K(m_h) = 1, \quad (6)$$

$$\int dm_h K(m_h) m_h^2 \leq m_B^2. \quad (7)$$

As a point of reference, we first consider the case of constant $K(m_h) = K$, for which (6) implies $K = 1/(m_h^{\max} - m_h^{\min})$, and (7) implies

$$m_B^2 \geq \frac{1}{3} ([m_h^{\max}]^2 + m_h^{\max} m_h^{\min} + [m_h^{\min}]^2). \quad (8)$$

Maximal spread (which makes detecting a signal most difficult) occurs for $m_h^{\min} = 0$, implying [Eq. (8)] $m_h^{\max} \leq \sqrt{3} m_B$. Full sensitivity to $0 \leq m_h \leq \sqrt{3} m_B$ requires $\sqrt{s}$ large enough that $\sigma_{\text{SM}}(m_h)$ is large for all $m_h \leq \sqrt{3} m_B$. For $m_B = 200$ GeV, $\sqrt{3} m_B \sim 340$ GeV and quite large $\sqrt{s}$ would be required to achieve this ideal. However, $\sqrt{s} \sim 500$ GeV would not be too bad. For this $\sqrt{s}$, $\sigma_{\text{SM}}(m_h = 0) \sim 70$ fb and $\sigma_{\text{SM}}(m_h = 340 \text{ GeV}) \sim 15$ fb. At $m_h \sim 200$ GeV, $\sigma_{\text{SM}}(m_h)$ is still ~ 42 fb. We will confine ourselves to the ≤ 200 GeV window in m_h in discussing experimental sensitivity.

Another important issue is the channel(s) in which we must detect the Higgs bosons. In the worst case, the Higgs bosons will decay to a large number of channels (possibly

including invisible channels such as two LSP's in a supersymmetric model). In this case, identification of the final state would not be useful because of the small signal-to-background ratio in any one channel. The only robust procedure is to look for $e^+e^- \rightarrow Zh$ and, possibly, $e^+e^- \rightarrow e^+e^-h$ (ZZ fusion) production in the $e^+e^- \rightarrow ZX$ (with $Z \rightarrow e^+e^-$ and $\mu^+\mu^-$) and $e^+e^- \rightarrow e^+e^-X$ channels, respectively—we employ cuts that separate $e^+e^- \rightarrow Zh$, $Z \rightarrow e^+e^-$ from $e^+e^- \rightarrow e^+e^-h$ via ZZ fusion—as an excess in the recoil M_X distribution due to the continuum of Higgs bosons. Our focus will be on the Zh channel.

We find it most convenient to use the analysis of Ref. [12]. Although the cuts employed there restrict the analysis to the region $m_h = 70\text{--}200$ (all masses in GeV units), since $m_B \leq 200$ this will be adequate to establish a no-lose theorem. We also consider the $m_h = 100\text{--}200$ interval that avoids the ZZ background. Roughly, a fraction $130/340 \sim 0.4$ ($100/340 \sim 0.3$) of the signal weight would be in the $70\text{--}200$ ($100\text{--}200$) regions for $m_h^{\min} = 0$ and m_h -independent K . Some $1/5$ of the continuum Higgs spectrum would lie in the $m_h \lesssim 70$ region to which LEP2 has sensitivity. More weight in this region can probably already be eliminated using current data. If the LEP limits prove even stronger, then we should increase $m_h^{\min}$ and our conclusions for a future e^+e^- machine would be improved. For example, for $m_h^{\min} = 70$ (i.e., very small K implied by LEP2 data below this), then $m_h^{\max} \sim 300$ is required for $m_B = 200$, and a fraction $130/230 \sim 0.55$ ($100/230 \sim 0.43$) of the uniform Higgs continuum signal would be in the $70\text{--}200$ ($100\text{--}200$) windows.

The results are best summarized in two tables. In Table I, we give the approximate signal and background (from Ref. [12]) rates for $L = 500 \text{ fb}^{-1}$ for the Zh and ZZ -fusion contributions to the $e^+e^- \rightarrow e^+e^-h$ channel, and the corresponding $S/\sqrt{B}$ values obtained by integrating over two different mass intervals: $70\text{--}200$, which includes the Z peak in the background; and $100\text{--}200$, which starts above the Z peak. The Table I numbers for the $70\text{--}200$ and $100\text{--}200$ intervals assume that the “continuum” of Higgs bosons in the mass interval have coupling-squared to ZZ whose sum is equal to that of the SM Higgs boson. The Zh with $Z \rightarrow \mu^+\mu^-$ channel would give S and B values that are essentially the same as for Zh with

TABLE I. Approximate S , B , and $S/\sqrt{B}$ values for the Zh and ZZ -fusion contributions to $e^+e^- \rightarrow e^+e^-h$, after integrating the M_X recoil mass spectrum from (a) 70 to 200 GeV and (b) 100 to 200 GeV, assuming that many Higgs bosons are distributed evenly throughout the interval and have $\sum_h g_{ZZh}^2 = g_{ZZh_{\text{SM}}}^2$. Results are for $\sqrt{s} = 500$ GeV, $L = 500 \text{ fb}^{-1}$, and assume the cuts and efficiencies of Ref. [12].

M_X interval	Zh			ZZ -fusion		
	S	B	$S/\sqrt{B}$	S	B	$S/\sqrt{B}$
70–200	675	3170	12.0	2615	80 100	9.2
100–200	678	1350	18.5	2300	39 600	11.5

$Z \rightarrow e^+e^-$. Summing the e^+e^- and $\mu^+\mu^- Zh$ channels yields $S \sim 1350$, and $S/\sqrt{B} \sim 17$ and ~ 26 for the 70–200 and 100–200 windows in M_X , respectively. Of course, both S and $S/\sqrt{B}$ must be reduced by the fraction f of the Higgs signal contained in the mass interval. Above, we argued that, for constant $K(m_h)$, $f \sim 0.4$ – 0.55 (0.3 – 0.43) is an appropriate range to consider for the 70–200 (100–200) window. Then, for $L = 500 \text{ fb}^{-1}$, we would have a net signal of $S \sim 1350f$ with a background of either $B = 6350$ or $B = 2700$, depending on which of the two mass intervals are considered. For the 70–200 (100–200) interval, we have to detect the presence of a broad $\sim 21\%$ f ($\sim 50\%$ f) excess over background, respectively. Understanding the absolute normalization of the backgrounds to the 20% f level of accuracy is probably feasible at an e^+e^- collider for $f > 0.4$; a 50% f excess would certainly be observable for $f > 0.3$.

We have included the $e^+e^- \rightarrow e^+e^-h$ ZZ-fusion channel in Table I to indicate how difficult its use would be in this situation. Even though ZZ fusion yields a large nominal $S/\sqrt{B}$, the S/B values are $0.032f$ and $0.058f$ for the 70–200 and 100–200 mass intervals, respectively. A 3.2% f broad excess would be impossible to detect. Even a $\sim 6\%$ ($f = 1$) excess over the latter interval would be difficult to be certain of.

If we decrease L to 50 fb^{-1} , we lose a factor of $\sqrt{10}$ in our $S/\sqrt{B}$ values, leaving effects at the $S/\sqrt{B} = 5.3f$ ($8.2f$) level in the Zh (with $Z \rightarrow e^+e^- + \mu^+\mu^-$) channel for the 70–200 (100–200) intervals, respectively, which is borderline for $f \sim 0.4$ – 0.55 ($f \sim 0.3$ – 0.43).

Let us now consider how the above results change if $K(m_h)$ is not constant. Because 70–200 includes the region $m_h \sim m_Z$ of largest background, and because $m_B \leq 200$, essentially the only way to reduce the 70–200 signal is to put extra weight in the $m_h \leq 70$ region (which would at the same time allow extra weight above 200). However, we believe that analysis of the LEP/LEP2 results will show that the resulting signal would have already been observed. Even if not, the background in the ≤ 70 GeV region at a $\sqrt{s} = 500$ GeV machine would be small enough that the resulting signal excess would be detectable.

If such a broad continuumlike signal is observed, the next important step will be to divide the M_X spectrum up

into bins, knowing that in each bin there is presumably an excess of events due to some set of Higgs bosons with some portion of the net SM ZZ coupling. Just to illustrate what the situation might be, suppose we distribute the roughly 1350 SM signal events (for Zh with $Z \rightarrow e^+e^-$ and $\mu^+\mu^-$) equally in the thirteen 10 GeV bins from 70 to 200 GeV. The level of signal and background, and corresponding $S/\sqrt{B}$ value, in each bin is given in Table II. Note that both S and $S/\sqrt{B}$ would more generally be reduced by the fraction f of the SM signal present in the 70–200 mass window.

Table II shows that $L = 500 \text{ fb}^{-1}$ would yield $S/\sqrt{B} > 3$ only for the $M_X \geq 120$ GeV bins when $f \sim 0.5$. With only $L = 100 \text{ fb}^{-1}$ (as might be achieved after a few years of running at a lower luminosity design), this bin-by-bin type of analysis would not be possible for 10 GeV bins if $f \sim 0.5$.

It is useful to determine the number of Higgs bosons that would necessitate considering the continuum limit. If there were 13 spread out over the 70–200 GeV interval, i.e., one every 10 GeV, the separate mass peaks would be impossible to resolve if $\Delta m \geq 10$ GeV. In fact, this is nearly twice as good as the typical M_X resolution found in Ref. [12] using “super-JLC (Japanese Linear Collider)” tracking [which, in turn, is substantially better than currently planned for first-round NLC (Next Linear Collider) detectors]. Thus, for current detector designs it would appear that a model with ≥ 6 Higgs bosons that can couple to ZZ would be in danger of leading to a continuum Higgs signal.

In summary, we have considered a “worst-case” Higgs scenario with a large number of Higgs bosons which cannot be separated from one another given expected detector resolutions, and which all decay to many final states. In this case, the only viable signal is a continuum excess in the $e^+e^- \rightarrow ZX$ (with $Z \rightarrow e^+e^-$ and $\mu^+\mu^-$) recoil M_X spectrum. We have shown that the assumption of perturbativity up to high scales and the bounds/inequalities derived therefrom, guarantee the observability of this signal provided that an integrated luminosity in excess of $L = 100 \text{ fb}^{-1}$ is accumulated. Bin-by-bin analysis of such a continuum excess would very possibly require $L \geq 500 \text{ fb}^{-1}$, implying that this situation provides a rather strong motivation for going to a very high L collider design.

TABLE II. Approximate S , B , and $S/\sqrt{B}$ values for Zh (with $Z \rightarrow e^+e^- + \mu^+\mu^-$) in each of the thirteen 10 GeV bins in M_X from 70 to 200 GeV, assuming that $S \sim 1350$ events, from Higgs bosons with $\sum_h g_{ZZh}^2 = g_{ZZh_{\text{SM}}}^2$, are distributed equally among these bins. We assume $\sqrt{s} = 500$ GeV, $L = 500 \text{ fb}^{-1}$, and use the results of Ref. [12].

Bin No.	1	2	3	4	5	6	7	8–13
S	104	104	104	104	104	104	104	104
B	1020	1560	1440	734	296	162	125	~ 130
$S/\sqrt{B}$	3.3	2.6	2.7	3.8	6.0	8.2	9.3	9.1

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- [1] J.F. Gunion, H.E. Haber, G.L. Kane, and S. Dawson, *The Higgs Hunters Guide* (Addison-Wesley Publishing, Redwood City, CA, 1990).
 - [2] P. Langacker and H. A. Weldon, Phys. Rev. Lett. **52**, 1377 (1984); H. A. Weldon, Phys. Lett. **146B**, 59 (1984).
 - [3] D. Comelli and J.R. Espinosa, Phys. Lett. B **388**, 793 (1996).
 - [4] T. Hambye and K. Riesselmann, Phys. Rev. D **55**, 7255 (1997).
 - [5] ECFA/DESY LC Physics Working Group, E. Accomando *et al.*, Phys. Rep. **299**, 1 (1998).
 - [6] See, e.g., J.A. Casas, J.R. Espinosa, M. Quirós, and A. Riotto, Nucl. Phys. **B436**, 3 (1995).
 - [7] R. Hempfling, Phys. Lett. B **296**, 121 (1992); J. Rosiek and A. Sopczak, Phys. Lett. B **341**, 419 (1995); OPAL Collaboration, K. Ackerstaff *et al.*, Eur. Phys. J. **C5**, 19 (1998).
 - [8] U. Ellwanger, M. Rausch de Traubenberg, and C.A. Savoy, Z. Phys. C **67**, 665 (1995); S. King and P. White, Phys. Rev. D **53**, 4049 (1996).
 - [9] J. Kamoshita, Y. Okada, and M. Tanaka, Phys. Lett. B **328**, 67 (1994).
 - [10] J.F. Gunion, H.E. Haber, and T. Moroi, in *Proceedings of the 1996 DPF/DPB Summer Study on High-Energy Physics, Snowmass, Colorado, 1996* (Report No. ph/96103337), p. 598.
 - [11] J.R. Espinosa and M. Quirós, Phys. Rev. Lett. **81**, 516 (1998).
 - [12] J.F. Gunion, T. Han, and R. Sobey, Phys. Lett. B **429**, 79 (1998).