EXCLUDED VOLUME IN MEAN FIELD FORMALISM
AND KAON CONDENSATION

V.K.Tiwari, Nirvikar Prasad and C.P.Singh

Department of Physics, Banaras Hindu University,
Varanasi 221 005, India

ABSTRACT

Sw09828

A mean-field formalism with excluded volume correction arising due to a hard-core repulsion between a pair of baryons has been extended for a multicomponent hadron gas involving nonstrange as well as strange hadrons. We incorporate the effects of chiral attraction as well as repulsion between the kaons. The resulting equation of state (EOS) is used to study the problem of kaon condensation as well as the medium modification of the properties of kaons. We find that the kaon condensation is model dependent and depends upon the hard-core radius of baryons in our excluded volume model and it can occur in the dense hadron gas when the radius is ≤ 0.6 fm.

1. INTRODUCTION:

Probing the nuclear matter equation of state (EOS) [1-3] in regions of high densities and temperatures ($n/n_0 \approx 2-6$, $T = 50-200$ MeV) is of fundamental importance not only in nuclear physics and field theory but is also a basic prerequisite for our understanding of many astrophysical problems. Here n_0 is the normal nuclear matter density $\approx 0.16 \text{ fm}^{-3}$. For the hadronic EOS, reliable empirical information is available only at low temperature near nuclear saturation density n_0 . The behaviour of the equation of state at high energy densities depends strongly on the hadrons actually included in the model and on their interactions. For example, pions, kaons and hyperons are not relevant at n_0 , but they produce a large qualitative and quantitative changes in the behaviour at high temperatures and densities. An EOS incorporating all the hadrons and their interactions will be a more realistic one which can be used to describe the hot and dense hadron gas produced in the ultra-relativistic heavy-ion collisions [4-6] or in the early universe or in the core of neutron stars [7-10].

It is well known that the interactions of kaons and antikaons with baryons are enormously different. The interaction of an antikaon with a nucleon is always attractive and yields a positive phase shift in contrast to a negative phase shift reported

for repulsive K^+N interactions .Another difference between a kaon and antikaon is that the latter can be absorbed by a nucleon to produce a hyperon and pion . Todate there is a debate regarding the KN interactions in a dense nuclear matter mainly in relation to the K^- -condensation .

The study of the medium modifications of the properties of kaons in highly compressed nuclear matter has generated a lot of current research interest [1,11-14].It started after the suggestion of Kaplan and Nelson [11-12] that kaon effective masses are surprisingly density dependent and a Bose-Einstein condensation of kaons should occur at densities between 2 to 5 times of normal nuclear density . The explicit breaking of the chiral symmetry $SU(3)_L \times SU(3)_R$ by the large strange quark mass generates an attractive kaon-baryon S wave interaction which drives kaons into a condensed state by reducing their effective mass at high densities . The kaon -condensation softens the equation of state for dense baryonic matter in the core of a neutron star and thus yields a lower critical mass for gravitational collapse . The strangeness condensation also helps in creating a 'strange' star which may be stable at pressures encountered in these stars .

Some authors [15-16] have observed that the arguments for kaon condensation do not need an underlying chiral symmetry but

are much more general . This phenomenon arises as a consequence of strangeness conservation in hadronic interactions and is independent of any details of hadron gas models . Strange-antistrange particle pairs can be spontaneously created in the nuclear matter produced in the ultra-relativistic heavy-ion collisions at high densities . It is energetically easier to produce K^+ because the production threshold of K^- is higher than that of K^+ . The difference in the number densities of kaons and antikaons necessitates the introduction of strangeness chemical potential μ_K in hadron gas models . To account for the excess of baryons over antibaryons , we have another non-zero chemical potential known as baryon chemical potential μ_B . As the baryon chemical potential μ_B is increased in the hadron gas , the production of strange baryons like Λ , Σ etc. becomes easier . In order to achieve the overall strangeness neutrality in the gas , the net strangeness of Λ and Σ has to be cancelled by the net antistrangeness of kaons . Thus the strangeness conservation constraint in the hadron gas models yields a relation between μ_K and μ_B in which μ_K increases corresponding to the increasing values of μ_B . As the baryon density is increased , a stage comes at a correspondingly higher value of μ_B when the strangeness chemical potential μ_K becomes equal to the mass of the kaon M_K . At this point, the diverging Bose-Einstein factor (at zero momentum) in the distribution function [17] yields the onset

of kaon condensation .

Contrary to the observations made by Cleymans et al [16] ,Davidson and Miller [17] showed that the occurrence of kaon-condensation is strongly dependent on the model used to describe the hadronic interactions in the hot and dense nuclear matter . In models where the baryon-baryon repulsive interaction has been taken into account through a thermodynamically inconsistent excluded volume approach , the treatment of kaons as an ideal gas in a reduced volume results into the condensation of kaons. The use of a thermodynamically consistent mean-field approximation for modelling the baryon-baryon and meson-meson repulsive interactions does not show the formation of a kaon-condensate .They assert that this highlights an important qualitative difference between these two phenomenological approaches in the description of the interactions existing in the hadron gas models [17].

Barz et al [3] have also studied the consequences of kaon condensation in the context of nuclear matter-quark matter deconfining phase transition. However, they have ignored the kaon-baryon interaction as proposed by Kaplan and Nelson and considered a phenomenological kaon-kaon repulsive interaction only. Davidson and Miller have also considered the role of repulsive interaction between a pair of baryons and a pair of mesons

,respectively in their mean- field model . The more logical approach would be to combine the kaon-baryon chiral (attractive as well as repulsive) interactions with nuclear mean field theories that model the known nuclear forces. In these theories, the short range repulsion between the baryons is modelled by the exchange of ω -vector mesons . In Walecka relativistic mean-field model [18-20],the repulsion caused by the ω -exchange is not sufficient to check the large increase of the attractive scalar field which drives the effective mass of the nucleons in the nuclear medium to unrealistically low values $M_N^*/M_N < 0.1$. In a thermodynamically consistent treatment of hard-core baryon-baryon repulsion, Rischke et al [21] modified the baryon chemical potential by a term proportional to the baryonic pressure. Recently Anchitskin and Suhonen [22] combined the Walecka mean-field approach and excluded volume model representing hard-core repulsion between nucleons in a thermodynamically consistent way .This synthesis is possible because of the fact that the hard-core interparticle repulsion can be effectively realized in terms of a mean-field $U_{VdW}(n,T)$ which depends not on the density alone but also on the temperature [22-23].However, they considered a hadron gas consisting of nucleons, antinucleons and pions only. Recently we have extended this formalism for a multicomponent hadron gas which includes hyperons and kaons also [24]

and we noticed a significant modification in the effective baryon masses in a dense and hot HG. The purpose of this paper is to study the phenomenon of kaon condensation and the medium modification of the properties of kaons, nucleons and hyperons at moderate as well as small temperatures and high baryon -densities in such a model. In our phenomenological model we have incorporated the following features :

1. An attractive interaction between baryons due to the exchange of a scalar σ - meson .
2. A repulsive interaction between baryons due to the exchange of a vector ω -meson .
3. The hard-core repulsion between baryons represented by a new thermodynamic mean-field potential $U_{\text{vdW}}(n,T)$.
4. An attractive kaon-baryon interaction because of explicit breaking of chiral symmetry .The mass of a kaon in the medium is reduced due to an effective scalar field .
5. A single time-derivative interaction arising from the standard vector- current coupling .This gives repulsion for kaons in the nuclear matter and results into attraction for antikaons due to G-parity [25].

In our model, the hard-core radius of the baryons is an additional parameter . So we have tried to show how the variation of the hard-core radius of a baryon gives rise to different amount of

hard-core repulsion between baryons represented by the U_{vdW} term . Moreover, it is worthwhile to study how the μ_K versus μ_B relationship is modified when different amount of in medium repulsion is considered between a pair of baryons and chiral attraction as well as repulsion between kaons are also taken into consideration . In this paper, we want to explore the model-dependence of kaon-condensation phenomenon. Hence we will study the significance of each of the above terms.

II METHOD OF CALCULATION

In the extended Walecka model [26-28] used for the description of a multicomponent hadron gas, the attractive interaction between baryons is given by the scalar σ -exchange while the exchange of the vector meson ω gives the repulsive force. We take the couplings for hyperons Y as related to those of nucleons as $g_{\sigma YY} = (2/3)g_{\sigma NN}$ and $g_{\omega YY} = (2/3)g_{\omega NN}$ [29] . The scalar field changes the baryon mass to an effective mass M^* and the time component of the vector field in the case of homogeneous and thermally equilibrated baryonic matter describes the repulsive force. We want to extend the Walecka model to account for the effects of the finite size of particles in a multicomponent hadron gas. Thus the expression for the total pressure of the hadron gas can be written as :

$$P = \frac{1}{3} \sum_j \int \frac{d^3 k}{(2\pi)^3} \frac{k^2}{(M_j^{*2} + k^2)^{1/2}} [f_j + f_{\bar{j}}] +$$

$$P_{VdW}(n, T) + P_B(n_B) + P_\sigma(\sigma) + \sum_m P_m(T) \quad (1)$$

In our calculation, we consider the multicomponent hadron gas consisting of mesons $m = \pi, K, K^* (890 \text{ MeV}), \phi, \rho, \omega, \eta, \eta' (950 \text{ MeV})$ and the low lying baryons $j = N, \Lambda, \Sigma$ and antibaryons $\bar{j} = \bar{N}, \bar{\Lambda}, \bar{\Sigma}$. We want to point out that the calculation becomes very much involved for many particles in the hadron gas spectrum. Therefore, we have ignored the contribution of other hyperons and higher nucleonic resonances in our calculation here. Since we are interested in small temperature and high density regions of the EOS in our calculation, this assumption appears appropriate. Further, the other terms involved in (1) are:

$$f_{j(\bar{j})} = \left[\exp \left(\frac{(M_j^{*2} + k^2)^{1/2} + U_{VdW}(n, T) \pm U_{Bj}(n_B) \pm \mu_j}{T} \right) + 1 \right]^{-1} \quad (2)$$

Here the upper (lower) sign refers to baryons (antibaryons), respectively. The expression for the Van-der Waals' hard-core repulsion terms P_{VdW} and U_{VdW} are:

$$P_{VdW}(n, T) = nT \frac{V_0 n}{1 - V_0 n} \quad (3)$$

$$U_{VdW}(n, T) = T \frac{V_0 n}{1 - V_0 n} - T \ln(1 - V_0 n) \quad (4)$$

with

$$n = \sum_j (n_j + n_j^-) \quad (5)$$

and V_0 is the hard core volume of all type of baryons. The baryon density dependent mean-field pressure P_B is given as:

$$P_B = (1/2) m_{\omega}^2 \omega_0^2 \quad (6)$$

where ω_0 is the time component of the ω -exchange field and in the mean field approximation it is given by:

$$\omega_0 = \frac{1}{m_{\omega}} [g_{\omega NN} n_{BN} + g_{\omega \Lambda \Lambda} n_{BA} + g_{\omega \Sigma \Sigma} n_{BE}] \quad (7)$$

where

$$n_{Bj} = n_j - n_j^- = g_j \int \frac{d^3 k}{(2\pi)^3} [f_j - f_j^-] \quad (8)$$

and

$$n_B = \sum_j n_{Bj}(\mu_j, T) \quad (9)$$

Here we have taken $g_N = 4$, $g_{\Lambda} = 2$ and $g_{\Sigma} = 6$ in Eq.(8). Furthermore, we get U_{Bj} in terms of the time component of vector field ω as:

$$U_{Bj}(n_B) = g_{\omega jj} \omega_0 \quad (10)$$

The vector interaction of ω -meson with nucleons and hyperons modifies their chemical potential as:

$$\mu_j = \mu_j - U_{Bj} \quad (11)$$

The attractive interaction of baryons with scalar field σ modifies their masses as:

$$M_j^* = M_j - g_{\sigma jj} \sigma \quad (12)$$

and the pressure is

$$P_{\sigma}(\sigma) = -(1/2) m_{\sigma}^2 \sigma^2 \quad (13)$$

In order to determine the effective mass of the different species, we have to determine the scalar field σ . For this purpose, we have to determine the thermodynamic consistency conditions[22]. We can make a straight forward generalisation and get the conditions as:

$$n \frac{\partial U_{VdW}(n, T)}{\partial n} = \frac{\partial P_{VdW}(n, T)}{\partial n} \quad (14)$$

$$\sum_i n_{Bi} \frac{\partial U_{Bi}(n_B, T)}{\partial n_B} = \frac{\partial P_B(n_B, T)}{\partial n_B} \quad (15)$$

The dependence of the effective masses on T and μ_j is determined by the condition:

$$\left(\frac{\partial P}{\partial M_j^*} \right)_{T, \mu_j} = 0 \quad (16)$$

Using thermodynamic consistency condition (14) and (15) we derive the following expression for the scalar field from Eq.(16):

$$\sigma = (1/m_{\sigma}^2) \sum_j g_{\sigma jj} (n_{\sigma j} + n_{\sigma j}^-) \quad (17)$$

where

$$n_{\sigma j}(j) = g_j \int \frac{d^3 k}{(2\pi)^3} \frac{M_j^*}{(M_j^{*2} + k^2)^{1/2}} f_j(j) \quad (18)$$

It is obvious to see that the solution for M^* involves a set of coupled and self consistent equations and hence it seems to be a difficult problem to study the variations of M^* either with T or with μ . Two parameters in this model are taken as the same as used by Anchishkin and Suhonen [22]:

$$\begin{aligned} c_s^2 &= g_{\sigma NN}^2 / m_\sigma^2 = 374.84 \text{ GeV}^{-2} \\ c_v^2 &= g_{\omega NN}^2 / m_\omega^2 = 283.59 \text{ GeV}^{-2} \end{aligned} \quad (19)$$

These values reproduce the properties of nuclei at zero temperature and normal nuclear density.

We have to solve the following set of five coupled equations self consistently in order to get the values of the baryon densities and effective masses of different species:

$$\begin{aligned} M_N^* &= M_N - c_s^2 \left[\frac{4}{(2\pi)^3} \int d^3k \frac{M_N^*}{(M_N^{*2} + k^2)^{1/2}} (f_N + f_N^-) \right. \\ &+ (2/3) \left\{ 2 \int \frac{d^3k}{(2\pi)^3} \frac{M_A^*}{(M_A^{*2} + k^2)^{1/2}} (f_A + f_A^-) \right. \\ &\left. \left. + 6 \int \frac{d^3k}{(2\pi)^3} \frac{M_\Sigma^*}{(M_\Sigma^{*2} + k^2)^{1/2}} (f_\Sigma + f_\Sigma^-) \right\} \right] \end{aligned} \quad (20)$$

and three equations for baryon densities n_{BN} , n_{BA} and $n_{B\Xi}$ as given by (8) and one more equation represented by (5). Further we get the effective masses of hyperons as:

$$M_{\Lambda, \Xi}^* = M_{\Lambda, \Xi} - (2/3)(M_N - M_N^*) \quad (21)$$

III. KAON-BARYON INTERACTION

From the chiral lagrangian, Kaplan and Nelson [11-12] showed that baryons act on kaons as an effective scalar field because of the explicit breaking of the chiral symmetry $SU(3)_L \times SU(3)_R$ i.e

$$\mathcal{L}_S = \frac{\bar{\Sigma}_{KN}}{f_K^2} \bar{B} B \bar{K} K \quad (22)$$

where f_K is the kaon decay constant and $\bar{\Sigma}_{KN}$ is the KN sigma term. We have taken $f_K = 93$ MeV and a conservative value of 350 MeV for $\bar{\Sigma}_{KN}$ [29]. This gives rise to an attractive S-wave interactions for the kaon. The term $\mathcal{L}_S$ when combined with the kaon mass term in the free-field lagrangian gives an effective mass M_K^* as follows :

$$M_K^* = M_K \left(1 - \frac{\bar{\Sigma}_{KN} \rho_S}{M_K^2 f_K^2} \right)^{1/2} \quad (23)$$

Where we have taken ρ_S as the total scalar density of the baryons and it is related to the scalar-field σ as follows:

$$\sigma = (g_{\sigma NN} / m_\sigma^2) \rho_S$$

Similarly for incorporating the repulsive vector interaction between the kaons we consider another term in the chiral lagrangian [12,25]:

$$\mathcal{L}_V = \frac{-3i}{8f_K^2} \bar{N} \gamma_0 N \bar{K} \vec{\sigma}_t K \quad (24)$$

This term gives repulsion for kaons in nuclear matter while it is attractive for antikaons [25] due to G-parity .

Combining the scalar and vector interactions with the kaon mass term in the lagrangian we can define the kaon (antikaon) effective energy at zero momentum as:

$$E_{K(\bar{K})}^* = M_K^* \pm (3/8f_K^2) n_B \quad (25)$$

For calculating the kaon (antikaon) number densities, we use the following equation:

$$n_{K(\bar{K})} = g_K \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\exp \left[\frac{(M_K^{*2} + k^2)^{1/2} + \frac{3}{8f_K^2} n_B \pm \mu_K}{T} \right] - 1} \quad (26)$$

where g_K is the degeneracy for kaons, μ_K is the kaon chemical potential . In the absence of chiral repulsion ,the effective energy $E_{K(\bar{K})}^*$ becomes equal to the kaon effective mass M_K^* .The chemical

potential for hyperons is defined as:

$$\mu_{\Lambda}(\Sigma) = \mu_N - \mu_K \quad (27)$$

Furthermore, we require another constraint equation regarding the strangeness neutrality to be satisfied in order to solve the above set of equations. We use

$$n_{B\Lambda} + n_{B\Sigma} (n_K - n_{\bar{K}} + n_K^* - n_{\bar{K}}^*) = 0 \quad (28)$$

IV. RESULTS AND CONCLUSIONS

We have already emphasized that the property of kaon condensation is closely linked with the values of the strangeness chemical potential μ_K in the hadron gas. The $\mu_K \approx M_K^*$ starts kaon-condensation. In Fig 1, we have shown the variation of strangeness chemical potential μ_K and the effective mass of the kaon M_K^* in nuclear medium with respect to the baryon chemical potential μ_B at a temperature $T=100$ MeV. We have considered the kaon-baryon attractive chiral interaction as suggested by Kaplan and Nelson. For comparison, we have also plotted the results obtained in the Walecka model. In the Walecka model, the decrease of M_K^* is faster (curve A) whereas μ_K also shows a faster increase (curve A') and both are almost equal for $\mu_B = 1900$ MeV and $T=100$ MeV. Thus kaon condensation can occur in the Walecka model. When we take hard-core radius as $r_N = 0.6$ fm, the consideration of excluded volume baryon-baryon

repulsion in our model modifies the μ_K versus μ_B relationship (curve B') and the effective mass M_K^* (curve B) decreases more slowly than the curve A. The kaon-condensation would now begin at a higher value of $\mu_B = 2400$ MeV and $T=100$ MeV. However, if we take a somewhat higher value of the hard-core radius as $r_N = 0.8$ fm, our model shows a distinct difference in comparison to that of the Walecka model and μ_K saturates (curve C') to a constant value of around 260 MeV at $\mu_B > 1500$ MeV. Moreover, M_K^* suffers a smaller decrease for higher μ_B (curve C) and we find that Kaon condensation can not occur in this case.

In order to see the effects of smaller temperatures in our calculation, we have shown in Fig 2 the variations of μ_K and M_K^* with the μ_B at $T = 40$ MeV. We notice that the kaon-condensation in the Walecka model occurs at a slightly higher value of μ_B , i.e., $\mu_B = 2000$ MeV. In our model with an excluded volume correction and for $r_N = 0.6$ fm, kaon-condensation starts at $\mu_B \approx 2100$ MeV. However, for $r_N = 0.8$ fm, the kaon-condensation fails to occur in our model.

In Fig 3 we have shown the variation of strangeness chemical potential μ_K and the effective energies E^* for kaons and antikaons, respectively at zero momentum with respect to the baryon chemical potential μ_B . Here in addition to the kaon-baryon chiral attraction in medium, we have also considered a vector chiral

interaction which is repulsive for the kaons and attractive for the antikaons in the nuclear matter. The kaon effective energy E_K^* in Walecka model calculation (curve A) increases from about 490 MeV at $\mu_B = 600$ MeV to about 752 MeV at $\mu_B = 2900$ MeV while the strangeness chemical potential (curve A') increases from 1 MeV to 750 MeV as the baryon chemical potential changes from 450 MeV to 2900 MeV. The intersection of the curves A and A' near $\mu_B \approx 2900$ MeV hints at the occurrence of the condensation of kaons in the Walecka model. Curves D and E show the effective energy E_K^* for antikaons in Walecka as well as our model for $r_N = 0.6$ fm case, respectively. When we consider the hard-core baryon-baryon repulsion in our calculation with $r_N = 0.6$ fm, the curves B and B' representing E_K^* and μ_K start showing the trend of saturation for a much larger value of $\mu_B > 3500$ MeV and μ_K versus μ_B relationship is modified so much so that kaon-condensation does not occur even if we increase the baryon chemical potential μ_B upto a value of 4500 MeV. Thus, if we consider the chiral repulsion as well as chiral attraction between the kaons and further incorporate the excluded volume repulsion between the baryons even with the reduced hard-core radius as $r_N = 0.6$ fm in place of $r_N = 0.8$ fm, we find that the strangeness balance alters the result in such a way that we do not get the kaon-condensation in our model.

Fig 4 shows the variation of the net baryon density

ratio n_B/n_0 (where $n_0 = 0.16 \text{ fm}^{-3}$) with respect to the baryon chemical potential μ_B at $T=100 \text{ MeV}$. The curve A in the Walecka model shows the fast increasing trend of n_B/n_0 whereas the incorporation of the van-der Waals' baryon-baryon repulsion in our model with $r_N = 0.6 \text{ fm}$ moderates the increasing trend as shown by the curve B. If we further increase the hard-core radius to $r_N = 0.8 \text{ fm}$, the n_B/n_0 is significantly modified as compared to that of Walecka model and we notice that the trend of saturation appears for $\mu_B > 1500 \text{ MeV}$. These results clearly indicate the important role of hard-core radius of baryons in the mean-field excluded-volume model.

In order to show the effects of chiral attraction as well as chiral repulsion between kaons, we show in Fig 5 the variation of ratio n_B/n_0 with μ_B at $T=40 \text{ MeV}$. Curves A, B and C represent the n_B/n_0 with the chiral attraction for kaons. The Walecka model calculation yields the curve A which has an increasing trend while the curve B represents our model with $r_N = 0.6 \text{ fm}$. For $r_N = 0.8 \text{ fm}$, the curve C shows the saturation for n_B/n_0 after $\mu_B > 1500 \text{ MeV}$ yielding a maximum value for $n_B/n_0 \approx 2.7$ at $\mu_B = 2000 \text{ MeV}$. If we further take into account the in-medium chiral repulsion for kaons, we get two curves A' and B' for this case. A' shows the calculation in Walecka model and B' represents our model with $r_N = 0.6 \text{ fm}$. We obtain a significant change in comparison to the curve A' and it

shows a saturation behaviour for $n_B/n_0 \approx 6.6$ at $\mu_B = 4500$ MeV. The saturation trends in our model arise from the fact that the Van-der Waals' repulsion becomes very dominant as we start approaching the close packing limit $V_0 n \approx 1$ in high density region.

In Fig 6 and Fig 7 we show the variations of the effective mass M_N^*/M_N and M_A^*/M_A in the medium with respect to the baryon chemical potential μ_B at $T=100$ MeV and $T=40$ MeV, respectively. In Walecka model, we get a large mass reduction such that $M_N^*/M_N \approx 0.09$ at $\mu_B = 1900$ MeV as shown in the curve A. This large mass-drop is significantly arrested if we consider the Van-der Waals' baryon repulsion in our model with $r_N = 0.8$ fm and we start approaching nearly a constant value of $M_N^*/M_N \approx 0.28$ for the values of $\mu_B > 1500$ MeV as shown by the curve C. If we consider the hard-core radius as $r_N = 0.6$ fm, we get only slight modification in the results of the Walecka model as shown by the curve B and we get $M_N^*/M_N \approx 0.09$ at a higher value of chemical potential i.e $\mu_B = 2400$ MeV. Curves D and E show M_A^*/M_A in the case of Walecka and our model calculation ($r_N = 0.8$ fm), respectively.

In order to use these results in the astrophysical situations like the core of a neutron star, we need a study of these effects at a lower temperature. At $T=40$ MeV we find a larger mass drop at somewhat smaller values of the baryon chemical potential μ_B .

in comparison to the curves obtained in the case of $T=100$ MeV . Moreover, the difference in the results of our model and that of Walecka model is somewhat reduced at $T=40$ MeV when compared to the corresponding results at $T=100$ MeV .

Fig 8 shows the variation of the attractive scalar field potential $g_{\sigma NN}$ with respect to the baryon chemical potential μ_B for both the cases of $T=40$ and $T=100$ MeV . This can be used to account for the medium modifications of the kaon , nucleon and Λ particle masses .

In Fig.9, we have explicitly shown the relative importance of the two repulsive potential terms U_{BN} and U_{VdW} at $T=100$ MeV . The ω -exchange gives the in medium repulsion among the nucleons and is given by the potential $U_{BN} = g_{\omega NN} \omega_0$. The consideration of Van-der Waals' repulsion in our model gives an extra repulsive mean-field potential term U_{VdW} which is absent in pure Walecka model calculation . The curve A shows the value of U_{BN} . The Van-der Waals' repulsive term U_{VdW} is represented by the curve B' when we take $r_N = 0.6$ fm . However the inclusion of Van-der Waals repulsion reduces the value of U_{BN} also in our model as shown by the curve A' . When we take the hard-core radius as $r_N = 0.8$ fm in our model , the curve A'' represents U_{BN} term and is drastically modified in comparison to that of pure Walecka model . Similarly a large increase in the Van-der Waals'

repulsion term U_{VdW} as shown by the curve B'' results in the dominance of U_{VdW} potential over the U_{BN} potential around the value of $\mu_B \approx 2000$ MeV . As a consequence, the decrease of the nucleon effective mass is stopped and yields a trend of saturation .

In Fig 10, we demonstrate the ω -exchange repulsive potential U_{BN} and Van-der Waals repulsive potential U_{VdW} in our calculation with $r_N = 0.8$ fm as well as chiral attraction for kaons at $T = 40$ MeV . The trend of saturation for U_{BN} is observed after $\mu_B = 1500$ MeV whereas the Van-der Waal potential U_{VdW} still increases but it is not able to take over the ω -exchange potential U_{BN} at $\mu_B = 2000$ MeV as we find in the case of $T = 100$ MeV (Fig 9) . We have included the chiral repulsion term for kaons and also shown the results obtained in Walecka model and our model calculations with $r_N = 0.6$ fm.

The corresponding calculation for the Van-der Waals' repulsive potential U_{VdW} has also been shown . The dominant contribution of hard-core repulsion again explains the saturation trend of the result in our model .

In conclusion , we find that the kaon condensation in Walecka model occurs at $T = 100$ MeV and $\mu_B = 1900$ MeV when $n_B/n_0 = 5.22$ but the decrease in the effective mass of the nucleon is much faster as $M_N^*/M_N \approx 0.09$ at $\mu_B = 1900$ MeV . If we include the Van-der Waals' repulsion between baryons in the mean-field formalism with $r_N = 0.6$ fm

,we can still get kaon condensation but at a higher value of μ_B (≈ 2400 MeV)and $n_B/n_0 = 5.32$.The reduction in the effective mass of nucleon is still considerable but we get $M_N^*/M_N \approx 0.09$ at $\mu_B = 2400$ MeV .However, if we increase the value of r_N to 0.8 fm ,we do not get the kaon condensation at all but the drastic reduction of the effective mass M_N^*/M_N is remedied as ≈ 0.28 at $\mu_B = 2600$ MeV . When we move down towards lower temperatures eg. $T=40$ MeV ,we still get similar results when we consider chiral attraction for kaons .We get kaon condensation at $n_B/n_0 \approx 4.7$ and $\mu_B = 2000$ MeV in the Walecka model. Similarly in our model with $r_N = 0.6$ fm, we get the onset of kaon condensation at $\mu_B = 2100$ MeV and $n_B/n_0 \approx 4.7$. However, even with the consideration of only an attractive chiral interaction between kaons we do not get such condensation when we take $r_N = 0.8$ fm in our model.

In such a model , n_B/n_0 increases to a maximum value of 2.7 while the kaon condensation requires a larger density for its occurrence. Moreover, with chiral repulsion, our model can not yield the kaon condensation even for smaller hard-core radius ≈ 0.6 fm .The chiral repulsion term for kaons is different than its value for antikaons . Our results indicate that if we ignore the chiral repulsion term and take a moderate excluded volume correction with a hard-core radius around 0.6 fm ,we shall witness the onset of kaon condensation in the dense hadron gas. Thus the consideration of chiral repulsion for

kaons does not give a realistic situation for kaon condensation at a finite but small temperature. In summary , we have amply demonstrated that the kaon condensation is a model dependent phenomenon and its occurrence in a dense hadron gas depends on the values of the interactions between baryons as well as kaons and it also depends on the hard-core size of the baryons in the nuclear medium . The excluded-volume correction in the hadron gas makes the EOS a stiff one and the possibility of kaon condensation decreases considerably.

ACKNOWLEDGEMENTS

V.K.T. and N.Prasad are thankful to University Grants Commission (UGC), Govt. of India for financial assistance.

FIGURE-CAPTIONS

FIGURE-CAPTIONS

FIG. 1. The variations of the effective mass M_K^* and the strangeness chemical potential μ_K with respect to the baryon chemical potential μ_B at $T=100$ MeV when we have considered the chiral attraction for kaons. Curves A ,B and C represent the values of M_K^* as calculated in 1) Walecka model, 2) our model with hard-core radius as 0.6 fm and 3) our model with hard-core radius as 0.8 fm, respectively. Curves A' ,B' and C' show the values of μ_K as calculated in the above three cases, respectively .

FIG. 2. The variations of the same quantities as shown in Fig.1 at a lower temperature $T=40$ MeV .

FIG. 3. The variations of the kaon (antikaon) effective energies E_K^* ($E_{\bar{K}}^*$) at zero momentum and strangeness chemical potential μ_K with respect to the baryon chemical potential μ_B at $T= 40$ MeV . Here in addition to the chiral attraction we have also incorporated the chiral repulsion between kaons .Curves A and B represent the calculation for E_K^* in the Walecka model and our model with $r_N = 0.6$ fm, respectively . A' and B' show the corresponding curves in the above cases for μ_K . D and E represent the calculation of $E_{\bar{K}}^*$ in Walecka model and in our model with $r_N = 0.6$ fm .

FIG. 4. The variations of the baryon density n_B/n_0 with respect to the baryon chemical potential μ_B at $T = 100$ MeV where we have considered only the in medium chiral attraction for kaons. Curves A ,B and C show the calculations in Walecka model and in our model with $r_N = 0.6$ and 0.8 fm, respectively .

FIG. 5. Curves A ,B and C represent the same variations as shown in Fig.4 at a lower temperature $T = 40$ MeV . A* and B* represent the calculation for n_B/n_0 in Walecka model and in our model with $r_N = 0.6$ fm, repectively where in addition to chiral attraction we have also incorporated the chiral repulsion between kaons.

FIG. 6. The variations of the effective mass ratios M_N^*/M_N and M_Λ^*/M_Λ with respect to the baryon chemical potential μ_B at $T = 100$ MeV .A ,B and C represent M_N^*/M_N in Walecka model and in our model with $r_N = 0.6$ and 0.8 fm, respectively .D and E show the variations for M_Λ^*/M_Λ as calculated in Walecka model and in our model with $r_N = 0.8$ fm, respectively .

FIG. 7. The variations of the same quantities as presented in Fig. 6 have been shown at a lower temperature $T = 40$ MeV .

FIG. 8. The variations of the scalar-field potential $g_{\sigma NN}$ with respect to the baryon chemical potential μ_B . A ,B and C represent the calculations in Walecka model and in our model

with $r_N = 0.6$ and 0.8 fm, respectively at $T=100$ MeV . A', B' and C' show the corresponding values at a lower temperature $T = 40$ MeV .

FIG. 9. The variations of the ω -exchange repulsive mean-field potential U_{BN} and Van-der Waals' mean-field potential U_{VdW} with respect to the baryon chemical potential μ_B at $T= 100$ MeV . A, A^* , and A^{**} represent the values for U_{BN} as calculated in Walecka model and in our model with $r_N = 0.6$ and 0.8 fm, respectively . B^* and B^{**} represent U_{VdW} when we have taken the hard-core radius as $r_N = 0.6$ and 0.8 fm , respectively in our model calculation .

FIG. 10. The variations of U_{BN} and U_{VdW} with respect to the baryon chemical potential μ_B at $T = 40$ MeV . A'' and B'' represent the values of U_{BN} and U_{VdW} as calculated in our model with $r_N = 0.8$ fm . Curves A and A' represent the values of U_{BN} as calculated in Walecka model and in our model with $r_N = 0.6$ fm respectively when chiral repulsion between kaons (in addition to chiral attraction) has also been considered. B' represent the values for U_{VdW} in our calculation with $r_N = 0.6$ fm.

REFERENCES

1. G.E.Brown, K. Kubodera and M. Rho, Phys. Lett. B 192, 273 (1997).
2. T.Tatsumi, Prog. Theor. Phys. 80, 22 (1988) ;
T.Muto, Prog. Theor. Phys. 89, 415 (1993).
3. H.W. Barz, B.L. Friman, J.Knoll and H. Schulz, Phys. Rev.D 40, 157
(1989)
4. R.Mattiello, H.Sorge, H.Stöcker and W. Greiner, Phys. Rev. Lett.
63, 1459 (1989).
5. X.S. Fang, C.M. Ko, G.E. Brown and V.Koch, Phys.Rev. C 47, 1678
(1993).
6. V.K.Tiwari, S.K.Singh, S.Uddin and C.P.Singh, Phys.Rev. C 53 ,
2388 (1996) .
7. N.K. Glendenning, Phys.Lett. B 114, 392 (1982).
8. F.Weber and M.K. Weigel, Nucl.Phys. A 505,779 (1989).
9. G.E. Brown, K.Kubodera, M.Rho and V.Thorsson, Phys.Lett. B 291,
355 (1992) ;
10. G.E. Brown, C.H. Lee, M.Rho and V. Thorsson, Nucl.Phys. A 567,
937 (1994) .
11. D.B. Kaplan and A.E. Nelson, Phys.Lett. B 175, 57 (1986) ;B 179,
409 (1986) .
12. A.E. Nelson and D.B. Kaplan, Phys.Lett. B 192, 193 (1987) .
13. V.Thorsson, M.Prakash and J.M. Lattimer, Nucl.Phys. A 572, 937

- (1994) .
14. P.J. Ellis, R.Knorren, M.Prakash, Phys.Lett. B 349, 11 (1995) .
 15. U.Heinz, K.S.Lee and M.Rhoades-Brown, Mod.Phys.Lett. A 2, 153 (1987).
 16. J.Cleymans and D.W.Von Oertzen, Phys.Lett B 249, 511 (1990) .
 17. N.J.Davidson and H.G.Miller, Phys.Lett. B 255, 110 (1990) .
 18. J.D.Walecka, Ann.Phys. (N.Y.) 83, 491 (1974) .
 19. B.D.Serot and J.D.Walecka, Adv.Nucl.Phys. 16, 1 (1986) .
 20. B.D.Serot, Rep.Pro.Phys. 55, 1855 (1992) .
 21. D.H.Rischke, M.I.Gorenstein, H.Stocker, W.Greiner
Z.Phys.C 51, 485-489 (1991)
 22. D.Anchiskin, E.Suhonen, Nucl.Phys. A 586, 734 (1995) .
 23. D.V.Anchiskin, Sov. Phys. JETP 75, 195 (1992)
 24. V.K.Tiwari, K.K.Singh, Nirvikar Prasad and C.P.Singh, Nucl.Phys. .
A (Submitted) .
 25. J.Schaffner, A.Gal, I.N.Mishustin, H.Stöcker and W.Greiner,
Phys.Lett. B, 334, 268 (1994) .
 26. S.I.A.Garpman, N.K.Glendenning and Y.J.Krant, Nucl.phys. A 322,
382 (1979) .
 27. N.K.Glendenning, Nucl.Phys. A 469, 600 (1987) .
 28. F.Weber and M.K.Wiegel, Z.Phys. A 330, 249 (1988) .
 29. X.S.Fang, C.M.Ko, G.Q.Li, Y.M.Zheng, Nucl.Phys.A 575, 766 (1994).

FIG. 5

$T = 40 \text{ MeV}$

FIG. 7

FIG. 8