Abstract

It has recently been shown that by extending the minimal standard model to include a right-handed partner to $\nu_\tau$, it is possible to gauge the $B - 3L_\tau$ quantum number consistently. If we add two scalar triplets, one trivial ($\xi_1$) and one nontrivial ($\xi_2$) under $B - 3L_\tau$, it is possible also to have desirable neutrino masses and mixing for neutrino oscillations. At the same time, a lepton asymmetry can be generated in the early universe through the novel mechanism of the decay of the heavier $\xi_1$ into the lighter $\xi_2$ plus a neutral singlet ($\zeta^0$). This lepton asymmetry then gets converted into a baryon asymmetry at the electroweak phase transition.
It has recently been shown[1] that the minimal standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge model of quarks and leptons may be extended to include an anomaly-free gauge factor $U(1)_{B-3L_\tau}$ if $\nu_\tau$ has a right-handed singlet partner, but not $\nu_e$ or $\nu_\mu$. The scale of symmetry breaking of this $B-3L_\tau$ gauge group may even be lower[2] than that of electroweak symmetry breaking. In the minimal standard model, there are dimension-six baryon-number nonconserving operators[3] of the form $Q^3L$ which would induce the proton to decay. In the $B-3L_\tau$ gauge model, the lowest dimensional baryon-number nonconserving operator is of the form $Q^9L_\tau$, hence such processes are suppressed by 22 powers of some higher energy scale and become totally negligible. To obtain a baryon asymmetry of the universe in this model, it is natural to propose instead that a primordial lepton asymmetry is generated[4, 5] which then gets converted into a baryon asymmetry during the electroweak phase transition[6].

In order to generate a lepton asymmetry in the early universe, we must have lepton-number nonconserving interactions which also violate $C$ and $CP$, as well as the existence of an epoch when such processes are out of thermal equilibrium[7]. The canonical way[4] of achieving this is to use heavy right-handed singlet neutrinos which also allow the known left-handed neutrinos ($\nu_e, \nu_\mu, \nu_\tau$) to acquire small seesaw masses. An equally attractive scenario was recently proposed[5] where heavy Higgs triplets are used. In the $B-3L_\tau$ gauge model, since $\nu_e$ and $\nu_\mu$ have no right-handed singlet partners, the natural thing to do is to adopt a variation of the latter mechanism. In fact, as we see below, the requirement of a desirable neutrino mass matrix and the absence of an unwanted pseudo-Goldstone boson together imply a successful leptogenesis scenario in this model without any further extension.

In the standard model, a general neutrino mass matrix may be obtained with the addition of one heavy Higgs triplet, but to generate a lepton asymmetry, two such triplets are required[5]. In the minimal $B-3L_\tau$ gauge model, only $\nu_\tau$ gets a mass. Furthermore, because $B-3L_\tau$ is a gauge symmetry, it is not obvious a priori how that will affect the conversion
of a primordial $L_e + L_\mu$ asymmetry into a baryon asymmetry during the electroweak phase transition. In this paper we will address both issues, \textit{i.e.} neutrino masses and baryogenesis, and show how they may have a common solution.

The fermion content of our model is identical to that of the original model\cite{1}. The quarks and leptons transform under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-3L_\tau}$ as follows:

$$
\begin{pmatrix}
  u_i \\
  d_i
\end{pmatrix}_L \sim (3, 2, 1/6; 1/3), \quad
\begin{pmatrix}
  u_i \\
  d_i
\end{pmatrix}_R \sim (3, 1, -1/3; 1/3); \quad
\begin{pmatrix}
  \nu_e \\
  e
\end{pmatrix}_L, \quad
\begin{pmatrix}
  \nu_\mu \\
  \mu
\end{pmatrix}_L \sim (1, 2, -1/2; 0), \quad
\begin{pmatrix}
  \nu_\tau \\
  \tau
\end{pmatrix}_L \sim (1, 2, -1/2; -3), \quad
\begin{pmatrix}
  \nu_{\tau R} \\
  \tau_R
\end{pmatrix} \sim (1, 1, -1; -3). \quad (1)
$$

The $U(1)_{B-3L_\tau}$ gauge boson $X$ does not couple to $e$ or $\mu$ or their corresponding neutrinos. It can thus escape detection in most experiments. Although it does couple to quarks as in a previously proposed model\cite{8}, such signatures are normally overwhelmed by the enormous quantum-chromodynamics (QCD) background. On the other hand, $X$ does couple to $\tau$ and $\nu_\tau$, so it could be observed through its decay into $\tau^+\tau^-$ pairs\cite{2}. Furthermore, the $\nu_\tau$-quark interactions in this scenario may affect the oscillations of neutrinos inside the sun and the earth and contribute\cite{9} to the zenith-angle dependence of the atmospheric neutrino deficit\cite{10}. Since the interactions of $X$ violates $e - \mu - \tau$ universality, present experimental constraints limit its coupling and mass\cite{2}. For example, $g_X < 0.1$ is required for $m_X < 50$ GeV.

The minimal scalar sector of this model consists of the standard Higgs doublet,

$$
\begin{pmatrix}
  \phi^+ \\
  \phi^0
\end{pmatrix}_L \sim (1, 2, 1/2; 0) \quad (4)
$$

which breaks the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ down to $U(1)_{\text{em}}$ and a neutral singlet,

$$
\chi^0 \sim (1, 1, 0; 6) \quad (5)
$$
which couples to $\nu_{\tau R}\nu_{\tau R}$ and breaks the $U(1)_{B-3L_{\tau}}$ gauge symmetry. The resulting theory allows $\nu_{\tau L}$ to obtain a seesaw mass[11] of order 1 eV and retains $B$ as an additively conserved quantum number and $L_{\tau}$ as a multiplicatively conserved quantum number.

In the original model[1], the other two neutrinos ($\nu_e, \nu_\mu$) acquire masses and mix with $\nu_\tau$ radiatively. In our present work, we propose instead to use the mechanism of Ref.[5] and add a couple of Higgs triplets:

$$\begin{pmatrix}
\xi_{1}^{++} \\
\xi_{1}^{+} \\
\xi_{1}^{0}
\end{pmatrix} \sim (1, 3, 1; 0) \quad \text{and} \quad \begin{pmatrix}
\xi_{2}^{++} \\
\xi_{2}^{+} \\
\xi_{2}^{0}
\end{pmatrix} \sim (1, 3, 1; 3).$$

(6)

Now $\xi_{1}$ will give small masses to $\nu_e$ and $\nu_\mu$, and will also generate a $\xi_{2}$-asymmetry of the universe when it decays at a very high temperature. Furthermore, $\xi_{2}$ will mix $\nu_e$ and $\nu_\mu$ with $\nu_\tau$, and its interactions will convert the $\xi_{2}$-asymmetry into a $L_e + L_\mu$ asymmetry of the universe. Since $B - 3L_{\tau}$ may well be an unbroken gauge symmetry during the electroweak phase transition, there may not be any $B - 3L_{\tau}$ asymmetry. In that case, the total $B - L$ asymmetry is the same as the $-(L_e + L_\mu)$ asymmetry, which will get converted into the baryon asymmetry of the universe during the electroweak phase transition.

The above scalar sector contains a pseudo-Goldstone boson which comes about because there are 3 global $U(1)$ symmetries in the Higgs potential and only 2 local $U(1)$ symmetries which get broken. In addition, there is no $CP$-violating complex phase which is necessary for generating a lepton asymmetry of the universe. However, if an extra neutral scalar $\zeta^0 \sim (1, 1, 0, -3)$ is added, then the Higgs potential will have additional terms which eliminate the unwanted global $U(1)$ symmetry, and one of these couplings will also have to be complex, allowing thus enough $CP$ violation to generate a lepton asymmetry, as explained below.

The triplet scalar fields $\xi_a, \quad (a = 1, 2)$ do not acquire any vacuum expectation value ($vev$)
to start with. At the tree level, we can write down the relevant part of the Lagrangian as

\[ -\mathcal{L} = \sum_{a=1,2} M_a^2 \xi_a^\dagger \xi_a + \sum_{i,j=e,\mu} f_{1ij} [\xi_i^0 \nu_i \nu_j + \xi_i^+ (\nu_i l_j + l_i \nu_j) / \sqrt{2} + \xi_i^+ l_j] \]

\[ + \sum_{i=e,\mu} f_{2i\tau} [\xi_i^0 \nu_i \nu_\tau + \xi_i^+ (\nu_i \tau + l_i \nu_\tau) / \sqrt{2} + \xi_i^+ l_\tau] + f_{\tau \nu_\tau \nu_\tau} \chi^0 \]

\[ + \mu_1 [\xi_1^0 \phi^0 \phi^0 + \sqrt{2}\xi_1^- \phi^+ \phi^- + \xi_1^- \phi^+ \phi^-] + \mu_2 [\xi_2^0 \phi^0 + g\xi_1^0 \phi^0 \chi^0 \]

\[ + h\xi_1^0 [\xi_2^0 \phi^0 \phi^0 + \sqrt{2}\xi_2^- \phi^+ \phi^- + \xi_2^- \phi^+ \phi^-] + h.c. \]  

(7)

Interactions of the scalar triplets \( \xi_a \), \((a = 1, 2)\) break the \( L_e \) and \( L_\mu \) numbers explicitly, whereas \( L_\tau \) is conserved. Since there is no spontaneous breaking of the lepton numbers, there are no massless Goldstone bosons (majorons) which can contribute to the invisible width of the Z boson. After electroweak symmetry breaking when the doublet acquires a nonzero vev, and the breaking of \( B - 3L_\tau \), there will be induced vev’s for these scalar triplets,

\[ \langle \xi_1^0 \rangle \simeq -\frac{\mu_1 v^2}{M_1^2} \quad \text{and} \quad \langle \xi_2^0 \rangle \simeq -\frac{h\langle \xi_1^0 \rangle v^2}{M_2^2}. \]

Since the masses of these scalar triplets and the would-be majorons are very large, they cannot contribute to the width of the Z boson.

At low energies, we can integrate out the heavier triplet fields and write down an effective neutrino mass matrix in the basis \{\bar{\nu}_e_L \ \bar{\nu}_\mu_L \ \bar{\nu}_\tau_L \ \nu_\tau_R\} as,

\[
M_\nu = \begin{pmatrix}
    f_{1ee} \langle \xi_1^0 \rangle & f_{1ep} \langle \xi_1^0 \rangle & f_{1e\tau} \langle \xi_2^0 \rangle & 0 \\
    f_{1ep} \langle \xi_1^0 \rangle & f_{1\mu\mu} \langle \xi_1^0 \rangle & f_{2e\tau} \langle \xi_2^0 \rangle & 0 \\
    f_{2e\tau} \langle \xi_2^0 \rangle & f_{2\mu\tau} \langle \xi_2^0 \rangle & 0 & m^D_\tau \\
    0 & 0 & m^D_\tau & f_\tau \langle \chi^0 \rangle
\end{pmatrix}
\]

(8)

where \( m^D_\tau \) is the Dirac mass term for \( \nu_\tau \) and \( f_\tau \langle \chi^0 \rangle \) is the Majorana mass of \( \nu_\tau R \). The left-handed \( \nu_\tau L \) will then get a seesaw mass, which can be of order \( 1 \) eV. The out-of-equilibrium condition for the generation of a lepton asymmetry dictates that the mass of the triplet \( \xi_1 \) be of order \( 10^{13} \) GeV, which implies that the \( e \) and \( \mu \) mass elements are also of order \( 1 \) eV or less. As we will see later, \( \nu_e \) and \( \nu_\mu \) may mix with \( \nu_\tau \) to form a desirable phenomenological
mass matrix for neutrino oscillations. The constraints on these elements come from the consideration of a realistic baryon asymmetry of the universe as we show below.

Around the time of the electroweak phase transition, we assume that \( B - 3 L_\tau \) is conserved; hence there cannot be any \( B - 3 L_\tau \) asymmetry. In particular, \( L_\tau \) is exactly conserved. However, \( L_e \) and \( L_\mu \) numbers are broken explicitly at some high scale \( M_1 \) in the decays of the triplets \( \xi_1 \). At such high energies, \( SU(2)_L \) gauge invariance means that we need only consider one of its components, say \( \xi_1^{++} \), which has the following decay modes:

\[
\xi_1^{++} \rightarrow \begin{cases} l_i^+ l_j^+ & (L_e + L_\mu = -2; \ n_{\xi_2} = 0) \\ \xi_2^{++} \xi^0 & (L_e + L_\mu = -1; \ n_{\xi_2} = 1) \\ \phi^+ \phi^+ & (L_e + L_\mu = 0; \ n_{\xi_2} = 0). \end{cases}
\]

(9)

Here we assumed that most of the time \( \xi_2^{++} \rightarrow l_i^+ l_j^+ \), and the other decay mode of \( \xi_2 \) never comes to equilibrium so that \( L_e + L_\mu = -1 \) for \( \xi_2 \). We will discuss this point later. In the following we will first explain how \( \xi_1 \) decay generates a \( \xi_2 \) asymmetry.

The first decay mode does not play any role in the generation of a lepton asymmetry. Here \( CP \) violation comes from the interference of the tree-level and one-loop diagrams of Figures 1 and 2. The scalar potential has one \( CP \)-violating phase in the product \( \mu_1^* \mu_2 h \), which cannot be absorbed by redefinitions and produces a \( \xi_2 \)-asymmetry when \( \xi_1 \) decays. As a result, the decays of \( \xi_1^{++} \) and \( \xi_1^{--} \) will create more \( \xi_2^{++} \) than \( \xi_2^{--} \) or vice versa; hence a \( \xi_2 \)-asymmetry \( \delta = (n_{\xi_2} - n_{\xi_2}^*)/n_\gamma \) will be created, given by

\[
\delta \simeq \frac{\text{Im}[\mu_1^* \mu_2 h]}{16\pi^2 g_* M_1^2} \left[ \frac{M_1}{\Gamma_1} \right],
\]

(10)

where \( g_* \) is the total number of relativistic degrees of freedom and

\[
\Gamma_1 = \frac{1}{8\pi} \left( \frac{|\mu_1|^2 + |\mu_2|^2}{M_1} + \sum_{ij} |f_{1ij}|^2 M_1 \right)
\]

(11)

is the decay rate of the triplet \( \xi_1 \). This \( \xi_2 \)-asymmetry will also have an apparent charge asymmetry, which will be compensated by an asymmetry in \( \phi^+ \) and \( \phi^- \). In earlier models
of leptogenesis, a lepton asymmetry is generated when the heavy particles decay into light
leptons and \( CP \) violation enters in the vertex corrections\[4\] or in the mass matrix\[5, 12\]. In
contrast, we generate in the present scenario an asymmetry in \( \xi_2 \) through the quartic scalar
couplings, which then generate a lepton asymmetry.

For the generation of the \( \xi_2 \)-asymmetry, this decay rate should also satisfy the out-of-
equilibrium condition\[13\]
\[
\Gamma_1 < \sqrt{1.7 g_* \frac{T^2}{M_{Pl}}} \quad \text{at } T = M_1, \tag{12}
\]
where \( M_{Pl} \) is the Planck scale. We assume \( M_1 \gg M_2 \), so that when \( M_1 \) decays, \( \xi_2 \) is
essentially massless. Taking \( \mu_{1,2}/M_1 \sim 0.1, f_{1ij} \sim 1 \), the out-of-equilibrium condition is
satisfied with \( M_1 > 10^{14} \) GeV. However, even if we choose \( M_1 \sim 10^{13} \) GeV, the generated
\( \xi_2 \)-asymmetry will be only less by a factor \( S \sim 10^{-2} \), which is still large enough to explain
the baryon asymmetry of the universe for a value of \( h \sim 10^{-4} \). This gives us the \( e \) and \( \mu \)
neutrino mass matrix elements to be of order 1 eV.

At a temperature \( T < M_1 \), there will be a \( \xi_2 \)-asymmetry. The decays of \( \xi_2 \) also break
lepton number,
\[
\xi_2^{++} \to \begin{cases} 
  l_i^+ l_r^+ & (L_e + L_\mu = -1) \\
  \phi^+ \phi^+ \zeta^0 \ast & (L_e + L_\mu = 0).
\end{cases} \tag{13}
\]
If both of these decay modes are in equilibrium at any time, that will erase the lepton
asymmetry of the universe\[6, 13, 14\]. We must therefore require that at least one of these
interactions and the scattering process,
\[
l_i^+ l_r^+ \to \phi^+ \phi^+ \zeta^0 \ast
\]
to satisfy the out-of-equilibrium condition till the electroweak symmetry breaking phase
transition is over.

For the choice \( h \sim 10^{-4} \), we may take \( M_2 > 10^5 \) GeV to ensure that
\[
\Gamma_2(\xi_2 \to \phi \phi \zeta^0) = \frac{h^2 M_2}{16 \pi^2} \frac{T^2}{8 \pi} < \sqrt{1.7 g_* \frac{T^2}{M_{Pl}}} \quad \text{at } T \geq M_2 \tag{14}
\]
so that $\xi_2$ can hardly decay into three scalars at any time. However, we would like the other
decay mode of $\xi_2$ to be fast, so that the $\xi_2$-asymmetry generated during the $\xi_1$ decay gets
converted into a lepton asymmetry. In other words, since the number of $\xi_2$ is different from
the numbers of $\xi^\dagger_2$, the number of leptons generated in decays of $\xi_2$ will be different from the
number of antileptons generated in decays of $\xi^\dagger_2$.

We take $f_{2i} \sim 0.1$, so that the two-lepton decay mode of $\xi_2$ is in equilibrium for most of
the time,

$$\Gamma_2(\xi_2 \rightarrow l_i l_\tau) = \frac{\sum_i |f_{2i}|^2}{8\pi} M_2 > \sqrt{1.7 g_* \frac{T^2}{M_{Pl}}} \quad \text{during } M_c \geq T \geq M_2 \quad (15)$$

where $M_c \simeq 10^9$ GeV. During this period from $M_c$ to $M_2$, the $\xi_2$-asymmetry will get converted
into a $L_e + L_\mu$ asymmetry of the universe. The interaction $\xi_2 l_i l_\tau$ will also be in equilibrium,
which will relate their chemical potentials: $\mu_{\xi_2} = \mu_{L_i} + \mu_{L_\tau}$ (notations will be explained later).
The generated $L_e + L_\mu$ asymmetry is accompanied by an equal amount of $L_\tau$ asymmetry.
However, that is compensated exactly by the $\zeta$-asymmetry created at the time of $\xi_1$ decay.

This $\zeta$-asymmetry generates an equal and opposite amount of $L_\tau$ asymmetry through $\zeta + \zeta \rightarrow
\chi^* \rightarrow \nu_{\tau R} + \nu_{\tau R}$, which compensates the $L_\tau$ asymmetry in $\xi_2$ decay. As a result, there will
not be any net $L_\tau$ asymmetry as expected, since $B - 3L_\tau$ is exactly conserved at this time.
The generated $L_e$ and $L_\mu$ asymmetries together give the $B - L$ asymmetry

$$n_L = \frac{1}{2} \delta S.$$ 

The above choice of parameter values will give us the neutrino mass mixing of the $e$ and the
$\mu$ to the $\tau$ neutrinos of order 1 eV.

We will now see how this lepton asymmetry can get converted into the baryon asymmetry
of the universe during the electroweak phase transition[6]. We consider all the particles to be
ultrarelativistic, which is the case above the electroweak scale, but at lower energies, although
we understand that a careful analysis has to include the mass corrections, we ignore them.
since they are small and cannot change the conclusion drastically. The particle asymmetry, i.e. the difference between the number of particles \((n_+)\) and the number of antiparticles \((n_-)\) can be given in terms of the chemical potential of the particle species \(\mu\) (for antiparticles the chemical potential is \(-\mu\)) as
\[
n_+ - n_- = n_d \frac{g T^3}{6} \left( \frac{\mu}{T} \right),
\]
where \(n_d = 2\) for bosons and \(n_d = 1\) for fermions.

In the rest of this discussion we will assume that after the triplets \(\xi_1\) and \(\xi_2\) have decayed, enough lepton asymmetry was generated. This will give nonvanishing \(\mu_{\nu e}\) and \(\mu_{\nu\mu}\), which are directly related to \(n_L\). When these neutrinos interact with other particles in equilibrium, the chemical potentials get related by simple additive relations, and that will allow us to relate this lepton asymmetry \(n_L\) to the baryon asymmetry during the electroweak phase transition.

At energies near the electroweak phase transition, most of the interactions are in equilibrium. These include the sphaleron\[15\] induced electroweak \(B + L\) violating interaction arising due to the nonperturbative axial-vector anomaly\[16\]. In Table 1, we give the interactions and the corresponding relations between the chemical potentials. In the third column we give the chemical potential which we eliminate using the given relation. We start with chemical potentials of all the quarks \((\mu_{uL}, \mu_{dL}, \mu_{uR}, \mu_{dR})\); the \(e\) and \(\mu\) leptons \((\mu_{\nu eL}, \mu_{\nu eR}, \mu_{\nu\mu L}, \mu_{\nu\mu R})\); the \(\tau\) leptons \((\mu_{\tau L}, \mu_{\nu\tau L}, \mu_{\nu\tau R}, \mu_{\nu\tau R})\); the gauge bosons \((\mu_W\) for \(W^-,\) and \(0\) for all others); and the Higgs scalars \((\mu^\phi, \mu^\phi_0, \mu^x, \mu^\xi)\). The triplets have decayed away much before the electroweak phase transition and have decoupled; hence they do not contribute to the present analysis.

We can then express all the chemical potentials in terms of the following independent chemical potentials only,
\[
\mu_0 = \mu^\phi_0; \quad \mu_W; \quad \mu_a = \mu_{aL}; \quad \mu_{\nu e L} = \mu_{\nu e R}; \quad \mu_{\nu \mu L}; \quad \mu_{\tau} = \mu_{\nu \tau L}.
\]
Table 1: Relations among the chemical potentials

<table>
<thead>
<tr>
<th>Interactions</th>
<th>µ relations</th>
<th>µ eliminated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_\mu \phi^\dagger D_\mu \phi$</td>
<td>$\mu_W = \mu_-^0 + \mu_0^0$</td>
<td>$\mu_-^0$</td>
</tr>
<tr>
<td>$q_L \gamma_\mu q_L W^\mu$</td>
<td>$\mu_4L = \mu_{aL} + \mu_W$</td>
<td>$\mu_{dL}$</td>
</tr>
<tr>
<td>$\overline{l}<em>L \gamma</em>\mu l_L W^\mu$</td>
<td>$\mu_{iL} = \mu_{\nu iL} + \mu_W$</td>
<td>$\mu_{iL}, i = e, \mu, \tau$</td>
</tr>
<tr>
<td>$\overline{q}_L u_R \phi^\dagger$</td>
<td>$\mu_uR = \mu_0 + \mu_{aL}$</td>
<td>$\mu_{aR}$</td>
</tr>
<tr>
<td>$\overline{q}_L d_R \phi$</td>
<td>$\mu_{dR} = -\mu_0 + \mu_{dL}$</td>
<td>$\mu_{dR}$</td>
</tr>
<tr>
<td>$\overline{l}_aL e_aR \phi$</td>
<td>$\mu_{aR} = -\mu_0 + \mu_{aL}$</td>
<td>$\mu_{aR}, a = e, \mu$</td>
</tr>
<tr>
<td>$\overline{l}<em>R \nu</em>\tau R \phi$</td>
<td>$\mu_{\nu R} = -\mu_0 + \mu_\tau L$</td>
<td>$\mu_{\nu R}$</td>
</tr>
<tr>
<td>$\overline{\nu}<em>\tau R^c \nu</em>\tau R \chi$</td>
<td>$\mu_x = -2\mu_{\nu \tau R}$</td>
<td>$\mu_x$</td>
</tr>
<tr>
<td>$\overline{l}<em>R \nu</em>\tau R \phi^\dagger$</td>
<td>$\mu_{\nu R} = \mu_0 + \mu_\tau L$</td>
<td>$\mu_{\nu \tau R}$</td>
</tr>
<tr>
<td>$\zeta \zeta \chi^0$</td>
<td>$\mu_x = 2\mu^\zeta$</td>
<td>$\mu^\zeta$</td>
</tr>
</tbody>
</table>

We can further eliminate one of these five potentials by making use of the relation given by the sphaleron processes. Since the sphaleron interactions are in equilibrium, we can write down the following $B + L$ violating relation among the chemical potentials for three generations,

$$9\mu_a + 6\mu_W + 2\mu_\alpha + \mu_\tau = 0. \quad (18)$$

We then express the baryon number, lepton numbers and the electric charge and the hypercharge number densities in terms of these independent chemical potentials,

$$B = 12\mu_a + 6\mu_W \quad (19)$$

$$L_e = L_\mu = 3\mu_a + 2\mu_W - \mu_0 \quad (20)$$

$$L_\tau = 4\mu_\tau + 2\mu_W \quad (21)$$

$$Q = 24\mu_a + (12 + 2m)\mu_0 - (4 + 2m)\mu_W \quad (22)$$

$$Q_3 = -(10 + m)\mu_W \quad (23)$$

where $m$ is the number of Higgs doublets $\phi$.  

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At temperatures above the electroweak phase transition, $T > T_c$, both $Q$ and $Q_3$ must vanish. In addition, since $B - 3L_\tau$ is also a gauge symmetry, this charge must also vanish. These three conditions and the sphaleron induced $B - L$ conserving, $B + L$ violating condition can be expressed as

$$<Q> = 0 \implies \mu_0 = \frac{-12}{6 + m}\mu_u$$  \hspace{1cm} (24)$$

$$<Q_3> = 0 \implies \mu_W = 0$$  \hspace{1cm} (25)$$

$$<B - 3L_\tau> = 0 \implies \mu_\tau = \mu_u$$  \hspace{1cm} (26)$$

$$\text{Sphaleron transition} \implies \mu_a = -5\mu_u$$  \hspace{1cm} (27)$$

Using these relations we can now write down the baryon number, lepton number, and their combinations in terms of the $B - L$ number density, which remains invariant under all electroweak phase transitions. They are

$$B = \frac{36 + 6m}{102 + 19m}(B - L)$$  \hspace{1cm} (28)$$

$$L_e = L_\mu = \frac{-78 - 15m}{204 + 38m}(B - L)$$  \hspace{1cm} (29)$$

$$L_\tau = \frac{12 + 2m}{102 + 19m}(B - L)$$  \hspace{1cm} (30)$$

$$B + L = \frac{-30 - 7m}{102 + 19m}(B - L)$$  \hspace{1cm} (31)$$

We will now consider two possibilities. In the first, the $B - 3L_\tau$ gauge symmetry is broken after the electroweak phase transition. Then, at temperatures below the electroweak phase transition, the other relations remain the same, but it is no longer neccessary to make $Q_3$ vanishing. Since $\phi$ acquires a vev, we require $\mu_0 = 0$. With this change we can now relate all the chemical potentials in terms of $\mu_u$ as

$$<Q> = 0 \implies \mu_W = \frac{12}{2 + m}\mu_u$$  \hspace{1cm} (32)$$

$$\langle \phi_0 \rangle \neq 0 \implies \mu_0 = 0$$  \hspace{1cm} (33)$$
\[ B - 3L_\tau > = 0 \implies \mu_\tau = \mu_a \]  

\textbf{Sphaleron transition} \implies \mu_a = -3\mu_W - 5\mu_u \hfill (35)

This will then allow us to write down the baryon number, lepton number, and their combinations in terms of the \( B - L \) number density as

\[ B = \frac{48 + 6m}{146 + 19m} (B - L) \]  \hfill (36)

\[ L_e = L_\mu = \frac{-114 - 15m}{292 + 38m} (B - L) \]  \hfill (37)

\[ L_\tau = \frac{16 + 2m}{146 + 19m} (B - L) \]  \hfill (38)

\[ B + L = \frac{-50 - 7m}{292 + 38m} (B - L) \]  \hfill (39)

Thus after the electroweak phase transition, the \( L_e + L_\mu \) asymmetry \( n_L \) generated after the scalar triplets \( \xi_1 \) and then \( \xi_2 \) have decayed, which is equal to the \( B - L \) asymmetry at that time, will get converted into a \( B \) asymmetry during the electroweak phase transition. Although any existing \( B + L \) asymmetry gets washed out, we still get a non-zero \( B + L \) asymmetry after the electroweak phase transition from the same \( B - L \) asymmetry. For consistency we check that the \( B - L \) asymmetry remains the same during the electroweak phase transition and there is no \( B - 3L_\tau \) asymmetry.

We will now consider the other possibility when \( B - 3L_\tau \) is broken before the electroweak phase transition. In this case the electroweak symmetry is unbroken and we still have \( <Q_3> = 0 \), but \( <B - 3L_\tau> \neq 0 \) so that \( \mu_\chi = 0 \). The constraints in this case are

\[ <Q> = 0 \implies \mu_0 = \frac{-12}{6 + m} \mu_u \]  \hfill (40)

\[ <Q_3> = 0 \implies \mu_W = 0 \]  \hfill (41)

\[ \langle \chi \rangle \neq 0 \implies \mu_\chi = 0 \implies \mu_{\nu R} = 0; \ \mu_\tau = -\mu_0 \]  \hfill (42)

\textbf{Sphaleron transition} \implies \mu_a = \mu_0 - \frac{9}{2} \mu_u \]  \hfill (43)
The baryon and lepton asymmetries are now related by

\[ B = \frac{24 + 4m}{66 + 13m} (B - L) \]  
\[ L_e = L_\mu = \frac{-58 - 9m}{132 + 26m} (B - L) \]  
\[ L_\tau = \frac{16}{66 + 13m} (B - L) \]  
\[ B + L = \frac{-18 - 5m}{66 + 13m} (B - L) \]  

Finally we give the relations between the baryon and lepton asymmetries after both the electroweak symmetry and the $B - 3L_\tau$ symmetry are broken. The final asymmetry will not depend on whether the electroweak symmetry was broken before or after the $B - 3L_\tau$ symmetry is broken. Now we have $\mu_\chi = 0$ but $<B - 3L_\tau> \neq 0$ and $\langle \phi \rangle = 0 \Rightarrow \mu_0 = 0 \Rightarrow \mu_\tau = 0$. We can then express the other chemical potentials in terms of $\mu_u$ as

\[ <Q> = 0 \implies \mu_W = \frac{12}{2 + m} \mu_u \]  
\[ \langle \phi \rangle \neq 0 \implies \mu_0 = 0 \]  
\[ \langle \chi \rangle \neq 0 \implies \mu_\chi = 0 \Rightarrow \mu_{\nu_R} = 0; \mu_\tau = -\mu_0 = 0 \]  

Sphaleron transition $\implies \mu_a = -3\mu_W - \frac{9}{2}\mu_u$  

which then let us write the baryon and lepton numbers as some combinations of $B - L$ as

\[ B = \frac{32 + 4m}{98 + 13m} (B - L) \]  
\[ L_e = L_\mu = \frac{-74 - 9m}{196 + 26m} (B - L) \]  
\[ L_\tau = \frac{8}{98 + 13m} (B - L) \]  
\[ B + L = \frac{-34 - 5m}{98 + 13m} (B - L) \]

The final baryon asymmetry of the universe is about 1/3 that of the $B - L$ asymmetry. Hence in the present scenario, the generated $B - L$ asymmetry $n_L$ will get converted into
a baryon asymmetry after the electroweak and $B - 3L_\tau$ symmetries are broken, and the present baryon asymmetry of the universe will be given by

$$n_b \sim \frac{1}{6} S\delta$$  \hspace{1cm} \text{(56)}$$

which is of the order of $10^{-10}$, for the choice of parameter values we have considered earlier.

To summarize, we studied an extension of the standard model to include a $B - 3L_\tau$ gauge symmetry, which may be broken below the electroweak symmetry breaking scale. The Higgs structure is modified to explain the baryon asymmetry of the universe, which comes about in an unconventional way. We first generate an asymmetry in the number of scalars ($n_\xi$), through only scalar interactions. This $n_\xi$ generates the $L_e + L_\mu$ asymmetry when these scalars decay. During the electroweak phase transition, the latter gets converted into a baryon asymmetry of the universe. Neutrino masses and mixing are obtained naturally in this scenario.

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References


Figure 1: Tree-level and one-loop diagrams for $\xi_1 \to \phi \phi$.

(a) $\xi_1^{++}$

(b) $\xi_1^{++}$

Figure 2: Tree-level and one loop diagrams for $\xi_1 \to \xi_2 \zeta$.

(a) $\xi_1^{++}$

(b) $\xi_1^{++}$