Supersymmetric FRW model and the ground state of supergravity

V.I. Tkach\textsuperscript{a}, J.J. Rosales \textsuperscript{a,b}, and J. Socorro \textsuperscript{a,}\textsuperscript{†}
\textsuperscript{a}Instituto de Física de la Universidad de Guanajuato, Apartado Postal E-143, C.P. 37150, León, Guanajuato, México
\textsuperscript{b}Ingeniería en computación, Universidad del Bajío Av. Universidad s/n Col. Lomas del Sol, León, Gto., México

Abstract

In this work we construct the vacuum configuration of supergravity interacting with homogeneous complex scalar matter fields. The corresponding configuration is of the FRW model invariant under the $n = 2$ local conformal time supersymmetry, which is a subgroup of the four dimensional space-time supersymmetry. We show, that the potential of the scalar matter fields is a function of the Kähler potential and of the arbitrary parameter $\alpha$. This parameter enumerates the vacuum states. The scalar matter potential induces the spontaneous breaking of supersymmetry in supergravity. On the quantum level our model is a specific supersymmetric quantum mechanics, which admits quantum states in supergravity, and the states with zero energy are described by the wave function of the FRW universe.

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\textsuperscript{E-mail}: vladimir@ifug1.ugto.mx
\textsuperscript{E-mail}: juan@ifug3.ugto.mx
\textsuperscript{E-mail}: socorro@ifug4.ugto.mx
I. INTRODUCTION

There are several reasons for studying local supersymmetric theories rather than nonsupersymmetric ones. Local gauge supersymmetry or supergravity leads to a constraint, which can be thought of as the “square root” of the Wheeler-DeWitt constraint, and it is related to it in the same way as the Dirac equation is related to the Klein-Gordon equation. This fact has appeared in [1]. Starting with a Hamiltonian treatment of classical supergravity [2–4] the canonical quantum theory of supergravity has appeared in [5] using two-component spinor notation. To obtain quantum physical solutions of the general theory of supergravity is a laborious assignment, this is due to the fact, that in the theory there are infinite degrees of freedom. One option to the search of quantum physical solution is the so-called supersymmetric minisuperspace models [6–9], which suppose that space-time is homogeneous. In other words, the gravitational and matter fields have been reduced to a finite number of degrees of freedom. Thus, the study of the associated quantum supersymmetric cosmology becomes analogous to a supersymmetric quantum mechanical problem. The hope we have in these models is that they could give us the notion of the full quantum theory of supergravity. However, not all the results obtained in quantum supersymmetric cosmology have their counterpart in the full theory of supergravity. Some of these problems have already been presented in two comprehensive and organized works: a book and an extended review [10].

In the last three years we have proposed a new approach to investigate the supersymmetric quantum cosmology [11]. The main idea is as follows: one knows that the action of the cosmological models obtained from four dimensional Einstein-Hilbert action by spatially reduction preserves the invariance under the local time reparametrization. Hence, the more extended symmetry must be local. In order to have a more extended symmetry in [11] the odd “time” parameter $\eta$ and their complex conjugate $\bar{\eta}$ were introduced, which are the superpartners of the usual time parameter $t$. Then, the new functions become superfunctions depending on $t, \eta, \bar{\eta}$, which in the field theory are called superfields. This generalization of the local time reparametrization is well-known in the new formulation of the spinning particles and superparticles [12,13]. This procedure allowed us to formulate in the superfield representation the supersymmetric action for all the Bianchi-type cosmologies [14], which are invariant under the $n=2$ local conformal time supersymmetry transformations. The inclusion of the real scalar supermatter fields, as well as the spontaneous breaking of local supersymmetry, were obtained in [15] following this procedure.

In the previous work [16] we have constructed the most general interacting action for the supersymmetric FRW model with a set of spatially homogeneous complex scalar matter superfields. This scheme of interaction gives a general mechanism of spontaneous breaking of the local supersymmetry analogous to the case of interaction of supergravity with chiral matter in four space-time dimensions [17]. Following these investigations in the present work we have shown, that in the quantum version the supersymmetric action in [16] describes the vacuum configuration of supergravity interacting with homogeneous complex scalar matter supermultiplets. The corresponding configuration is of the FRW model invariant under the $n=2$ local supersymmetry, which is a subgroup of the four space-time supersymmetry.

The steps to follow this line of research are: 1) the construction of the general-type superfield action interacting with two superfunctions of the kinetic term $\Phi(Z^A, \bar{Z}^{\bar{A}})$ and of the superpotential $g(Z)$; 2) Weyl rescaling of the superfields; 3) recombination of the
function \( \Phi(Z^A, \bar{Z}^\dot{A}) \) and \( g(Z) \) in the Kähler superfunction \( G(Z^A, \bar{Z}^\dot{A}) \); 4) elimination of the auxiliary fields and analysis of spontaneous breaking of supersymmetry and 5) classical and quantum Hamiltonian and supercharges.

The first three steps may be realized if we see, that the simplest way to construct the classical Lagrangian is considering the superfields on the superspace \((t, \eta, \bar{\eta})\). This classical Lagrangian describes the evolution of the bosonic and additional Grassmann degrees of freedom, which after quantization become generators of the Clifford algebra.

In the fourth step we will see, that the scalar field potential is described by Kähler function and by one arbitrary parameter. We show, that we have not one parametrical family theory with new parameter, and we have a family of vacuum states in supergravity invariant under \( n = 2 \) conformal supersymmetry, which is subgroup of space-time supersymmetry.

The plan of this paper is the following: in the second section the \( n = 2 \) local superfield formulation of the FRW model interacting with a set of spatially homogeneous complex scalar matter superfields [16] is reviewed in order to fix notation. In the third section the supersymmetric lagrangian without auxiliary fields is written, and the spontaneous breaking of local supersymmetry is analyzed. In the section four we consider the canonical formalism on the classical level, and the fundamental supersymmetric charges and the Hamiltonian are explicitly written. Section five is devoted to discussion of quantization in particular, we will see that the supersymmetric still allows ambiguity on the factor ordering of the quantum operators, as well as we note, that the specific supersymmetric quantum mechanics of our model is due to the fact, that the particles-like excitation doesn’t correspond to the scale factor \( R \).

II. SUPERFIELD FORMULATION AND SYMMETRY

We begin with the superfield action [16]

\[
S = \int \left\{ \Phi \left[ I_N^{-1} I_R D_\eta I_R D_\eta I_R - \sqrt{k} \bar{R}^2 - \frac{1}{2} \left\{ D_\tilde{\eta} \left( I_N^{-1} I_R^2 D_\eta I_R \right) - D_\eta \left( I_N^{-1} I_R^2 D_\eta I_R \right) \right\} \right] \\
+ \frac{1}{4} I_N^{-1} I_R^3 \Phi^{-1} D_\eta \Phi D_\eta \Phi + \frac{1}{2\alpha} I_N^{-1} I_R^3 \frac{\partial^2 \Phi}{\partial Z^A \partial \bar{Z}^B} \left[ D_\eta \bar{Z}^\dot{A} D_\eta Z^B + D_\eta Z^B D_\eta \bar{Z}^\dot{A} \right] \\
- \frac{1}{2\alpha} I_N^{-1} I_R^3 \Phi^{-1} \frac{\partial \Phi}{\partial Z^A} \frac{\partial \Phi}{\partial \bar{Z}^B} \left[ D_\eta \bar{Z}^\dot{A} D_\eta Z^B + D_\eta Z^B D_\eta \bar{Z}^\dot{A} \right] - \frac{2 I_R^3}{\kappa^2} |g(Z)|^\alpha \right\} d\eta d\bar{\eta} dt,
\]

(1)

with \( k = 0,1 \) stands for plane and spherical FRW, and \( \kappa^2 = 8\pi G_N \), where \( G_N \) Newton’s constant of gravity and \( \alpha \) is an arbitrary constant parameter. As it will be shown, this parameter is not fixed by the local conformal time supersymmetry. We can see from (1), that the interaction depends on two arbitrary superfunctions \( \Phi(Z^A, \bar{Z}^\dot{A}) \) and \( g(Z^A) \), which is the dimensionless superpotential. Making the following Weyl conformal transformations

\[
I_N \rightarrow \exp\left(\frac{\alpha I_K}{6}\right) I_N, \quad I_R \rightarrow \exp\left(\frac{\alpha I_K}{6}\right) I_R, \\
\Phi \exp\left(\frac{\alpha I_K}{3}\right) = -\frac{3}{\kappa^2}.
\]

we find, that the terms standing under the integration of (1) take the form
The third term in the expression (3) is identical to the right term of (4) and they are cancelled in the action. On the other hand, the last two terms in (3) are a total derivative and may be ignored. In the expression (6) we have introduced the superfunction \( G(Z^A, \bar{Z}^\bar{A}) \) as a special combination of the super-Kähler potential \( IK(Z^A, \bar{Z}^\bar{A}) \) and of the spatially homogeneous superpotential \( g(Z) \)

\[
G(Z, \bar{Z}) = IK(Z, \bar{Z}) + \log |g(Z)|^2,
\]

the Kähler superfunction \( G(Z^A, \bar{Z}^\bar{A}) \) is invariant under the super-transformations

\[
g(Z) \to g(Z) \exp f(Z), \\
IK(Z, \bar{Z}) \to IK(Z, \bar{Z}) - f(Z) - \bar{f}(\bar{Z}),
\]

where the super-Kähler potential \( IK(Z, \bar{Z}) \) is defined on the complex superfield \( Z^A \) related to \( \Phi(Z, \bar{Z}) \) (2). We denoted the derivatives of the Kähler superfunction by \( \frac{\partial G}{\partial Z^A} = G_A \equiv G_A, \frac{\partial G}{\partial Z^\bar{A}} = G_{\bar{A}} \equiv G_{\bar{A}}, \frac{\partial G}{\partial Z^B} = G_B \equiv G_B, \frac{\partial G}{\partial Z^{\bar{B}}} = G^{\bar{B}} \equiv G^{\bar{B}}, \) and the Kähler supermetric is \( G_{AB} = G_{BA} = K_{AB}, \) and their inverse \( G^{\bar{A}}_{\bar{B}} \) such as \( G^{\bar{A}}_{\bar{B}} G_{\bar{B}} = \delta^\bar{A}_\bar{B} \) can be used to define \( G^A_A G_B B = G^{\bar{A}}_{\bar{B}} G^B_{\bar{B}}. \)

So, the superfield action (1) becomes

\[
S = \int \left\{ -\frac{3}{\kappa^2} N^{-1} \mathcal{R} D_\eta \mathcal{R} D_\eta \mathcal{R} - \sqrt{\kappa} \mathcal{R}^2 - \frac{1}{2} [D_\eta (N^{-1} \mathcal{R}^2 D_\eta \mathcal{R}) - D_\eta (N^{-1} \mathcal{R}^2 D_\eta \mathcal{R})] \\
+ \frac{1}{2\kappa^2} N^{-1} \mathcal{R}^3 D_\eta \mathcal{R} [D_\eta Z^A D_\eta Z^B + D_\eta Z^B D_\eta Z^A] \right\} d\eta d\bar{\eta} dt.
\]
also possible to introduce the Weyl transformations \( \phi(z, \bar{z}) \rightarrow -\frac{3}{z^2} \exp(-\frac{z}{3}K(z, \bar{z})) \) and the vierbein \( e^\mu_a \rightarrow \exp(\frac{\alpha}{3}K(z, \bar{z})) \) with an arbitrary parameter \( \alpha \). However, the terms in the supergravity action can not be represented by the Kähler function \( G(z, \bar{z}) \), which is due to the scalar curvature term, the kinetic term in the complex fields and auxiliary fields \( A_\mu \) in the supergravity multiplet are eliminated only if \( \alpha = 1 \) [17].

The superfield action (9) is invariant under the \( n = 2 \) local conformal supersymmetric transformations of \((t, \eta, \bar{\eta})\). These transformations can be written as

\[
\begin{align*}
\delta t &= \mathcal{I}I(t, \eta, \bar{\eta}) + \frac{1}{2} \eta \mathcal{D}_\eta \mathcal{I}I(t, \eta, \bar{\eta}) - \frac{1}{2} \eta \mathcal{D}_\eta \mathcal{I}I(t, \eta, \bar{\eta}), \\
\delta \eta &= \frac{i}{2} \mathcal{D}_\eta \mathcal{I}I(t, \eta, \bar{\eta}), \quad \delta \bar{\eta} = -\frac{i}{2} \mathcal{D}_\eta \mathcal{I}I(t, \eta, \bar{\eta}),
\end{align*}
\]

with the superfunction \( \mathcal{I}I(t, \eta, \bar{\eta}) \) defined in the superspace \((t, \eta, \bar{\eta})\) as

\[
\mathcal{I}I(t, \eta, \bar{\eta}) = a(t) + i \eta \bar{\psi}'(t) + i \bar{\eta} \psi'(t) + b(t) \eta \bar{\eta},
\]

where \( D_\eta = \frac{\partial}{\partial \eta} + i \eta \frac{\partial}{\partial \bar{\eta}} \) and \( D_{\bar{\eta}} = -\frac{\partial}{\partial \bar{\eta}} - i \eta \frac{\partial}{\partial \eta} \) are the supercovariant derivatives of the global conformal supersymmetry, which have dimension \([D_\eta] = l^{-1/2}\), \(a(t)\) is a local time reparametrization parameter, \(\beta(t) = N^{-1/2} \beta(t)\) is the Grassmann complex parameter of the local conformal supersymmetric transformations (10), and \(b(t)\) is the parameter of the local \(U(1)\) rotations on the complex \(\eta\).

In order to have the component action for (9) we must expand in Taylor series the superfields \( \mathcal{N}(t, \eta, \bar{\eta}), \mathcal{R}(t, \eta, \bar{\eta}), \mathcal{Z}^A(t, \eta, \bar{\eta}) \) and \( \mathcal{Z}^A(t, \eta, \bar{\eta}) \) with respect to \(\eta, \bar{\eta}\). Due to the anticommuting properties of the \(\eta, \bar{\eta}\) we see, that this series expansion ends up with the second term, as in the case of the superfunction in (11). For the one-dimensional gravity superfield \( \mathcal{N}(t, \eta, \bar{\eta}) \) we have the following series expansion

\[
\mathcal{N}(t, \eta, \bar{\eta}) = N(t) + i \eta \bar{\psi}'(t) + i \bar{\eta} \psi'(t) + \eta \bar{\eta} V'(t),
\]

where we have introduced the reparametrization \(\psi(t) = N^{1/2} \psi(t), \bar{\psi}(t) = N^{1/2} \bar{\psi}(t)\) and \(V(t) = NV + \bar{\psi} \psi\). The superfield (12) transforms as one-dimensional vector field under the local conformal time supersymmetric transformations (10). This transformation law may be written as

\[
\delta \mathcal{N} = (\mathcal{I}I \mathcal{N}) + \frac{i}{2} D_\eta \mathcal{I}I \mathcal{N} + \frac{i}{2} D_{\bar{\eta}} \mathcal{I}I \mathcal{N}.
\]

The components of the superfield \( \mathcal{N}(t, \eta, \bar{\eta}) \) in (12) are gauge fields of the one-dimensional \(n = 2\) extended supergravity. The transformations law for the components \(N(t), \psi(t), \bar{\psi}(t)\) and \(V(t)\) of the superfield (12) may be obtained from (13)

\[
\begin{align*}
\delta N &= (aN) + \frac{i}{2} (\beta \psi + \bar{\beta} \bar{\psi}), \\
\delta \psi &= (a \psi) + D \beta - \frac{i}{2} b \psi, \\
\delta \bar{\psi} &= (a \bar{\psi}) + D \bar{\beta} + \frac{i}{2} b \psi, \\
\delta V &= (a \bar{\psi}) + \dot{b},
\end{align*}
\]

where \(D \beta = \beta + i \frac{1}{2} \beta V\) and \(D \bar{\beta} = \bar{\beta} - i \frac{1}{2} \bar{\beta} V\) are the \(U(1)\) covariant derivatives and \(\dot{b} = b - \frac{1}{2N} (\beta \bar{\psi} - \bar{\beta} \psi)\).
The Taylor series expansion for the superfield \( \mathcal{R}(t, \eta, \bar{\eta}) \) has the form
\[
\mathcal{R}(t, \eta, \bar{\eta}) = R(t) + i \eta \chi(t) + i \bar{\eta} \lambda(t) + \eta \bar{\eta} B(t),
\] (15)
where \( \chi(t) = \kappa N^{1/2} \hat{X}(t), \lambda(t) = \kappa N^{1/2} \hat{X}(t) \) and \( B(t) = \kappa N B + \frac{1}{2}(\bar{\psi} \lambda - \psi \bar{\lambda}) \). The transformation rule for the real scalar superfield \( \mathcal{R}(t, \eta, \bar{\eta}) \) under supersymmetric transformation is
\[
\delta \mathcal{R} = i \hat{R} \mathcal{R} + i \frac{1}{2} D\eta \mathcal{R} + i \frac{1}{2} D\bar{\eta} \mathcal{R}.
\] (16)
The component \( B(t) \) in (15) is an auxiliary degree of freedom (nondynamical variable), \( \lambda(t) \) and \( \bar{\lambda}(t) \) are the fermionic superpartners of the scale factor \( R(t) \). Their transformations law has the form
\[
\begin{align*}
\delta R &= a \hat{R} + i \kappa \left( \beta \hat{\lambda} + \bar{\beta} \bar{\lambda} \right), \\
\delta \lambda &= a \hat{\lambda} + i \kappa B, \\
\delta \bar{\lambda} &= a \hat{\bar{\lambda}} + \frac{\beta}{2 \kappa} \frac{D R}{N} - i \kappa B, \\
\delta B &= a \hat{B} + \frac{1}{2N} \left( \beta \hat{D} \lambda - \beta \bar{D} \bar{\lambda} \right),
\end{align*}
\] (17)
where \( DR = \hat{R} - \frac{i \kappa}{2} (\psi \hat{\lambda} + \bar{\psi} \bar{\lambda}) \), \( \hat{D} \lambda = DL\lambda - \frac{i \kappa}{2N} \psi \) and \( \hat{D} \bar{\lambda} = D\bar{\lambda} - \frac{i \kappa}{2N} \bar{\psi} \bar{\lambda} \) are the supercovariant derivatives, and \( DL\lambda = \hat{\lambda} + \frac{i}{2} \bar{V} \lambda \) and \( D\bar{\lambda} = \hat{\bar{\lambda}} - \frac{i}{2} V \bar{\lambda} \) are the \( U(1) \) covariant derivatives.

The spatially homogeneous complex scalar matter superfields \( Z^A(t, \eta, \bar{\eta}) \) and \( \bar{Z}^A(t, \eta, \bar{\eta}) \) consist of a set of spatially homogeneous matter fields \( z_A(t) \) and \( z_A(t) \) \((A = 1, 2, \ldots, n)\), four fermionic degrees of freedom \( \chi^A(t), \bar{\chi}^A(t) \), \( \phi^A(t) \) and \( \bar{\phi}^A(t) \), as well as bosonic auxiliary fields \( F^A(t) \) and \( \bar{F}^A(t) \).

The components of the matter superfields \( Z^A(t, \eta, \bar{\eta}) \) and \( \bar{Z}^A(t, \eta, \bar{\eta}) \) may be written as
\[
Z^A(t, \eta, \bar{\eta}) = z^A(t) + \eta \bar{\chi}^A(t) + i \bar{\eta} \phi^A(t) + \eta \bar{\eta} F^A(t),
\] (18)
\[
\bar{Z}^A(t, \eta, \bar{\eta}) = \bar{z}^A(t) + \eta \bar{\chi}^A(t) + i \bar{\eta} \phi^A(t) + \eta \bar{\eta} F^A(t),
\] (19)
where \( \chi^A(t) = N^{1/2} \chi^A(t), \phi^A(t) = N^{1/2} \phi^A(t), F^A(t) = NF^A - \frac{1}{2}(\psi \chi^A - \bar{\psi} \bar{\chi}^A) \) and \( \bar{F}^A(t) = NF^{\bar{A}} - \frac{1}{2}(\psi \bar{\chi}^A - \bar{\psi} \chi^A) \).

The transformation rule for the superfields \( Z^A(t, \eta, \bar{\eta}) \) and \( \bar{Z}^A(t, \eta, \bar{\eta}) \) may be written as
\[
\begin{align*}
\delta Z^A &= i \hat{Z}^A + \frac{1}{2} D\eta Z^A + \frac{1}{2} D\bar{\eta} Z^A, \\
\delta \bar{Z}^A &= i \hat{\bar{Z}}^A + \frac{1}{2} D\eta \bar{Z}^A + \frac{1}{2} D\bar{\eta} \bar{Z}^A.
\end{align*}
\] (20)
(21)

From this superfield transformations we can obtain easily the transformations law for the components of the matter superfields \( Z^A(t, \eta, \bar{\eta}) \) and \( \bar{Z}^A(t, \eta, \bar{\eta}) \). We get
\[
\begin{align*}
\delta z^A &= a \hat{z}^A + \frac{i}{2} (\beta \chi^A + \bar{\beta} \phi^A), \\
\delta \chi^A &= a \hat{\chi}^A + \frac{i}{2} \beta \chi^A + \frac{1}{2} \left( \frac{Dz^A}{N} - i F^A \right), \\
\delta \bar{\chi}^A &= a \hat{\bar{\chi}}^A + \frac{i}{2} \bar{\beta} \bar{\chi}^A + \frac{1}{2} \left( \frac{D\bar{z}^A}{N} + i F^A \right), \\
\delta \phi^A &= a \hat{\phi}^A - \frac{i}{2} \beta \phi^A + \frac{1}{2} \left( \frac{D\phi^A}{N} + i F^A \right), \\
\delta \bar{\phi}^A &= a \hat{\bar{\phi}}^A + \frac{i}{2} \bar{\beta} \bar{\phi}^A + \frac{1}{2} \left( \frac{D\bar{\phi}^A}{N} - i F^A \right), \\
\delta F^A &= a \hat{F}^A + \frac{1}{2N} \left( \beta \hat{D} \phi^A - \beta \bar{D} \bar{\chi}^A \right), \\
\delta \bar{F}^A &= a \hat{\bar{F}}^A + \frac{1}{2N} \left( \beta \hat{D} \bar{\phi}^A - \beta D \chi^A \right).
\end{align*}
\] (22)
where \( Dz^A = \frac{\partial}{\partial z^A} \), \( D\bar{z}^A = \frac{\partial}{\partial \bar{z}^A} \), \( D\phi^A = D\phi^A - \frac{i}{2}(N^{-1} Dz^A + iF^A)\psi \), and \( D\bar{\phi}^A = D\bar{\phi}^A - \frac{i}{2}(N^{-1} D\bar{z}^A - iF^A)\bar{\psi} \). The supercovariant derivatives, and \( D\phi^A = \frac{\partial}{\partial \phi^A} + \frac{i}{2}V\phi^A \), where \( \phi \) is the Kähler superfunction in the \( D\phi^A = \frac{\partial}{\partial \phi^A} + \frac{i}{2}V\phi^A \), and \( D\bar{\phi}^A = \frac{\partial}{\partial \bar{\phi}^A} + \frac{i}{2}V\bar{\phi}^A \), are the \( U(1) \) covariant derivatives.

### III. SUPERSYMMETRIC LAGRANGIAN AND SUSY BREAKING

It is clear, that the superfield action (9) is invariant under the \( n = 2 \) local conformal time supersymmetry. We can write the expression, which is found under the integral (9) by means of certain superfunction \( f(N, R, Z, \bar{Z}) \). Then, the infinitesimal small transformations of the action (9) under the superfield transformations (13,16,20,22) have the form

\[
\delta S = \frac{i}{2} \int \left\{ D\eta (IL D\eta f(N, R, Z, \bar{Z})) + D\eta (IL D\eta f(N, R, Z, \bar{Z})) \right\} d\eta dt, \tag{23}
\]

we can see, that under the integration it gives a total derivative, \( i.e \) the action (9) is invariant under the superfield transformations (13,16,20,22).

For the Kähler superfunction in (9) we have the following Taylor series expansion

\[
G(Z, \bar{Z}) = G(z, \bar{z}) + G_A(z, \bar{z})(Z^A - z^A) + G_A(z, \bar{z})(\bar{Z}^\alpha - \bar{z}^\alpha) + \frac{1}{2}G_{AB}(z, \bar{z})(Z^A - z^A)(Z^B - \bar{z}^B) + \frac{1}{2}G_{AB}(z, \bar{z})(\bar{Z}^\alpha - \bar{z}^\alpha)(\bar{Z}^\beta - \bar{z}^\beta), \tag{24}
\]

where the first term in the expansion is the ordinary Kähler function of the supergravity theories interacting with complex scalar matter supermultiplets [17]. So, making the corresponding operations in the superfield action (9), and in order to have the correct kinetic term for the “fermionic fields”, we make the following redefinition of the fields \( \lambda = \frac{\lambda}{\sqrt{R}} \), \( \chi^A = \frac{\chi^A}{\sqrt{R}} \), and \( \phi^A = \frac{\phi^A}{\sqrt{R}} \).

After integration over \( \eta, \bar{\eta} \) the superfield action (9) may be written in the usual form \( S = \int L dt \), the lagrangian \( L \) contains terms with auxiliary fields \( B(t) \), \( F(t) \) and \( \tilde{F}(t) \) of superfields \( R, Z \) and \( \bar{Z} \) respectively. We can write the lagrangian as the sum \( L = L + L_{aux} \) of lagrangian without auxiliary fields and lagrangian for the auxiliary fields respectively. Explicitly we have

\[
\tilde{L} = -\frac{3}{N\kappa^2}R(\phi^A)^2 + \frac{2i}{3}\sqrt{R} (\bar{\psi} \lambda - \psi \bar{\lambda}) + \frac{2N\sqrt{R}}{3R} (\bar{\lambda} \lambda) + \frac{R^3}{N\kappa^2} G_{AB} Dz^A Dz^B + \frac{i}{2\kappa} G_{AB} \bar{\psi}^A \bar{D} \phi^B - \frac{N}{2\kappa R^3} G_{AB} \bar{\psi}^A \bar{D} \phi^B - \frac{1}{4\kappa\sqrt{R}} (\bar{\psi} \lambda - \psi \bar{\lambda}) G_{AB} (\bar{\chi}^A \lambda^B + \phi^B \bar{\chi}^A) + \frac{N}{2\kappa R^3} (\bar{\lambda} G_{ABC} \bar{\psi}^C - \lambda G_{ABC} \phi^C) \phi^A + \frac{N}{2\kappa R^3} (\bar{\lambda} G_{ABC} \phi^C - \lambda G_{ABC} \bar{\psi}^C) \bar{\chi}^A \lambda^B
\]
\[ + \frac{N}{3R^3} G_{AB}(\tilde{\chi}^A \chi^B + \phi^B \tilde{\phi}^A) \tilde{\lambda} \lambda - \frac{4N}{3 \kappa} e^{\frac{\alpha \phi}{2}} \tilde{\lambda} \lambda - \frac{2N}{\kappa^3} (e^{\frac{\alpha \phi}{2}},_{AB} \chi^A \phi^B ) \]

\[ - 2 \frac{N}{\kappa^3} (e^{\frac{\alpha \phi}{2}},_{AB} \tilde{\phi}^A \tilde{\chi}^B - \frac{2N}{\kappa^3} (e^{\frac{\alpha \phi}{2}},_{AB} (\tilde{\chi}^A \chi^B + \phi^B \tilde{\phi}^A ) - \frac{2N}{\kappa^3} \tilde{\lambda} \left[ (e^{\frac{\alpha \phi}{2}},_{AB} \phi^A + (e^{\frac{\alpha \phi}{2}},_{AB} \tilde{\phi}^A ) \right] \]

\[ + \frac{2N}{\kappa^2} (e^{\frac{\alpha \phi}{2}},_{AB} \chi^A + (e^{\frac{\alpha \phi}{2}},_{AB} \tilde{\phi}^A )] + \sqrt{R^3} (e^{\frac{\alpha \phi}{2}},_A (\psi \tilde{\chi}^A - \tilde{\psi} \phi^A ) + \frac{\sqrt{R^3}}{\kappa^3} (e^{\frac{\alpha \phi}{2}},_A (\psi \tilde{\phi}^A - \tilde{\psi} \chi^A ) \]

\[ - \frac{1}{\kappa^2} (\psi \lambda - \tilde{\psi} \tilde{\lambda}) e^{\frac{\alpha \phi}{2}}, \]

and the lagrangian for the auxiliary fields has the form

\[ L_{aux} = L_{aux} \text{ FRW} + L_{aux} \text{ kinetic term} + L_{aux} \text{ potential term} = L_{aux} B + L_{aux} F,F; \quad (26) \]

explicitly the lagrangian for the auxiliary field \( B \)

\[ N^{-1} L_{aux} B = -3RB^2 + \left[ -\frac{\kappa}{3R} \lambda \lambda + \frac{6\sqrt{k}}{\kappa} R + \frac{3}{2\kappa R} G_{AB}(\tilde{\chi}^A \chi^B + \phi^B \tilde{\phi}^A ) - \frac{6}{\kappa^2} R e^{\frac{\alpha \phi}{2}}, \right] \quad (27) \]

and the lagrangian for the auxiliary fields \( F, \tilde{F} \) is

\[ N^{-1} L_{aux} F,\tilde{F} = F^B \left[ \frac{1}{2\kappa} G_{AB}(\lambda \tilde{\chi}^A - \lambda \tilde{\phi}^A ) + \frac{1}{\kappa} G_{AB} \tilde{\phi}^A \tilde{\chi}^A - \frac{2R^3}{\kappa^3} (e^{\frac{\alpha \phi}{2}},_B \right] \]

\[ + F^A \left[ \frac{1}{2\kappa} G_{AB}(\lambda \phi^B - \lambda \chi^B ) + \frac{1}{\kappa} G_{AB} \chi^B \phi^B - \frac{2R^3}{\kappa^3} (e^{\frac{\alpha \phi}{2}},_A \right] + \frac{R^3}{\kappa^2} G_{AB} F^A F^B \]  

The equations for the auxiliary fields \( B, F^A \) and \( \tilde{F}^A \) are algebraical and may be determined from the component action of (9) by taking the variation with respect to them. We get the following solutions for the auxiliary fields

\[ B = \frac{\kappa}{18R^2} \lambda \lambda + \frac{\sqrt{k}}{\kappa} + \frac{1}{4\kappa R^2} G_{AB}(\tilde{\chi}^A \chi^B + \phi^B \tilde{\phi}^A ) - \frac{R}{\kappa^2} e^{\frac{\alpha \phi}{2}}, \]

\[ F^D = -\frac{\kappa}{2R^3}(\lambda \phi^D - \lambda \chi^D ) - \frac{1}{R^3} G^{DA} G_{AB} \chi^B \phi^A + \frac{2}{\kappa} G^{DA} (e^{\frac{\alpha \phi}{2}},_A , \]

\[ \tilde{F}^D = -\frac{\kappa}{2R^3}(\lambda \phi^D - \lambda \chi^D ) - \frac{1}{R^3} G^{DA} G_{AB} \tilde{\phi}^A \tilde{\chi}^B + \frac{2}{\kappa} G^{DA} (e^{\frac{\alpha \phi}{2}},_B , \]

where \((\_),A\) and \((\_),\tilde{A}\) are derivatives with respect to \( z^A \) and \( \tilde{z}^A \). After substituting them into the component action we have the total supersymmetric action \( \int L dt = \int (\tilde{L} + L_{aux} B + L_{aux} F,F) dt \). We get the following expression for the total Lagrangian

\[ L = -\frac{3}{\kappa^2} R(D \tilde{R})^2 - N R^3 U(R, z, \tilde{z}) + \frac{2i}{3} \tilde{\lambda} D \lambda + \frac{N \sqrt{k}}{3R} \lambda \lambda - \frac{N}{\kappa^2} \lambda \lambda \]

\[ + \frac{\sqrt{k}}{\kappa} \sqrt{R} (\psi \lambda - \tilde{\psi} \tilde{\lambda}) + \frac{R^3}{\kappa^2} G_{AB} D \tilde{z}^A D z^B + \frac{i}{2\kappa} D z^B \left( \lambda G_{AB} \tilde{\chi}^A + \lambda G_{AB} \tilde{\phi}^A \right) \]

\[ + \frac{i}{2\kappa} D \tilde{z}^A \left( \lambda G_{AB} \phi^B + \lambda G_{AB} \chi^B \right) - \frac{i}{\kappa^2} G_{AB} \left( \tilde{\chi}^A D \chi^B + \phi^A D \phi^B \right) \]
\[-\frac{N}{k^2 R^3} R_{ABCD} \partial X^A \partial^B \partial^C \partial^D \phi^\alpha \phi^\beta - \frac{i}{4 k \sqrt{R^3}} (\bar{\psi} \lambda - \bar{\psi} \lambda) G_{\lambda \beta} \left( \bar{\lambda}^A \chi^B + \phi^B \bar{\phi}^A \right) \]

\[+ \frac{3N}{16 k^2 R^3} [G_{\lambda \beta} \left( \bar{\lambda}^A \chi^B + \phi^B \bar{\phi}^A \right)]^2 + \frac{3 \sqrt{k}}{2 k^2 R} G_{\lambda \beta} \left( \bar{\lambda}^A \chi^B + \phi^B \bar{\phi}^A \right) \]

\[- \frac{3N}{2 k^3} \partial^C G_{\alpha \beta} \left( \bar{\lambda}^A \chi^B + \phi^B \bar{\phi}^A \right) - \frac{2 N}{k^3} \left( e^{\frac{\alpha}{n^a}} \right) \chi^A \phi^B - \frac{2}{k^3} N(e^{\frac{\alpha}{n^a}}) \chi^A \bar{\phi}^B \]

\[+ \frac{N}{3 k^2} \left( e^{\frac{\alpha}{n^a}} A \chi^A + (e^{\frac{\alpha}{n^a}}) \bar{\phi}^A \right) - \frac{\sqrt{R^3}}{k^2} \left( \bar{\psi} \lambda - \bar{\psi} \lambda \right) e^{\frac{\alpha}{n^a}} \]

\[+ \frac{\sqrt{R^3}}{k^3} \left( e^{\frac{\alpha}{n^a}} \right) \left( \bar{\psi} \chi^A - \bar{\psi} \phi^A \right) + \frac{\sqrt{R^3}}{k^3} \left( e^{\frac{\alpha}{n^a}} \right) \left( \bar{\psi} \bar{\phi}^A - \bar{\psi} \chi^A \right) , \]

where \( DR, Dz^A, D\chi^B, D\phi^B \) and \( D\lambda \) are defined before \( \hat{D} \chi^B = D\chi^B + \Gamma^B_{CD} z^C \chi^D, \hat{D} \phi^B = D\phi^B + \Gamma^B_{CD} z^C \phi^D, \) and \( R_{ABCD} \) is the curvature tensor of the Kähler manifold defined by the coordinates \( z^A, \bar{z}^B \) with the metric \( G_{AB} \), and \( \Gamma^B_{CD} = G^{BA} G_{ACD} \) are the Christoffel symbols in the definition of covariant derivative and their complex conjugate. After elimination of the auxiliary fields the Lagrangian (30) only depends on covariant magnitudes, as we will show, it is a specific classical Lagrangian of supersymmetric quantum mechanics. In the Lagrangian (30) the potential term is written as

\[U(R, z, \bar{z}) = -\frac{3k}{k^2 R^2} + \frac{6 \sqrt{k}}{k^3 R} e^{\frac{\alpha}{n^a}} + V_{eff}(z, \bar{z}), \quad (31)\]

where the effective potential of the scalar matter fields is

\[V_{eff} = \frac{4}{\kappa^4} \left[ (e^{\frac{\alpha}{n^a}}) A G^{AB} (e^{\frac{\alpha}{n^a}}) D - \frac{3}{4} e^{\frac{\alpha}{n^a}} \right] = \frac{\kappa^2 G^A G_A - 3}{\kappa^4} . \quad (32)\]

In order to discuss the implication of spontaneous supersymmetry breaking we need to display the potential (31) in terms of the auxiliary fields

\[U(R, z, \bar{z}) = \frac{F^A G_{AB} F^B}{k^2 R^2} - \frac{3 B^2}{R^2} , \quad (33)\]

where the bosonic terms (29) are

\[F^A = \frac{\alpha}{\kappa} e^{\frac{\alpha}{2} \frac{z}{n^a}} G^A (z, \bar{z}) , \quad (34)\]

\[B = \frac{\sqrt{k}}{\kappa} - \frac{R}{k^2} e^{\frac{\alpha}{2} \frac{z}{n^a}} . \quad (35)\]

The supersymmetry is spontaneous breaking if the auxiliary fields (34) of the matter supermultiplets get non-vanishing vacuum expectation values. According to our assumption at the minimum in (32) for \( k = 0, V_{eff}(z_0, \bar{z}_0) = 0 \), but \( < F^A > = F^A(z_0, \bar{z}_0) \neq 0 \) and, thus, the supersymmetry is broken.
The scalar field potential (32) consists of two terms, one of them is the potential for the scalar fields in the case of the global supersymmetry. Unlike the standard supersymmetric quantum mechanics the potential is not positive-definite and allows spontaneous breaking of supersymmetry with vanishing classical vacuum energy. In this case the condition of the function \( G(z, \bar{z}) \) must be fulfilled in the minimum of the potential \( <G>=G(\bar{z_0}, z_0) \)

\[
\frac{\partial V(z, \bar{z})}{\partial z^A} \bigg|_{z^A=\bar{z}_0} = 0, \quad <G^A> <G_A> = \frac{3}{\alpha^2},
\]

where \( <G^A> = G^A(\bar{z_0}, z_0) \).

Due to the Lagrangian form (30) in the simplest case \( k=0 \), the fermion bilinear terms \( \lambda, \bar{\lambda}, \chi^A, \bar{\chi}^A, \phi^A \) and \( \bar{\phi}^A \) are also the mass terms.

If supersymmetry is broken the fermion fields \( \lambda, \bar{\lambda} \) will be described by

\[
\tilde{\lambda} = \lambda + \frac{1}{\kappa} e^{-\frac{\alpha <\bar{\phi}>}{2}} \tilde{\eta}, \quad \tilde{\bar{\lambda}} = \bar{\lambda} + \frac{1}{\kappa} e^{-\frac{\alpha <\bar{\phi}>}{2}} \tilde{\bar{\eta}},
\]

where

\[
\tilde{\eta} = <(e_{\phi}^{AB})_{A} > \phi^A + <(e_{\phi}^{AB})_{\bar{A}} > \bar{\phi}^A, \quad \tilde{\bar{\eta}} = <(e_{\phi}^{AB})_{\bar{A}} > \bar{\chi}^A + <(e_{\phi}^{AB})_{A} > \phi^A.
\]

After substitution the equations (37) and (38) into the mass term of Lagrangian (30) has the following form

\[
L_{\text{fermion mass term}} = -N \left[ \frac{1}{\kappa} e^{-\frac{\alpha <\bar{\phi}>}{2}} \tilde{\lambda} + m_{AB} \chi^A \phi^B + m_{\bar{A}B} \bar{\chi}^\bar{A} \bar{\phi}^B \right. \]
\[
+ m_{AB} (\bar{\chi}^\bar{A} \chi^B + \phi^B \bar{\phi}^\bar{A}]),
\]

where \( m_{AB}(\bar{z}_0, z_0), m_{AB}(\bar{z}_0, z_0) \) and \( m_{AB}(\bar{z}_0, z_0) \) are the mass matrices depending on \( z_0^A = <z^A(t)> \) vacuum expectation values of the scalar fields in the minimum of the potential \( V_{eff}(\bar{z}_0, z_0) = 0 \).

For the mass matrices we have the representation

\[
m_{AB} = e^{-\frac{\alpha <\bar{\phi}>}{2}} \left[ \frac{\alpha^2}{4} <G_A> <G_B> + \alpha <G_{AB}> \right],
\]

\[
m_{\bar{A}B} = e^{-\frac{\alpha <\bar{\phi}>}{2}} \left[ \frac{\alpha^2}{4} <G_{\bar{A}}> <G_B> + \alpha <G_{\bar{A}B}> \right],
\]

\[
m_{\bar{A}B} = e^{-\frac{\alpha <\bar{\phi}>}{2}} \left[ \frac{\alpha^2}{4} <G_{\bar{A}}> <G_B> + (\alpha + \frac{3}{\alpha}) <G_{\bar{A}B}> \right].
\]

The ordering parameter of spontaneous breaking of supersymmetry is given by the coefficient of \( \bar{\lambda} \lambda \) term. From (39) we identify

\[
\frac{1}{\kappa} e^{-\frac{\alpha <\bar{\phi}>}{2}} = m_{3/2},
\]

as the gravitino mass in the effective supergravity theories [17].
IV. CANONICAL FORMULATION AND THE CONSTRAINTS

Now, we will proceed with the Hamiltonian analysis of this system. For this purpose we need to write the momenta conjugate to dynamical variables \( R(t), z^A(t) \) and \( \bar{z}^\dagger(t) \)

\[
\pi_R = -\frac{6}{N\kappa^2} R(\partial R)
\]

\[
\pi_A(z) = \frac{R^3}{N\kappa^2} G_{BA} Dz^B + \frac{i}{2\kappa} (\lambda G_{BA} \chi^B + \lambda G_{BA} \phi^B) - \frac{i}{2\kappa^2} G_{MB} (\Gamma^B_{AD} \chi^D + \Gamma^B_{AD} \phi^D),
\]

\[
= p_A(z) - \frac{i}{2\kappa^2} G_{MB} (\Gamma^B_{AD} \chi^D + \Gamma^B_{AD} \phi^D) \tag{42}
\]

\[
\bar{\pi}_A(z) = \frac{R^3}{N\kappa^2} G_{AB} Dz^B + \frac{i}{2\kappa} (\lambda G_{AB} \phi^B + \lambda G_{AB} \chi^B) + \frac{i}{2\kappa^2} G_{MB} (\Gamma^B_{AD} \chi^D + \Gamma^B_{AD} \phi^D) \]

\[
= \bar{p}_A(z) + \frac{i}{2\kappa^2} G_{MB} (\Gamma^B_{AD} \chi^D + \Gamma^B_{AD} \phi^D),
\]

where \( p_A(z) \) and \( \bar{p}_A(z) \) are the covariant momenta. With respect to the canonical Poisson brackets we have

\[
\{ R, \pi_R \} = -1, \quad \{ z^B, \pi_A \} = -\delta_A^B, \quad \{ \bar{z}^\dagger, \bar{\pi}_A \} = -\delta_A^B. \tag{43}
\]

For the dynamical Grassmann variables \( \lambda(t), \chi(t) \) and \( \phi(t) \) we have the following constraints

\[
\Pi_{\lambda} \equiv \pi_\lambda + \frac{i}{3} \bar{\lambda} \approx 0, \quad \Pi_{\lambda} \equiv \pi_\lambda + \frac{i}{3} \bar{\lambda} \approx 0,
\]

\[
\Pi_A(\chi) \equiv \pi_A(\chi) - \frac{i}{2\kappa^2} G_{BA} \chi^B \approx 0, \quad \Pi_A(\chi) \equiv \pi_A(\chi) - \frac{i}{2\kappa^2} G_{BA} \chi^B \approx 0, \tag{44}
\]

\[
\Pi_A(\phi) \equiv \pi_A(\phi) - \frac{i}{2\kappa^2} G_{BA} \phi^B \approx 0, \quad \Pi_A(\phi) \equiv \pi_A(\phi) - \frac{i}{2\kappa^2} G_{BA} \phi^B \approx 0,
\]

where \( \pi_\lambda = \frac{\partial}{\partial \lambda} \), \( \pi_A(\chi) = \frac{\partial}{\partial \chi} \) and \( \pi_A(\phi) = \frac{\partial}{\partial \phi} \) are the momenta conjugate to the anticommuting variables \( \lambda(t), \chi(t) \) and \( \phi(t) \) respectively. The constraints (44) are of the second class, and, therefore, they can be eliminated by the Dirac procedure. We define the matrix constraints

\[
C_{\lambda\lambda} = \frac{2}{3} i, \quad C_{\lambda B}(\chi) = -\frac{i}{\kappa^2} G_{\lambda B}, \quad C_{\lambda B}(\phi) = -\frac{i}{\kappa^2} G_{\lambda B}, \tag{45}
\]

and their inverse matrices as

\[
C_{\lambda\lambda}^{-1} = (\frac{3}{2}) i, \quad C_{\lambda B}(\chi) = i\kappa^2 G_{\lambda B}, \quad C_{\lambda B}(\phi) = i\kappa^2 G_{\lambda B}. \tag{46}
\]

The Dirac brackets \( \{ , \}^\dagger \) are then defined by

\[
\{ f, g \}^\dagger = \{ f, g \} - \{ f, \Pi_i \} (C_{i k}^{-1})^{ik} \{ \Pi_k, g \}. \tag{47}
\]

The result of the Dirac procedure is the elimination of the momenta conjugate to the fermionic variables leaving us with the following non-zero Dirac bracket relations
\{ R, \pi_R \}^* = \{ R, \pi_R \} = -1, \quad \{ z_A, \pi^*_A \} = \{ z_A, \pi^*_A \} = -\delta^B_A,
\{ \bar{z}_A, \bar{\pi}^*_A \} = \{ \bar{z}_A, \bar{\pi}^*_A \} = -\delta^B_A, \quad \{ \lambda^A, \bar{\lambda}^B \} = -i\kappa^2 G^{AB},
\{ \phi^A, \bar{\phi}^B \}^* = -i\kappa^2 G^{AB}, \quad \{ \lambda, \bar{\lambda} \}^* = \frac{3}{2}i. \tag{48}

The canonical Hamiltonian is the sum of all the first-class constraints

\[ H_c = NH + i\frac{\psi}{2} \bar{S} + i\frac{\bar{\psi}}{2} S + \frac{V}{2} \mathcal{F}, \tag{49} \]

where \( H \) is the classical Hamiltonian of the system written as

\[
H = -\frac{\kappa^2}{12R^2} \pi^2_R + R^3 U(R, z, \bar{z}) - \frac{\sqrt{k}}{3R} \bar{\lambda} \lambda + \frac{\kappa^2}{R^3} \bar{p}_A G^{AB} p_B - \frac{i\kappa}{2R^3} \bar{\lambda} \bar{\lambda} + \lambda \bar{\lambda}^A + \lambda \bar{\lambda}^A - \frac{3}{16\kappa^2 R^3} [G^{AB} \lambda^A \lambda^B + \phi^A \phi^B]^2
\]
\[
- \frac{3\sqrt{k}}{2k^2 R} G^{AB} \bar{\lambda} \lambda^B + \phi^A \phi^B + \frac{3}{2k^3} e^{\frac{a_b G}{2}} G^{AB} \lambda^A \lambda^B + \phi^A \phi^B + \frac{1}{k^2} [G^{AB} \lambda^A \lambda^B + \phi^A \phi^B]^2
\]
\[
+ \frac{2}{k^3} \left( e^{\frac{a_b G}{2}} \right) \bar{\lambda} \lambda^B + \frac{2}{k^3} \left( e^{\frac{a_b G}{2}} \right) \lambda \lambda^B + \frac{2}{k^2} \left( e^{\frac{a_b G}{2}} \right) \lambda \lambda^B
\]
\[
- \frac{1}{k^2} \left[ \lambda \left( e^{\frac{a_b G}{2}} \right) \lambda \lambda^B + \frac{1}{k^2} \left( e^{\frac{a_b G}{2}} \right) \lambda \lambda^B \right]. \tag{50}
\]

\( S \) and \( \bar{S} \) are the classical supersymmetric generators of our model

\[
S = \left[ \frac{\kappa}{3\sqrt{R}} \pi_R + \frac{2i}{\kappa} \frac{\sqrt{k}}{R} \sqrt{\bar{\lambda} \lambda^A} \right] \lambda
+ \frac{1}{\sqrt{R^3}} \left[ p_C + \frac{2i}{\kappa} \frac{\sqrt{k}}{R} \sqrt{\bar{\lambda} \lambda^A} \right] \phi^C
+ \left[ \frac{1}{\sqrt{R^3}} \bar{p}_C + \frac{2i}{\kappa} \frac{\sqrt{k}}{R} \sqrt{\bar{\lambda} \lambda^A} \right] \bar{\lambda}^C, \tag{51}
\]

\[
\bar{S} = \left[ \frac{\kappa}{3\sqrt{R}} \pi_R + \frac{2i}{\kappa} \frac{\sqrt{k}}{R} \sqrt{\bar{\lambda} \lambda^A} \right] \lambda
+ \frac{1}{\sqrt{R^3}} \left[ \bar{p}_C + \frac{2i}{\kappa} \frac{\sqrt{k}}{R} \sqrt{\bar{\lambda} \lambda^A} \right] \bar{\phi}^C
+ \left[ \frac{1}{\sqrt{R^3}} p_C + \frac{2i}{\kappa} \frac{\sqrt{k}}{R} \sqrt{\bar{\lambda} \lambda^A} \right] \bar{\lambda}^C. \tag{52}
\]

and \( \mathcal{F} \) is the classical \( U(1) \) rotation generator

\[
\mathcal{F} = -\frac{2}{3} \bar{\lambda} \lambda + \frac{G^{AB}}{\kappa^2} \left( \lambda \lambda^A \lambda^B + \phi^A \phi^B \right). \tag{53}
\]

These first-class constraints are obtained from the component action (9) varying \( N(t), \psi(t), \bar{\psi}(t) \) and \( V(t) \).

We see from (42), that the canonical momenta are written as

\[
p_A(z) = \pi_A + \frac{i}{2\kappa^2} G_{AB} (\Gamma^B_A \bar{\lambda} \lambda^D + \Gamma^B_A \bar{\phi} \phi^D),
\]
\[
\bar{p}_A(z) = \bar{\pi}_A - \frac{i}{2\kappa^2} G_{AB} (\Gamma^B_A \bar{\lambda} \lambda^D + \Gamma^B_A \bar{\phi} \phi^D), \tag{54}
\]

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for which we have the following non-zero Dirac brackets

\[
\{p_A, \bar{p}_B\}^s = -\frac{i}{\kappa^2} R_{ABCD} (\chi^C \chi^D + \phi^C \phi^D),
\]

\[
\{p_A, \chi^C\}^s = \frac{1}{2} G_{CB} \Gamma^B_A \chi^D, \quad \{\bar{p}_A, \chi^C\}^s = \frac{1}{2} G_{AB} \Gamma^B_A \chi^D, \quad (55)
\]

\[
\{p_A, \phi^C\}^s = \frac{1}{2} G_{CB} \Gamma^B_A \phi^D, \quad \{\bar{p}_A, \phi^C\}^s = \frac{1}{2} G_{AB} \Gamma^B_A \phi^D.
\]

In terms of those Dirac brackets (48) and (55) we have a closed super-algebra of the conserving even charges \(H, \mathcal{F}\) and odd supercharges \(S, \bar{S}\),

\[
\{S, \bar{S}\}^s = -2iH, \quad \{S, H\}^s = \{\bar{S}, H\}^s = 0, \quad \{\mathcal{F}, S\}^s = i\bar{S},
\]

\[
\{\mathcal{F}, \bar{S}\}^s = -iS, \quad \{\mathcal{F}, H\}^s = 0, \quad \{S, S\}^s = 0, \quad \{\bar{S}, \bar{S}\}^s = 0.
\]

\[
\text{V. QUANTIZATION OF THE MODEL}
\]

On the quantum level we replace the Dirac brackets (48) by anticommutators if both arguments are odd, and by commutators if they are both even

\[
\{O_1, O_2\} = i\{O_1, O_2\}^s, \quad [E, O] = i[E, O]^s, \quad [E_1, E_2] = i[E_1, E_2]^s, \quad (57)
\]

in particular, this gives the following non-zero commutation relations for the Dirac brackets

\[
[R, \pi_R] = -i, \quad [\bar{z}^A, \pi_B] = -i\delta^A_B, \quad [\bar{z}^A, \bar{\pi}_B] = -i\delta^A_B,
\]

\[
\{\lambda, \bar{\lambda}\} = -\frac{3}{2}, \quad \{\chi^A, \bar{\chi}_B\} = \kappa^2 \delta^A_B, \quad \{\phi^A, \bar{\phi}_B\} = \kappa^2 \delta^A_B.
\]

where \(\bar{\lambda}, \bar{\chi}_B\) and \(\bar{\phi}_B\) are hermitian conjugates to \(\lambda, \chi_B\) and \(\phi_B\) with respect to operation \((\chi^A)^i = \bar{\chi}_A, (\phi^B)^i = \bar{\phi}_B, \bar{\chi}_A G^{A\tilde{A}} = \bar{\chi}^\tilde{A}\) and \(\bar{\phi}_B G^{A\tilde{A}} = \bar{\phi}^\tilde{A}\). In the quantum theory the first class-constraints (50-53) associated with the invariance of the action (30) become conditions on the wave function \(\Psi(R, z, \lambda, \chi, \phi)\). So, that any physically allowed states must obey the following quantum constraints

\[
H|\Psi\rangle = 0, \quad S|\Psi\rangle = 0, \quad \bar{S}|\Psi\rangle = 0, \quad \mathcal{F}|\Psi\rangle = 0, \quad (59)
\]

where the first equation in (59) is the so-called Wheeler-DeWitt equation for minisuperspace models.

The quantum generators \(H, S, \bar{S}\) and \(\mathcal{F}\) form a closed super-algebra of the supersymmetric quantum mechanics under the Dirac brackets (48) and (55).

\[
\{S, \bar{S}\} = 2H, \quad [S, H] = [\bar{S}, H] = 0, \quad [\mathcal{F}, S] = -S,
\]

\[
[\mathcal{F}, \bar{S}] = -S, \quad [\mathcal{F}, H] = 0, \quad S^2 = \bar{S}^2 = 0, \quad (60)
\]

where \(H\) is the Hamiltonian, \(S\) is the single complex supersymmetric operator and \(\mathcal{F}\) is the fermion number operator. The super-algebra (60) doesn’t define a positive-definite Hamiltonian.
In the usual canonical quantization the even canonical variables are replaced by operators

\[ R \rightarrow R, \quad \pi_R = i \frac{\partial}{\partial R}, \quad z^A \rightarrow z^A, \quad \pi_A = i \frac{\partial}{\partial z^A}, \tag{61} \]

doing the odd variables \( \lambda, \bar{\lambda}, \chi^A, \bar{\chi}^A, \phi^A \) and \( \bar{\phi}^A \), which obey the Dirac brackets (48) after quantization become anticommutators. We can fulfill them on the Fock space representation with \( \bar{\lambda}, \bar{\chi}^A, \bar{\phi}^A \) as a creation, and \( \lambda, \chi^A \) and \( \phi^A \) as annihilation operators with \( |0 > \) vacuum of the Fock space, so that \( \lambda|0 > = \chi^A|0 > = \phi^A|0 > = 0 \) and the general quantum states can be written as the vectors depending on \( R, z_A, \) and \( z_A^* \) in the corresponding Fock space. There is other approach [18], which ensures the canonical anticommutation rules (58). Now, we can write \( \lambda, \bar{\lambda}, \chi^A, \bar{\chi}^A, \phi^A \) and \( \bar{\phi}^A \) in the form of the direct product of \( 1 + 2n \) matrix \( 2 \times 2 \), then, we obtain the following matrix realization for the case of \( n \) complex matter supermultiplets

\[
\lambda = \sqrt{\frac{3}{2}} \sigma^{(\pm)}_1 \otimes 1_2 \otimes \ldots \otimes 1_{2n+1}, \quad \lambda\dagger = \sqrt{\frac{3}{2}} \sigma^{(\pm)}_1 \otimes 1_2 \otimes \ldots \otimes 1_{2n+1}, \tag{62}
\]

down, the odd index in the direct product of the matrix shows the place of the matrix \( i = 1, 2, \ldots, n \), \( \sigma^\pm = \frac{\sigma^1 \pm \sigma^2}{2} \) with \( \sigma^1, \sigma^2 \) and \( \sigma^3 \) Pauli matrices.

When classical variables \( H, S, S\dagger \) and \( \mathcal{F} \) become operators on the quantum level we must consider the nature of the Grassmann variables \( \lambda, \lambda^\dagger, \chi^A, \bar{\chi}^A, \phi^A \) and \( \bar{\phi}^A \), and with respect to those ones we perform the anticommutarization, \( i.e. \), we can write the bilinear combination in the form of commutators, \( \lambda^\dagger \lambda = \frac{1}{2}[\lambda^\dagger, \lambda] \). To obtain the quantum expression for the hermitian \( H \) and for the supercharges \( S \) and \( S\dagger \) we must solve the operator ordering ambiguity. Such ambiguities always arise, when the operator expression contains the product of non-commuting operators \( R, \pi_R, z^A \) and \( \pi_A \) in our case [18]. Technically it means the following: for the quantum supercharges we take the same order that for the operator in (50). Then, we must integrate with measure \( R^{1/2}(\det G_{AB})^{-1/2}dR^p \alpha^p \), \( \beta^a \) in the inner product of two states. In this measure the momenta hermitian-conjugate \( \pi_R = i \frac{\partial}{\partial R} \) is non-hermitian with \( \pi_R^\dagger = R^{-1/2} \pi_R R^{1/2}, \) however, the combination \( (R^{-1/2} \pi_R^\dagger)^\dagger = \pi_R^\dagger R^{-1/2} = R^{-1/2} \pi_R \) is hermitian. The canonical momenta \( \pi^\dagger \) hermitian-conjugate to \( \pi_A = i \frac{\partial}{\partial x^A} \) have the form \( (\pi_A)^\dagger = g^{-1/2}(\pi_A)g^{1/2} \), where \( g = \det G_{AB} \) and \( \pi_A = i \frac{\partial}{\partial x^A} \) such a procedure leads in our case to the following expression for supercharges \( S, S\dagger \) and the fermionic number operator \( \mathcal{F} \)

\[
S = \{ \frac{K}{3} R^{-1/2} \pi_R + \frac{2i}{K} \sqrt{k} R R^{1/2} - \frac{2i}{K^2} R^{3/2} e^{\frac{\alpha g}{2}} + \frac{i R^{3/2}}{4K} [\bar{\chi}^A, \chi^A] + \frac{i R^{3/2}}{4K} [\phi^A, \bar{\phi}^A] \} \lambda 
\]
\[
+ [R^{-3/2} \mathcal{F}_C - \frac{2i R^{3/2}}{K^3} (e^{\frac{\alpha g}{2}}) C] \phi^C + [R^{-3/2} \mathcal{F}_C - \frac{2i R^{3/2}}{K^3} (e^{\frac{\alpha g}{2}}) C] \bar{\phi}^C, \tag{63}
\]

\[
S\dagger = \{ \frac{K}{3} R^{-1/2} \pi_R - \frac{2i}{K} \sqrt{k} R R^{1/2} + \frac{2i}{K^2} R^{3/2} e^{\frac{\alpha g}{2}} \} \lambda 
\]
\[
+ [R^{-3/2} \mathcal{F}_C + \frac{2i R^{3/2}}{K^3} (e^{\frac{\alpha g}{2}}) C] \phi^C + [R^{-3/2} \mathcal{F}_C + \frac{2i R^{3/2}}{K^3} (e^{\frac{\alpha g}{2}}) C] \bar{\phi}^C, \tag{64}
\]
\[ \mathcal{F} = \frac{1}{2} \left( \frac{2}{3} [\lambda^\dagger, \lambda] + \frac{1}{\kappa^2} [\bar{\chi}, \chi^A] + \frac{1}{\kappa^2} [\phi^A, \bar{\phi}_A] \right), \]  

(65)

where \( p_A = i \frac{\partial}{\partial x^A} + \frac{i}{\kappa^2} \Gamma^D_A [\bar{\chi}, \chi^D] + [\bar{\phi}_B, \phi^B] \), \( \bar{p}^C = \bar{p}_B G^{BC} \) and \( \{ \cdot \}^C = ( \cdot ) B G^{BC} \). Note, that \( (p_A)^\dagger = g^{-1/2} \bar{p}_A g^{1/2} \), then, the anticommutation relation \( \{ S, S^\dagger \} = 2H \) and \( S^2 = S^\dagger = 0 \) fix all additional terms and define the quantum Hamiltonian, but in this case the operational expression \( \frac{\kappa^2}{12} (R^{-1/2} \pi_R R^{-1/2} \pi_R) \) corresponding to the energy of the scale factor \( R \) contributes to positive in the Hamiltonian, as well as to the energy of the scalar fields. As we can see from classical Hamiltonian (50) the energy of the scale factor is negative, this is due to the fact, that the particle-like fluctuations don’t correspond to the scale factor. This is reflected in the fact, that the anticommutator value \( \{ \lambda, \bar{\lambda} \} = -\frac{3}{2} \) of superpartners \( \lambda, \bar{\lambda} \) of the scale factor \( R \) is negative, unlike anticommutation relations (48), which are positive. Anticommutation relations may be fulfilled under the conditions

\[ \bar{\lambda} = -\lambda^\dagger, \quad (\chi^A)^\dagger = \bar{\chi}_A, \quad (\phi^A)^\dagger = \bar{\phi}_A, \]  

(66)

where \( \{ \lambda, \lambda^\dagger \} = \frac{3}{2} \). Then, the equation may be written in the form

\[ \lambda = \xi^{-1} \lambda^\dagger \xi, \quad \bar{\chi}_A = \xi^{-1} (\chi^A)^\dagger \xi, \quad \bar{\phi}_A = \xi^{-1} (\phi^A)^\dagger \xi. \]  

(67)

In order to have a consistence with the expression (66) and (67) it is necessary, that the operator \( \xi \) possess the following properties

\[ \lambda^\dagger \xi = -\xi \lambda^\dagger, \quad (\chi^A)^\dagger \xi = \xi (\chi^A)^\dagger, \quad (\phi^A)^\dagger \xi = \xi (\phi^A)^\dagger. \]  

(68)

The operators \( \lambda, \bar{\chi}_A \) and \( \bar{\phi}_A \) will be conjugate to operators \( \lambda, \chi^A \) and \( \phi^A \) under the inner product of two states

\[ < \Psi_1, \Psi_2 >_\xi = \int \Psi_1^\dagger \xi \Psi_2 R^{1/2} g^{1/2} dR d^nz d^n \bar{z}, \]  

(69)

which in general is non-positive. In the matrix realization the operator \( \xi \) has the form

\[ \xi = \sigma_1^{(3)} \otimes 1_2 \otimes \ldots \otimes 1_{2n+1}, \]  

(70)

and it can be written as a difference of two projection operators \( p_+ = \frac{1}{2}(1 + \xi) \) and \( p_- = \frac{1}{2}(1 - \xi) \). On the other hand, when the states fulfill equation (59) with zero energy on subspace of Fock space with vacuum \( p_+ |0> = 0 > \), the inner product (69) is positive-definite.

So, for the supercharge operator \( S \) we can construct conjugation (69) under the operator \( \bar{S} \) with the help of the following equation

\[ \bar{S} = \xi^{-1} S^\dagger \xi. \]  

(71)

In the general case any arbitrary operator \( L \) conjugate with respect to (69) has the form \( \bar{L} = \xi^{-1} L^\dagger \xi \), the same operator \( \bar{\xi} \) is conjugate \( \bar{\xi} = \xi^{-1} \bar{\xi}^\dagger \xi \), if the condition \( \xi = \xi^\dagger \) is fulfilled.

We can see, that the anticommutator of supercharge \( S \) and their conjugate \( \bar{S} \) under our conjugate operation has the form

\[ \{ S, \bar{S} \} = \xi^{-1} \{ S, \bar{S} \}^\dagger \xi = \{ S, \bar{S} \}, \]  

(72)
and it is a self-conjugate operator.

As a consequence of the algebra (60) we obtain, that the Hamiltonian $H$ is a self-conjugate operator $\hat{H} = \xi^{-1}H\xi = H$ and its value will be real. Then, the quantum Hamiltonian will have the form of the classical Hamiltonian (50) with antisymmetrization under fermionic operator and with representation (55) for the momenta covariant operator $p_\alpha$ and $\bar{p}_\alpha$. The kinetical term for the scale factor $R$ and matter fields will have the following order form

$$-\frac{\kappa^2}{12}R^{-1/2}\pi_R R^{-1/2}\pi_R + \frac{\kappa^2}{8\pi}g^{-1/2}\bar{p}_A g^{1/2}G^{AB}p_B.$$  

(73)

Note, that the super-algebra (60) does not define positive-definite Hamiltonian in a full agreement with the circumstance, that the potential $V(z, \bar{z})$ of scalar fields is not positive semi-definite in contrast with the standard quantum mechanics.

VI. CONCLUSION

The Grassmann components of the vacuum configuration with the FRW metric may be obtained by decomposition of the Rarita-Schwinger field in the following way: commuting covariant constant spinors $\lambda_\alpha(x^i)$ and $\bar{\lambda}_\alpha(x^i)$ are fixed on the configuration space, and on the other hand, time-like depending variables are not spinors. The time-like components of the Rarita-Schwinger field may be written as

$$\psi_0^\alpha(x^i, t) = \lambda^\alpha(x^i)\psi(t), \quad \bar{\psi}_0^\alpha(x^i, t) = \bar{\lambda}^\alpha(x^i)\bar{\psi}(t).$$

(74)

The spatially components of the Rarita-Schwinger field has the following representation corresponding to the direct product time-subspace on three-space of the fixed spatial configuration (in our case it is a plane or a three sphere). Explicitly we get

$$\psi_\alpha(x^i, t) = e^{(\mu)}_m \sigma_{(\mu)}^\alpha \lambda_\beta(x^i)\bar{\lambda}(t), \quad \bar{\psi}_m^\alpha(x^i, t) = e_{(\mu)}^m \sigma_{(\mu)}^\alpha \lambda_\beta(x^i)\lambda(t),$$

(75)

where $e^{(\mu)}_m(x^i, t)$ are the tetrad for the FRW metric. Those fermion representation are solutions of the supergravity equations.

Hence, specific quantum supersymmetric mechanics corresponding to quantum level in our models define the structure, which permits the fundamental quantum states invariant under the $n = 2$ local conformal supersymmetry in $N = 1$ supergravity interacting with a set of matter fields [17]. In our case the small supersymmetry is a subgroup of the space-time supersymmetry. In the small supersymmetry the parameter $\alpha$ is not necessarily $\alpha = 1$, and the mechanism of spontaneous breaking of local small supersymmetry induces the general mechanism of spontaneous breaking of supersymmetry in the supergravity theories.

In our case the constraints on the wave function of the universe permit the existence of non-trivial solutions, unlike the standard formulation on minisuperspace models, in which the Lorentz constraint is present [10]. The Lorentz constraints imposes many conditions on the wave function, which can lead to trivial solutions [10,19].

The next step is to find non-trivial wave function of the universe for different set of fields including dilaton in the spontaneous breaking phase, as well as to establish dependence of the universe parameters with parameter of spontaneous breaking of supersymmetry gravitino
mass. This wave function will be a vector-state with zero energy in the supergravity theories or in the effective superstring theory.

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REFERENCES