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Experiments on Nucleon Polarizabilities

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Abstract. Virtual Compton Scattering allows us to measure the generalized polarizabilities of the proton. For the first time, some VCS experiments are investigated at MAMI, MIT-Bates and Jefferson Lab in different ranges of Q^2 . The experimental method used to extract these new observables is presented in these different experimental cases. A complete experiment requires double polarization measurements.

Virtual Compton Scattering (VCS) is a fundamental exclusive reaction off the proton which provides us with a new insight of the internal structure of the nucleon. Below π^0 production threshold ($M_p < \sqrt{s} < M_p + M_\pi$, but arbitrary Q^2), this experiment measures the generalized polarizabilities of the proton defined for the first time by Guichon *et al.* [1]. They are fundamental electromagnetic observables of the nucleon, functions of Q^2 , and are complementary to the elastic form factors. At $Q^2 = 0$ with real photons, we get the usual electric (α) and magnetic (β) polarizabilities. Many real Compton experiments have been devoted to these measurements, and improvement on the accuracy (notably on β) is still under way. This paper deals only with virtual Compton scattering which is now investigated at MIT-Bates [2], MAMI [3] and CEBAF [4] in different ranges of Q^2 : 0.05, 0.33, 1. to 2. GeV^2 respectively. The experimental method and the advantages of the different kinematical parameters chosen in these experiments are discussed. A complete experiment to extract these new observables would need double polarization measurements and is presented in the last part.

1 Virtual Compton Scattering and Generalized Polarizabilities

The process of virtual Compton scattering off the proton is presently reached by the reaction $ep \rightarrow e'p'\gamma'$, which gives a background of Bethe-Heitler (BH) scattering together with the VCS signal.

This exclusive reaction $ep \rightarrow e'p'\gamma'$ is defined by five kinematical independent variables : ϵ , Q^2 , s , (these three ones depend only on the electron kinematics), $\theta_{\gamma\gamma}$ (angle between the two photons) and ϕ (azimuthal angle of the electron plane with respect to the hadron plane). Since we are in a kinematical regime defined by a center of mass (CM) energy $\sqrt{s}$ of the final proton-photon system close to threshold and below pion photoproduction

threshold, it is natural to work in the CM frame and to use q and q' the modulus of the momentum of the real and virtual photon respectively, instead of Q^2 and s . q' varies from 0 to ~ 125 MeV/c. The relations between the two sets of variables are :

$$q' = \frac{(s - m^2)}{2\sqrt{s}} \quad q = (Q^2 + \frac{(s - Q^2 - m^2)^2}{4s})^{\frac{1}{2}} \quad (1)$$

The cross section has the form :

$$\frac{d^5\sigma}{dk'_{lab}[d\Omega_e]_{lab}[d\Omega_p]_{CM}} = \frac{(2\pi)^{-5}}{64m} \left(\frac{k'_{lab}}{k_{lab}} \right) \frac{s - m^2}{s} \times \mathcal{M}^{exp} \quad (2)$$

where $\mathcal{M}^{exp}$ is invariant under a Lorentz transformation.

Fig. 1. The $p(e, e'p)\gamma$ reaction. The initial, final electron and initial, final proton quadri-momenta are k, k' and p, p' respectively. The final photon quadri-momentum is q' . In the one photon exchange approximation, a) and b) correspond to the Bethe-Heitler (BH) process. c) corresponds to the Virtual Compton Scattering (VCS) process. We note q the quadri-momentum of the virtual photon exchanged in the VCS process, that is $q = k - k'$ ($Q^2 = -q^2$).

In this reaction, the final photon can be emitted either by the electron or by the proton (see Fig. 1). The first process is described by Bethe-Heitler (BH) amplitude, calculable from Quantum Electro-Dynamics (QED) if the proton form factor are well known. The second process is described by the Virtual Compton Scattering (VCS) amplitude that can be split in two parts : the Born term containing only the nucleon and anti-nucleon contributions

(exactly calculable) and the Non-Born term related to the excited states. The probability $\mathcal{M}^{exp}$ (which is the differential cross section divided by a phase space factor) is therefore a coherent sum of the different amplitudes,

$$\mathcal{M}^{exp} = \frac{1}{4} \sum_{spin} |T^{BH} + T^{VCS}|^2 = \frac{1}{4} \sum_{spin} |T^{BH} + T^{Born} + T^{NonBorn}|^2 \quad (3)$$

The low energy theorem from Low [5] says that in an expansion in powers of the outgoing photon energy (q') in the final photon-proton center of mass system, the first term of the amplitudes T^{BH} and T^{Born} is of the order q'^{-1} (well-known infra-red divergence), while the first term of $T^{NonBorn}$ is of the order q'^1 . Then the two first terms of the probability $\mathcal{M}^{exp}$, of the order q'^{-2} and q'^{-1} are only due to the interference of the BH and Born amplitudes and are completely calculable. They may be expressed by the global properties of the nucleon, i.e. charge, mass and anomalous magnetic moment. The effect of the internal structure of the nucleon appears from the q'^0 term. The leading order of the Non-Born amplitude is parametrized by Guichon *et al.* [1] with 10 generalized polarizabilities (GPs) as functions of the initial photon momentum q . The restriction to terms in q'^1 corresponds to the dipole approximation, i.e. the outgoing real photons have $E1$ or $M1$ radiation. Selection rules of parity and angular momentum conservation give only 10 GPs, 3 scalar $P^{(01,01)0}$, $P^{(11,11)0}$, $\hat{P}^{(01,1)0}$ and 7 vector (or spin) polarizabilities $P^{(01,01)1}$, $P^{(01,12)1}$, $\hat{P}^{(01,1)1}$, $P^{(11,11)1}$, $P^{(11,00)1}$, $P^{(11,02)1}$, $\hat{P}^{(11,2)1}$. According to Guichon *et al.* they are noted $P^{(\rho'L',\rho L)S}$ or $\hat{P}^{(\rho'L',L)S}$, where $L(L')$ denotes the initial (final) photon orbital angular momentum, $\rho(\rho')$ the type of multipole transition [$0 = C$ (Coulomb), $1 = M$ (magnetic), $2 = E$ (electric)], and S distinguishes between non-spin-flip ($S = 0$) and spin-flip ($S = 1$) transitions and gives the scalar GPs and the vector or spin GPs respectively. Since the electric ($\rho = 2$) and Coulomb ($\rho = 0$) multipoles are related in the low energy limit by gauge invariance, Guichon *et al.* expressed the electric transition ($\rho = 2$) by the Coulomb ($\rho = 0$) one plus a remainder, which leads to the so-called mixed polarizabilities $\hat{P}^{(\rho'L',L)S}$. However it has been proved by Drechsel *et al.* [6], [7] that only six of the above 10 GPs are independent, if one requires combined crossing and charge-conjugation symmetries to hold. For example we have a relation between the 3 scalar GPs :

$$-\frac{3q^2}{2\tilde{q}_0} \hat{P}^{(01,1)0}(q) = \sqrt{3/2} P^{(01,01)0}(q) + \sqrt{3/8} P^{(11,11)0}(q) \quad (4)$$

where

$$\tilde{q}_0 = m - \sqrt{m^2 + q^2} \quad (5)$$

is the virtual photon center of mass energy when $q' = 0$.

Thus we can choose the 6 following polarizabilities in order to describe the low energy regime :

$P^{(01,01)S=0,1}(q)$ corresponding to the C1/C1 transition
 where $P^{(01,01)0}(0) = -\sqrt{2/3} \alpha/e^2$
 and $P^{(01,01)1}(0) = 0$

$P^{(11,11)S=0,1}(q)$ corresponding to the M1/M1 transition
 where $P^{(11,11)0}(0) = -\sqrt{8/3} \beta/e^2$
 and $P^{(11,11)1}(0) = 0$

$P^{(01,12)1}(q)$ corresponding to the M2/C1 transition (N- D_{13} transition)
 where $P^{(01,12)1}(0) \sim \gamma_3$

$P^{(11,02)1}(q)$ corresponding to the C2/M1 transition (N- Δ transition)
 where $P^{(11,02)1}(0) \sim (\gamma_2 + \gamma_4)$

The continuity of $P^{(01,01)0}$ to α and of $P^{(11,11)0}$ to β at $q = 0$ has been checked by Guichon *et al.* [1]. Note that α , β and e^2 in these equations are expressed in the same system of units.

The values of $P^{(01,01)1}$, $P^{(11,11)1}$, $P^{(01,12)1}$, $P^{(11,02)1}$ at $q = 0$ have been proved by Drechsel *et al.* [7], [8]. $P^{(01,12)1}$, $P^{(11,02)1}$ which correspond to multipole in $q'q^2$, are probed in an expansion of the VCS amplitude to leading order in q' , but have a limit linked to the spin polarizabilities $\gamma_{2,3,4}$ obtained in the real Compton amplitude at upper order.

Several calculations have been performed in order to evaluate the GPs :

- Guichon, Liu, and Thomas [1] made a first estimate of generalized polarizabilities in a non relativistic quark model (NRQCM) in order to guide the first generation of experiments. In a later publication, Liu, Thomas and Guichon [9] extended their calculation to include recoil effects. Nevertheless the 4 relations for crossing and charge-conjugation symmetries between the 10 GPs are not verified.
- Vanderhaeghen [10] performed a calculation in an effective Lagrangian approach (ELM) including nucleon resonance effects. The polarization of the pion cloud is also taken into account by direct coupling of the photon to the pion or pion pair. The advantage of this phenomenological model is to take into account the nucleon structure effects not only in the first order q' in the outgoing photon energy, but to all the orders. It is the only model which can really predict the deviation of $\sigma^{BH} + Born + NonBorn$ from $\sigma^{BH} + Born$.
- Metz and Drechsel [11] performed a one-loop calculation of the polarizabilities in the linear sigma model (LSM) in the limit of an infinite sigma mass. The 4 relations between the 10 GPs were first found in the framework of that model.
- A calculation in the heavy-baryon chiral perturbation theory (HBChPT) has also been performed to third order in the initial and final photon

momentum expansion by Hemmert, Holstein, Knöchlein and Scherer [12]. The range in momentum transfer over which this expansion is valid is limited by the order to which the expansion is carried. This calculation is expected to work up a momentum transfer of $q = 300$ MeV/c.

The absolute values of the GPs and their dependence with the momentum transfer Q^2 are quite different in the various models. In particular pion cloud contributions lead to large transition radii and GPs decreasing much more rapidly with Q^2 than in the case of quark and phenomenological models.

So it is very useful to get data for these polarisabilities in different ranges of Q^2 ; it is the goal of the three following experiments.

2 The VCS experiment at MAMI

The first experiment which is now realized and is being analyzed, has been performed at MAMI [3] at $Q^2 = 0.33$ GeV² or $q = 600$ MeV/c, $\epsilon = 0.62$, $\phi \sim 0$ deg.

Figure 2 presents the differential cross section, and the different contributions $|T^{BH}|^2$, $|T^{Born}|^2$, $|T^{BH} + T^{Born}|^2$, $|T^{BH} + T^{Born} + T^{NonBorn}|^2$. It shows the dominant contribution of the Bethe-Heitler amplitude in the direction of the incident or outgoing electron, a significant effect of the polarizabilities can be measured only far from the electron lines. So we clearly have to detect the photon in the half-plane opposite to the electron directions, under backward angles.

The experiment is performed in the A1 Hall at MAMI. The scattered proton and electron are detected in coincidence in two magnetic spectrometers and the final photon is tagged by the reconstructed missing mass: $M_X^2 = 0$. With the high duty cycle available at MAMI, one is not limited by the accidental counting rates and one can use a typical luminosity of $\mathcal{L} = 2.10^{37}$ cm⁻²s⁻¹. Thanks to the excellent resolution of the facilities ($\Delta E/E = 10^{-4}$ for the energy of the incident electrons, $\Delta P/P = 10^{-4}$ for the momenta of the two spectrometers and an angular resolution better than 3 mrad) the contribution of the π^0 in the missing mass is very clearly separated from the events characterized by the photon mass (see Fig. 3).

The radiative corrections have been calculated for the first time by Vanderhaeghen *et al.* taking into account all the diagrams of order α^4 in the cross section [13]. The dimensional regularization scheme to treat both Ultra-Violet and Infra-Red divergences is used. The expression of the virtual photon correction contains analytical terms and a numerical term which has been computed. The real photon correction is splitted in two parts, one which depends

Fig. 2. The contributions $|T^{BH}|^2$, $|T^{Born}|^2$, $|T^{BH} + T^{Born}|^2$, $|T^{BH} + T^{Born} + T^{NonBorn}|^2$ are indicated in a logarithmic scale in the kinematical domain studied at MAMI. The "generalized" polarizabilities introduced in the Non Born term are estimated in a static Δ model.

on the cut in missing mass and the other one which depends on the electron kinematics (electron angle, velocity, and Q^2). After exponentiation of the radiative corrections, one gets -16% for the virtual one, and -3% for the real one as shown in Fig. 4. Continuity between the radiative corrections in the elastic case and the VCS case has been checked.

Figure 5 shows preliminary results obtained at $q' = 45$ MeV/c. Data points are corrected for radiative effects. The two sets of black and squared points are obtained at two different periods, and the triangle points are obtained at a different position for the proton spectrometer. They are in rather good agreement in the region of same kinematics. The analysis is still under way. The LET prediction is also indicated by the curve. As q' is small, the deviation from the LET prediction has to be very small and these data provide first test of the low energy theorem for VCS. This represents a severe check

Fig. 3. Experimental spectrum of the missing mass in the reaction $p(e, e'p)X$ with a very good separation between Compton and π^0 electroproduction events.

of our analysis method and of the radiative corrections. It will ensure a reliable extraction of the polarizabilities in the energy domain $q'=100$ MeV/c. This is necessary to have a very good control of the absolute value of the experimental cross section, in order to measure the expected small deviation from the LET prediction.

The experiment near threshold ($q' \leq m_\pi$) has to determine $\mathcal{M}^{exp}$ in the form :

$$\mathcal{M}^{exp} = \frac{\mathcal{M}_{-2}^{exp}}{q'^2} + \frac{\mathcal{M}_{-1}^{exp}}{q'} + \mathcal{M}_0^{exp} + o(q') \quad (6)$$

where the coefficients $\mathcal{M}_i^{exp}$, $i=-2,-1,0$ are functions of $(q, \epsilon, \theta, \varphi)$. The experimental method (see Fig. 6) is to study the evolution of $q'^2 \mathcal{M}^{exp}$ at small values of q' . The ordinate and slope at $q' = 0$ are due only to the interference between BH and Born and their experimental values have to be in perfect agreement with the low energy theorem predictions (LET). This is a crucial part of the experiment because it validates the following step which is the extraction of some GPs from the next order of the expansion. In fact with an **unpolarized experiment**, the difference $\mathcal{M}_0^{exp} - \mathcal{M}_0^{LET}$ provides some linear combinations of GPs :

Fig. 4. Effect of the real and virtual radiative corrections on the total BH+Born cross section according the calculation of Ref.[13].

$$\begin{aligned}
 \mathcal{M}_0^{exp} - \mathcal{M}_0^{BH+Born} &= \frac{4me^6 q}{\tilde{Q}^2(1-\epsilon)} \sqrt{\frac{2E(q)}{E(q)+m}} \\
 &\{ \sin\theta (\omega'' \sin\theta - \omega' k_T \cos\varphi \cos\theta) (\epsilon \mathbf{P}_{LL}(q) - \mathbf{P}_{TT}(q)) \\
 &+ \left[\frac{\tilde{q}_0}{q} (\omega'' \sin\theta \cos\theta \cos\varphi - \omega' k_T (1 - \cos^2\varphi \sin^2\theta)) \right. \\
 &\left. - (\omega'' \sin\theta \cos\varphi - \omega' k_T \cos\theta) \right] \sqrt{2\epsilon(1+\epsilon)} \mathbf{P}_{LT}(q) \}
 \end{aligned} \tag{7}$$

where the various kinematic coefficients are : $\tilde{Q} = [Q]_{q'=0}$

$$\begin{aligned}
 \omega &= - \left[q' \left(\frac{1}{p \cdot q'} + \frac{1}{k \cdot q'} \right) \right]_{q'=0} ; \quad \omega' = + \left[q' \left(\frac{1}{k' \cdot q'} - \frac{1}{k \cdot q'} \right) \right]_{q'=0} \\
 k_T &= \tilde{Q} \sqrt{\frac{\epsilon}{2(1-\epsilon)}} ; \quad \omega'' = + \left[\omega q - \omega' \sqrt{k'^2 - k_T^2} \right]_{q'=0}
 \end{aligned} \tag{8}$$

Fig. 5. Preliminary results at $q'=45$ MeV/c. Different sets of data are reported. The curve presents the low energy theorem prediction.

The amplitudes $P_{LL}(q)$, $P_{TT}(q)$, $P_{LT}(q)$ are the following combinations of 5 generalized polarizabilities (where G_E and G_M are the form factors applied at $\tilde{Q}^2$) :

$$P_{LL}(q) = -2\sqrt{6}mG_E P^{(01,01)0}(q) \quad (9)$$

$$P_{TT}(q) = 3G_M(\tilde{Q})q^2 \left(\sqrt{2}P^{(01,12)1}(q) - \frac{1}{\tilde{q}_0}P^{(11,11)1}(q) \right)$$

$$P_{LT}(q) = \sqrt{\frac{3}{2}}m\frac{q}{\tilde{Q}}G_E(\tilde{Q})P^{(11,11)0}(q) + \frac{3}{2}\frac{q}{\tilde{q}_0}\tilde{Q}G_M(\tilde{Q})P^{(01,01)1}(q)$$

We can remark that at $\theta = 0^\circ \quad \forall \varphi$:

$$\mathcal{M}_0^{exp} - \mathcal{M}_0^{BH+Born} = \omega'k_T \sqrt{2\epsilon(1+\epsilon)} \left(1 - \frac{\tilde{q}_0}{q}\right) P_{LT} \quad (10)$$

Fig. 6. Evolution of $q'^2 \times \mathcal{M}^{exp}$ from calculations in a static Δ model. The ordinate and slope at $q' = 0$ are due only to the interference between BH and Born, while the polarizability effect appears in the second order of the expansion.

In order to extract $\epsilon P_{LL} - P_{TT}$ and P_{LT} we have to play with different sets (θ, φ) . The last equation shows that an experiment performed at $\theta = 0$ (whatever φ) will provide directly P_{LT} (mainly related to the polarizability $P^{(11,11)0}$ if we assume that the spin polarizability $P^{(01,01)1}$ is small as it has to go to 0 at $q = 0$).

To separate P_{LL} (or $P^{(01,01)0}$) from P_{TT} we need at least two values of ϵ . Even if it is the case we get only 3 combinations of 5 GPs, it is not enough to solve the problem. We are obliged to make some assumptions, as the spin GPs, $P^{(01,01)1}$ and $P^{(11,11)1}$ are small as they have to go to 0 at $q = 0$.

The values of $\mathcal{M}_0^{exp} - \mathcal{M}_0^{BH+Born}$ predicted in the different models are presented in Fig. 7 as a function of $\theta_{\gamma\gamma}$. Their predictions are rather quite different. At $\theta_{\gamma\gamma} = 0$ deg we have mainly the effect of the value of the polarizability $P^{(11,11)0}$ as it was said before. For negative values of $\theta_{\gamma\gamma}$ in the range $[-125, -75 \text{ deg}]$, the effects of all the polarizabilities seem to compensate themselves in the full ELM model and in the NRQCM model, while the HBChPT gives large and negative predictions.

Nevertheless extracting the third term $\mathcal{M}_0^{exp}$ as it is presented in Figure 6, is not so easy as we do not know the polynomial order in the fitting procedure, and as we have only five measurements in q' . The investigated values in

Fig. 7. $\mathcal{M}_0^{exp} - \mathcal{M}_0^{BH+BORN}$ estimated in the different models : 1) thin solid line : in the ELM model with only the contribution of the Δ resonance, 2) thick solid line : in the full ELM model with π^0 and σ exchange in the t channel, and all the other resonance D_{13} , S_{11} , ..., 3) thin dotted line : NRQCM, 4) thick dotted line : HBChPT

q' are 33, 45, 67.5, 90, 111.5 MeV/c. These values are selected to explore all the range in q' from 0 to pion threshold. The minimal value of q' (33 MeV/c) is chosen not so small in order to be larger than the energy of an extra radiative photon (see next section). The difficulty of extracting the information $\mathcal{M}_0^{exp} - \mathcal{M}_0^{BH+BORN}$ is described on Figure 8. We represent the value of $\mathcal{M}^{exp} - \mathcal{M}^{BH+BORN}$ estimated in the ELM model (which is the only model to give a complete calculation for the Non-Born amplitude with all the orders of the expansion in q'). $\mathcal{M}_0^{exp} - \mathcal{M}_0^{BH+BORN}$ is now the ordinate at $q' = 0$, but the lack of accuracy at small q' prevents its determination. To understand this lack of accuracy even the experiment is performed very precisely ($\Delta\sigma/\sigma = \Delta\mathcal{M}^{exp}/\mathcal{M}^{exp}$ better than 3%), we have to note that the deviation $\mathcal{M}^{exp} - \mathcal{M}^{BH+BORN}$ is almost constant (of the order of 0.1×10^{-6}

Fig. 8. $\mathcal{M}^{exp} - \mathcal{M}^{BH+Born}$ estimated in the ELM model from Marc Vanderhaeghen. The two upper curves are calculated by including only the Δ resonance. The two lower curves are calculated with the full model, with π^0 and σ exchange in the t channel, and all the other resonance D_{13} , S_{11} , ... The thicker and thinner curves in the two sets correspond to $\theta_{\gamma\gamma} = -120$ deg and -60 deg respectively. The error bars of the expected experimental values $\mathcal{M}^{exp} - \mathcal{M}^{BH+Born}$ (estimated at $\theta_{\gamma\gamma} = -120$ deg in the ELM model including only the Δ resonance) are determined with a statistical error on the measured cross section of 3 %.

MeV^{-2}) while the value of $\mathcal{M}^{BH+Born}$ varies from a large value $174 \times 10^{-6} \text{ MeV}^{-2}$ at $q' = 10 \text{ MeV/c}$ to a smaller value $1.5 \times 10^{-6} \text{ MeV}^{-2}$ at $q' = 100 \text{ MeV/c}$. So to extract $\mathcal{M}_0^{exp} - \mathcal{M}_0^{BH+Born}$ how to do ? The hope is that the evolution of $\mathcal{M}^{exp} - \mathcal{M}^{BH+Born}$ in q' be only controlled by the evolution of $BH + Born$. This will assume that the internal structure effects in the upper order are negligible. So the evolution of $\mathcal{M}^{exp} - \mathcal{M}^{BH+Born}$ will be only due to the interference term of the $BH + Born$ amplitude (with an expansion with several orders in q') with the *NonBorn* amplitude (with only the first

order parametrized by the 6 investigated GPs). We are still working on this problem which is common to all the VCS experiments.

3 Comparison between the VCS experiments at MIT-Bates, MAMI and Jefferson Lab.

Three experiments are now under studied in different kinematical ranges :
 at Bates : $Q^2 = 0.05 \text{ GeV}^2$, $q = 240 \text{ MeV/c}$, $\epsilon = 0.90$, $\phi \sim 90 \text{ deg}$,
 at MAMI : $Q^2 = 0.33 \text{ GeV}^2$, $q = 600 \text{ MeV/c}$, $\epsilon = 0.62$, $\phi \sim 0 \text{ deg}$,
 at Jefferson Lab : $Q^2 = 1 \text{ to } 2 \text{ GeV}^2$, $q \geq 1 \text{ GeV/c}$, $\epsilon \geq 0.80$, $\phi \sim 0 \text{ deg}$.

The pilot experiment at MAMI has clearly demonstrated that the measurement of VCS is feasible, the 2 other experiments will take data in the next spring 1998.

3.1 role of Q^2 or q

The choice of a large value of q ($\geq 500 \text{ MeV/c}$) implies that the outgoing proton is better focused along the virtual photon direction. This Lorentz boost effect allows to intercept easily the proton in a magnetic spectrometer (of reasonable large solid angle). So the two experiments at MAMI and Jefferson Lab are realized by detecting electron and proton in two magnetic spectrometers of high resolution and large solid angle moving in the same plane ($\phi \sim 0 \text{ deg}$) and exploring all the values of $\theta_{\gamma\gamma}$.

The choice of a small value of q (240 MeV/c), guided by the wish to verify the Chiral Perturbation predictions, at MIT-Bates does not allow such an experimental method. Moreover at $\theta_{\gamma\gamma} = 0 \text{ deg}$, the value of the outgoing proton momentum is minimum and the proton would not escape the target. So the selected value of $\theta_{\gamma\gamma}$ is 90 deg, (the outgoing momentum is then 240 MeV/c), and 4 values in ϕ (0, 90, 180 and 270 deg) are explored thank to the OOPS modules (Out-Of-Plane-Spectrometers). Out-of-plane detection is an important capability for VCS and this is described further.

Nevertheless the multipole expansion used to analyse the experiment data implies that q has to remain larger than q' (see References [1], [6].) Moreover the investigated value of q' has not to be too small, but larger than the energy of the virtual photon (q'') of an extra radiative photon. Experimentally the positive radiative tail of the missing mass ΔM_X^2 is due to the presence of 2 undetected photons.

$$\Delta M_X^2 = 2q'q''(1 - \cos\theta_{q'q''}) \quad (11)$$

To have an idea of the values of $\vec{q}'$ and $\vec{q}''$, so to determine 6 quantities, we can use the measurement of the total missing momentum and the total missing energy plus two extra relations ($\vec{q}''$ in the direction of the incident electron, or $\vec{q}''$ in the direction of the outgoing photon). In these two cases, we can solve the problem. The two solutions (in the case of MAMI) are presented on Fig. 9 and compared to the modulus of the total missing momentum.

Fig. 9. Determination of two missing photons in the reaction $p(e,e'p)X$ for the set-up of smaller total missing momentum (around 33 MeV/c). The modulus are noted q' and q'' . A cut in missing mass on the MAMI data is imposed at 4000 MeV². The spectrum in q'' is a typical Bremsstrahlung spectrum. The real value of q' is not too much affected by the small value of q'' . The spectrum in q' is compared with the modulus of the total missing momentum (indicated in dotted line) on the left figures.

3.2 role of ϵ

The virtual photon flux is in $1/(1 - \epsilon)$, so it is better to choose ϵ close to 1. Nevertheless at fixed energy of the beam, if q is large, ϵ is small, and vice versa.

It is necessary to have two values of ϵ to separate P_{LL} from P_{TT} (see equation (7)). So it is better to have a first value close to 1, in order to compromise then the gain in lever arm and the time of counting with a second smaller value of ϵ .

3.3 role of ϕ

By going above and below the scattering plane, the BH process can be suppressed relative to VCS.

In the case of the MIT-Bates experiment, as they explore 4 values in ϕ at $\theta = 90^\circ$, they get :

$$\phi = \begin{pmatrix} 0 \\ 180 \end{pmatrix} \mathcal{M}_0^{exp} - \mathcal{M}_0^{BH+Born} = \omega''(\epsilon P_{LL} - P_{TT}) \begin{pmatrix} - \\ + \end{pmatrix} \omega'' \sqrt{2\epsilon(1+\epsilon)} P_{LT}$$

$$\phi = \begin{pmatrix} 90 \\ 270 \end{pmatrix} \mathcal{M}_0^{exp} - \mathcal{M}_0^{BH+Born} = \omega''(\epsilon P_{LL} - P_{TT}) - \omega' k_T \frac{\tilde{q}_0}{\tilde{q}} \sqrt{2\epsilon(1+\epsilon)} P_{LT}$$

They will take this advantage to measure directly in a same run, $\epsilon P_{LL} - P_{TT}$ and P_{LT} .

Nevertheless it is important to note that if the MAMI and Jefferson Lab experiments are using magnetic spectrometers moving in the reaction plane, they investigate a range in ϕ which is not so small and which increases when q increases. Remark that at MAMI, we can extend the measurement with the spectrometer moving outside the scattering plane.

4 A Complete Experiment with double polarisation

Fig. 10. Outline of the kinematics for the $p(e, e' p') \gamma$ reaction with polarized incident electron and recoil proton polarization measurement.

The double polarization asymmetry in the reaction $p(\vec{e}, e' \vec{p}') \gamma$ where the incident electron is polarized (with helicity $h = \pm \frac{1}{2}$) and where the polarization of the final nucleon is measured, is given by :

Fig. 11. VCS double polarization asymmetry (polarized electron, recoil proton polarization along the x - or z - directions) in MAMI kinematics ($Q^2 = 0.33 \text{ GeV}^2$) as function of the CM angle between real and virtual photon. The BH + Born contribution is shown by the dashed lines. The result of the BH + Born + Δ contribution is shown by the dotted lines, whereas the BH + Born + D_{13} + σ contribution is shown by the dashed-dotted lines. The total effect in the model is shown by the full lines. The figure is extracted from the reference [14].

$$\Sigma^{(i)} = \frac{[\sigma_{h=+\frac{1}{2}, s'_i \uparrow} - \sigma_{h=+\frac{1}{2}, s'_i \downarrow}] - [\sigma_{h=-\frac{1}{2}, s'_i \uparrow} - \sigma_{h=-\frac{1}{2}, s'_i \downarrow}]}{[\sigma_{h=+\frac{1}{2}, s'_i \uparrow} + \sigma_{h=+\frac{1}{2}, s'_i \downarrow}] + [\sigma_{h=-\frac{1}{2}, s'_i \uparrow} + \sigma_{h=-\frac{1}{2}, s'_i \downarrow}]}, \quad (12)$$

We can define the three new following combinations of the 6 Generalized Polarizabilities :

$$P_{LT}^z(q) = \frac{3\tilde{Q}q}{2\tilde{q}_0} G_M P^{(01,01)1}(q) - \frac{3mq}{\tilde{Q}} G_E P^{(11,11)1}(q) \quad (13)$$

$$P_{LT}'^z(q) = -\frac{3}{2}\tilde{Q} G_M P^{(01,01)1}(q) + \frac{3mq^2}{\tilde{Q}\tilde{q}_0} G_E P^{(11,11)1}(q)$$

$$P_{LT}'^\perp(q) = \frac{3q\tilde{Q}}{2\tilde{q}_0} G_M \left(P^{(01,01)1}(q) - \sqrt{\frac{3}{2}}\tilde{q}_0 P^{(11,02)1}(q) \right)$$

This new system plus the previous system (10) allow to determine the 6 GPs.

Fig. 12. Effect of the radiative corrections on the asymmetry according the calculation of Ref.[13]

Vanderhaeghen has demonstrated (see Ref.[14]) that the asymmetry when the proton is polarized parallel to the virtual photon direction (i.e. along the z -axis),

$$\Delta \mathcal{M}_0^{exp}(h, z) - \Delta \mathcal{M}_0^{BH+Born}(h, z) \quad (14)$$

is a combination of P_{TT} , P_{LT}^z , $P_{LT}^{'z}$. We can separate these three quantities with different sets of data using different values of $\theta_{\gamma\gamma}$.

Vanderhaeghen has also demonstrated (see Ref.[14]) that the asymmetry when the proton is polarized along the x -axis (i.e. perpendicular to the virtual photon direction and parallel to the reaction plane),

$$\Delta \mathcal{M}_0^{exp}(h, x) - \Delta \mathcal{M}_0^{BH+Born}(h, x) \quad (15)$$

will be sensitive also to P_{LL} , P_{LT} and $P_{LT}^{'\perp}$. The measurement with different values in $\theta_{\gamma\gamma}$ will provide the two first combinations P_{LL} , P_{LT} . To get $P_{LT}^{'\perp}$

and then $P^{(11,02)1}$ (which corresponds to the C2/M1 transition) we have to go outside the scattering plane ($\phi \neq 0$).

So in conclusion a complete experiment to extract the 6 GPs is the measurement of the double polarization asymmetry in the out-of-plane experiment $p(\vec{e}, e' \vec{p})\gamma$ where the incident electron is polarized (with helicity $h = \pm \frac{1}{2}$) and where the polarization of the final nucleon is measured (along the two directions x and z).

The two asymmetries along x and z are presented on Fig.11. For both polarization directions it is furthermore seen that the Δ (magnetic) contribution and the $D_{13} + \sigma$ (electric) contribution yield a large and opposite effect to the asymmetry relative to the BH+Born result.

From the experimental point of view, the measurement of such asymmetries has the advantage over an absolute cross section measurement that no absolute normalisation is required. Furthermore, in a previous section, we have seen that radiative corrections to the VCS are not at all negligible compared to the nucleon structure effects one hope to extract from the non polarized experiment. Figure 12 shows that an asymmetry measurement turns out to be promising as the effect of the radiative corrections are very small.

Unfortunately the efficiency of a polarimeter is only 5 to 10%, so the time of counting will increase considerably.

5 Conclusion

Polarizabilities are very clean observables to access the internal structure of the nucleon, but it is a long way to get these observables, as their effects are very small in the photon electroproduction and we need to perform absolute measurements with a very great precision.

We could think to observe the process of VCS off the proton by the (difficult, but may-be not too much more difficult) experiment $\gamma p \rightarrow pe^+e^-$. The real photon pair production will provide the polarizabilities of the nucleon in the time-like region, and could be complementary of the photon electroproduction approach.

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