Off-bucket proton losses during ramping

N. Catalan Lasherас

Abstract

In this paper, we report a study undertaken to determine whether longitudinal and transverse amplitudes become coupled before the loss of the off-bucket protons during the ramp. We compute the synchrotron as well as the betatron tune changes with momentum and determine if synchro-betatron resonances blow-up the transverse particle amplitude. A strong coupling might allow a betatron cleaning of these particles before they are outside the momentum acceptance of the machine. We show that this is not the case, justifying the need of momentum cleaning.
1 Introduction

In the LHC the bucket area \(1.46 \, eV \cdot s\) and bunch longitudinal emittance \(1.00 \, \text{eV} \cdot \text{s} \text{ at } 4\sigma\) are comparable at injection [1]. Even in good operational conditions, a finite fraction (estimated to \(f \approx 5\%)\) of the injected protons will lie outside the RF bucket at the beginning of the acceleration ramp. These uncaptured protons are not accelerated and drift towards the vacuum chamber in a short time when the magnetic field rises. The level of transient loss of protons per meter capable of quenching a magnet has been estimated to be \(n_{q,\text{inj}} \approx \frac{10^{10}}{\text{pm}^{-1}}\). With the number of protons stored in the machine being \(N_{\text{st}} = 3 \cdot 10^{14}\), the expected losses exceed the tolerable level by a factor \(r \approx 1500\). A dedicated momentum collimation section is being designed to cope with these transient losses together with other steady losses at injection and collision [2]. For particles with a large momentum deviation, synchro-betatron coupling might have an important effect and cause a blow-up of the betatron amplitude. In this way, off-bucket particles might reach the transverse machine aperture defined by the betatron collimators \((A_{\text{bet}} = 6\sigma)\) before reaching the momentum aperture \((A_{\delta p/p_0} \approx 3.5 \cdot 10^{-3})\) defined by the dispersion in the arcs of the ring [3].

Tracking of these off-bucket particles including acceleration has been performed with the MAD program by applying at each turn the exact longitudinal equations described in section 2. We applied FFT analysis of their transverse positions to calculate the transverse tune variations during ramping. The results for the two last optics versions of the LHC machine 4.2 and 5.0 are shown in sections 3 and 4. A summary of the most important results for both versions can be found in section 5.

2 Exact equations for the longitudinal motion.

The equations describing the longitudinal beam dynamics in their differential form (for a sinusoidal RF voltage) are [4, 5]

\[
\frac{\partial}{\partial t}(\delta \phi) = -2\pi \eta h(\delta p/p_0) \quad (1)
\]

\[
\frac{\partial}{\partial t}(\delta p/p_0) = eV/\beta p_0 (\sin \phi - \sin \phi_*) \quad (2)
\]

The variables are the longitudinal phase \(\phi\) and the momentum deviation \(\delta p/p_0\) while the parameters are the synchronous phase \(\phi_*\), the RF voltage \(V\), the harmonic number \(h\) and \(\eta = 1/\gamma^2 - \alpha\). In this last expression \(\alpha\) is the momentum compaction defined by \(\Delta L/L_0 = \alpha \delta p/p_0\).

Generally, the bunch occupies a small fraction of the bucket. We work then in the small oscillations range where \(\phi \rightarrow \phi_*\) and we can make the approximation \(\sin \phi - \sin \phi_* \rightarrow \delta \phi = \phi - \phi_*\). Rewriting (2) we have

\[
\frac{\partial}{\partial t}(\delta p/p_0) \approx (eV/\beta p_0) \delta \phi \quad (3)
\]

Combining the equations (1) and (3), we are left with an harmonic oscillation whose frequency is completely determined by the RF parameters and the magnetic field.

\[
\frac{\partial^2}{\partial t^2}(\delta \phi) \approx -\Omega^2 \delta \phi \quad (4)
\]

\[
\Omega^2 = \omega_0^2 eV \eta h / 2\pi \beta p_0 \quad (5)
\]

\(\Omega\) is the so called synchrotron frequency which is related to the synchrotron tune through \(Q_* = \Omega / f_r\) where \(f_r\) is the revolution frequency. This synchrotron frequency is valid only
in the immediate vicinity of the synchronous particle. In the specific case of LHC at injection $\Omega = 66.14$ Hz and $Q_s = 0.0059$.

When we move away from the synchronous particle, the small phase approximation is no longer valid. Trajectories deform progressively up to the unstable point $\phi = -\phi_x$ which defines the separatrix [5]. (See figure 1). The phase acceptance is defined by $\phi_x$ and $\phi_2$ (for slow acceleration $\phi_2 \approx 2\phi_x$ [6]) and is infinite for no acceleration ($\phi_x = 0, \pi$). The synchrotron frequency also depends on the phase difference. From reference [4], we take the second order truncation for the frequency as a function of the initial longitudinal amplitude (equal to $\delta \phi$ when $\delta p/p_0 = 0$).

$$\Omega^2 = 1 - 1/2 \sin^2 \delta \phi/2 - 3/32 \sin^4 \delta \phi/2$$

(6)

The synchrotron frequency decreases when $\delta \phi$ increases and goes to zero in the separatrix. The real values, as well as the approximation of equation 6 are also shown in figure 1.

Figure 1: Left, different trajectories in the longitudinal phase space. (A), the particle lays inside the bucket and oscillates around the synchronous phase, (B) off-bucket particles do not follow closed trajectories. The separatrix (S) is the limit trajectory between both cases and corresponds to an unstable point. Right, synchrotron frequency as a function of the phase difference inside the bucket.

If the magnetic field ramps slowly, we can consider the phase acceptance as being infinite. Still there might be particles close to the unstable phase but with a relative momentum deviation different from zero. For the LHC where the bunch and bucket sizes are comparable, the fraction of these non-captured particles can be as large as 5% of the total bunch intensity. In order to study the evolution of these particles, we simulate the longitudinal motion using the exact equations (1) and (2) in their discrete form.

$$\Delta(\delta \phi) = -2\pi \hbar \eta(\delta p/p_0)$$

(7)

$$\Delta(\delta p/p_0) = eV/\beta p_0(\sin \phi - \sin \phi_s)$$

(8)

Some trajectories generated in this way are shown in figure 2 for different initial values of the momentum deviation. These off-bucket particles drift in phase and oscillate in momentum at a frequency which depends on the average momentum deviation as indicated in figure 3. During acceleration the average momentum deviation increases and as a consequence, the synchrotron frequency $\Omega_s$ and the tune $Q_s$ grow with time.

Once known the longitudinal evolution of off-bucket particles, we are interested in their transverse dynamics when introduced in a non-linear machine as the LHC. We made
Figure 2: Longitudinal trajectories with $\delta p/p_0$ versus $\phi$. Trajectory (A) lays inside the bucket while (B) is outside bucket but close to the separatrix. As off-bucket particles are not captured by the RF, they will not be accelerated and their momentum deviation will increase with time (It will be negative for all of them after a while).

Figure 3: Longitudinal frequency as a function of momentum deviation. As we move away from the central point in the bucket the oscillation frequency decreases to become zero at the separatrix. Once outside the bucket, the frequency of the momentum oscillations increases again. The horizontal line corresponds to the value of $\Omega$ for the harmonic approximation.
use of the MAD code [7] and added some lines to the tracking subroutine in order to give the exact energy kick for every turn. The phase increment is performed consistently by MAD when tracking around the ring.

3 Version 4.2

The LHC optics files used for tracking are the following:
- LHC ver 4.2: injection, thin lens from /afs/cern.ch/user/1/lhcring/optics/v4.2, lhc42.seq, lhc42.seq.more and lhc42.Kinj.
- Errors files from /afs/cern.ch/user/1/lhcring/optics/v4.2/errors/, lhc42.Einj and lhc42.Eset.
- Chromaticity corrected by the GLOBAL command in RGOMAD
- RF cavities removed.

All errors switches are set to ON except for A2S, A2R, B2S, B2R and BDOT for all magnet types. Those errors are supposed to be corrected by spool pieces or dynamically during the ramp.

We use RGOMAD to simulate acceleration [8]. The acceleration is performed within the same tracking subroutine by increasing the momentum deviation. The four RF cavities are considered as only one thin cavity. To avoid synchro-betatron coupling due to dispersion at the cavities, we simulate the RF system at IP1 where both $D_x$ and $D'_x$ are minimal. The ramping is assumed linear with an energy gain per turn of 485 KeV as foreseen in the LHC design [1]. The initial conditions correspond to a particle laying on the beam axis but outside the bucket separatrix (Close to the point $\phi = -\phi_{\text{sat}}$ in the separatrix). The betatron oscillation amplitude is set to very small values ($\approx 10^{-5}$m). It has been checked that the choice of different initial conditions do not have an incidence in the final results.

The tracking has been repeated for eleven different error realisations. In all cases, a slight drift of the particle oscillations centre due to residual dispersion is observed but its amplitude is not increased sensibly. After a while, the betatron amplitude in the $x - x'$ plane increases abruptly to some millimetres. Either the particle is lost or its amplitude is large enough to be captured by the betatron cleaning of the machine within some turns. The momentum deviation at which the losses take place is almost the same for all seeds and all initial conditions. For this version LHC 4.2, this value is around $\delta p/p_0|_{\text{loss}} = 6.5 \cdot 10^{-3}$. Figure 4 illustrates the typical behaviour before losses occur.

As it was shown in the first part of this note, when the momentum deviation changes, the synchrotron tune increases. The betatron tunes also change with time due to the residual chromaticity. A high precision frequency analysis becomes difficult because tunes are rapidly changing. Fast Fourier analysis of the $x$ and $y$ coordinates is done for short periods of time ($\approx 256$ turns). In this way we calculate the tune corresponding to the two oscillation modes $Q_1$ and $Q_2$ along the ramping. After a sufficient number of turns ($\approx 2000$), also the synchrotron tune can be identified in the spectrum.

Lines corresponding to $2Q_1$, $3Q_1$, and even $4Q_1$ in some seeds are identifiable. As $Q_1$ increases and $Q_x$ decreases, $3Q_1$ and $Q_x$ might overlap. Sidebands corresponding to the synchrotron tune are present almost in every case just before the particle is lost. This shows that we are dealing with synchro-betatron resonances (see fig. 4). The moving tunes do not allow to have a better insight of the resonance mechanism.

Still, the tune values computed via FFT analysis match those calculated using the TWISS MAD command (Figure 5 shows the comparison between both methods for version 4.2). The tune computation by FFT can be extended almost until the particle is lost.
Figure 4: Left, evolution of the particle transverse coordinates in the $x - x'$ phase space for two different seeds. Right, the frequency spectrum for the horizontal position just before the resonance. The two transverse tunes can be seen, as well as the synchrotron tune which forms satellites around them.

Figure 5: Comparison of the tunes calculated by the MAD TWISS command (solid line) and by FFT analysis of the transverse position (dots) for one seed. Both results agree very well.
4 Version 5.0

The same analysis was repeated for version v5.0 using the following machine description.
- LHC ver 5.0: injection, thin lens from /afs/cern.ch/eng/lhc/optics/V5.0/, Ring1.seq081097 and K0450IsNn4.081097.
- Errors files from /afs/cern.ch/eng/lhc/optics/V5.0/toolkit/errors, Einj and Esub-routines.
- Residual chromaticity corrected using sextupoles by the GLOBAL command in RGOMAD ($Q1' = 2.0, Q2' = 2.0$).
- RF voltage at zero.

All errors switches are set to ON except for A2S, A2R, B2S, B2R and BDOT for all magnet types. The MQW errors were switched off even if their effect was negligible in the amplitude range we are interested in.

4.1 Linear ramping

We first apply a linear ramping as for version v4.2 with an energy gain per turn of 485 KeV. The particle behaviour is similar to the one showed in version LHC 4.2. The centre of the oscillation drifts slowly with energy deviation due to second order dispersion but the oscillation amplitude stays small. When approaching $\delta p/p_0 \approx 7.0 \cdot 10^{-3}$ the betatron amplitude increases quickly and the particle is lost (figure 6). For this version, losses occur at slightly higher momentum deviation ($\approx 7.2 \cdot 10^{-3}$) definitely above the momentum aperture $A_{\delta p/p0} = 3.5 \cdot 10^{-3}$. In figure 6 the last FFT analysis before the overlapping of the some harmonics of $Q_s$ and certain spectral lines. The peaks corresponding to $2Q_s$, $3Q_s$ and $4Q_s$ are clearly visible.

![Horizontal trajectories](image1.png)

![Frequency analysis of the y-position](image2.png)

Figure 6: Left, evolution of the particle transverse coordinates in the $x - x'$ phase space. Units in meters and radians. Two different seeds are plotted. Right, the frequency spectrum for the horizontal position before the resonance for one seed. The line for $Q_s$ is outside the scale. Besides $Q_x$ and $Q_y$ we can also distinguish $2Q_s$, $3Q_s$ and $4Q_s$.

To improve the quality of the frequency analysis, we stop acceleration slightly before the particle is lost. Tracking enough number of turns without acceleration, allows a fine computation of the spectral lines and their identification with SUSSIX [9]. The results of this frequency analysis are presented in figure 7. There are small sidebands at each side of $Q_x$ and $Q_y$ that indicate synchro-betatron resonances in both transverse planes. As the
synchrotron tune is still growing, the particle is going to cross the resonance condition $Q_x + Q_s = Q_y$.

Figure 7: Main spectral lines from the frequency analysis of the x coordinate. The synchrotron sidebands almost overlap with the transverse tunes.

4.2 Parabolic ramping

The proposed baseline current ramping rate for LHC dipole magnets is not linear but a combination of flat, parabolic and exponential segments [10]. Proper acceleration starts with a parabolic ramping of the form

$$I(t) = I_{inj}(1 + \alpha t^2)$$

where the parameter $\alpha = 6.087 \cdot 10^{-6}$ is chosen to minimise the dynamical errors. The total time for this ramping segment is $\Delta t_{rb} \approx 344$ s whereas the time to reach a momentum deviation of $\delta p / p_0 = 7 \cdot 10^{-3}$ is $\approx 34$ s. Thus the off-momentum particle losses take place during the parabolic segment of the ramp.

Figure 8: Parabolic energy ramping as proposed for the LHC dipoles. Close to the loss point $\delta p / p_0 = 7 \cdot 10^{-3}$, we approximate the parabola by a linear ramping with the same energy gain per turn ($\approx 17$ KeV).

To simulate the acceleration we take a linear ramping tangent to the parabola at the presumed loss point ($\delta p / p_0 = 7 \cdot 10^{-3}$). This is shown in figure 8. The initial momentum
deviation used in the simulation is chosen close to the resonance to shorten the tracking time. It was checked also that the point where the resonance appears does not depend on the transverse initial conditions. Results for the parabolic ramping vary statistically but their mean values do not differ essentially from the linear ramping case.

5 Results

Figure 9 shows the maximum momentum deviation reached by the particles before being lost or captured by the betatron cleaning system. In all cases, this value is outside the limits fixed by the momentum aperture \((A \delta p/p_0 = 3.5 \cdot 10^{-3})\) [3]. For version 5.0, parabolic and linear ramping where simulated with the same final results.

![Figure 9: Minimum momentum deviation reached by the particle for both LHC optics versions. In abscissa, the seed number and in ordinate the minimum momentum deviation reached before being lost. For both cases and all ten seeds this momentum deviation is larger than the momentum acceptance of the ring \((A \delta p/p_0 = 3.5 \cdot 10^{-3})\). For LHC version 5.0, linear and parabolic ramping have been simulated. The difference between them is negligible.](image)

Conclusions

A tracking with MAD, modified to allow for acceleration with exact formulae has been done for the LHC versions 4.2 and 5.0. Different kinds of ramps were simulated at the end of the injection plateau.

With version 4.2, the transverse amplitude diverges near \(\delta p/p_0 \approx 6.5 \cdot 10^{-3}\), while for version 5.0, this occurs at \(\delta p/p_0 \approx 7 \cdot 10^{-3}\). A FFT analysis of the tracking data indicates that the loss is in fact related to synchro-betatron resonances and that it is a direct consequence of the increasing synchrotron tune. The momentum at which the loss takes place depends only on \(\delta p/p_0\) and not on the type of ramp. Once the particles arrive to the critical value of \(\delta p/p_0\) they are lost in a few turns. Thus the betatron cleaning might become ineffective to clean these particles.

For every seed and in both versions, the six-dimensional motion is still regular when the off-bucket particles reach the critical momentum deviation which defines the longitudinal acceptance for LHC. There is therefore no hope that the coupling of the longitudinal and transverse amplitudes induces a betatron cleaning before. This justifies the need of a momentum cleaning insertion at injection energy.
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References