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PSEUDO INVARIANT MASSES OF HADRONS IN NUCLEI

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Abstract

How to obtain experimental information on the hadronic mass modified in nuclear medium is discussed. It is shown that a pseudo invariant mass (PIM), which can be reconstructed as if the hadron were an isolated on-shell particle, is related to the average potential energy and thus has no direct connection with the modified hadronic mass, as demonstrated in typical examples. Furthermore, a PIM spectrum of an unbound short-lived hadron which is subject to rapid collisions with surrounding nucleons is expected to be kinematically broadened and shifted toward the lighter mass region, and thus it is difficult to extract a genuine mass shift of dynamical origin.

1. Introduction

Since hadrons are composite particles, their sizes, masses, *etc.*, are expected to change in nuclei. How the hadronic masses are modified in nuclei are extremely interesting questions related to quantum chromodynamics. There are a number of theories in this respect (see, for instance, Ref. [1-3]), whereas there is no clear experimental information yet. The reason is obvious; there is no convincing procedure as to how to deduce modified masses from experimental observables. There are many experimental efforts going on to attempt to determine the invariant masses of short-lived hadrons such as ρ meson [4,5] and ϕ meson [6] in high energy nuclear reactions. Recent results of the CERES [4] and the HELIOS-3 [5] experiments on dilepton invariant mass spectra have revealed an anomalous excess in the 0.3-0.6 MeV range, which are interpreted in terms of a decreased ρ meson mass in the nuclear medium [7-9]. One has, however, to examine the implication of such invariant mass spectra before connecting them to modified masses which are theoretically predicted. In the present paper we discuss and clarify the meaning of invariant masses of off-shell hadrons embedded in the medium, which we call "pseudo invariant masses".

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2. Mass Shift, Modified Mass and Invariant Mass

The mass shift (ΔM) of a hadron (with a free mass M_0) embedded in a nucleus is a result of the interaction between the hadron and the nucleus, and thus, is not a physical observable inherent to the hadron alone. In other words, the modified hadron mass M^* is not relativistic invariant and *cannot be measured directly*. It is a fictitious quantity defined as

$$M^* = M_0 + \Delta M, \quad (1)$$

once we know ΔM by some means.

The mass shift ΔM can be determined in an indirect way by studying the hadron-nucleus interaction, if the interaction is related to the mass shift. The mass shift occurs inside the nucleus and is expected to be proportional to the nuclear density $\rho(r)$ as

$$\Delta M(r) = m \frac{\rho(r)}{\rho(0)} = \frac{m}{1 + \exp[(r - R)/a]}. \quad (2)$$

In the relativistic formalism with the Dirac equation

$$\{\vec{\alpha}\vec{p} + \beta[M_0 + \Delta M(r)] + \beta U_S(r) + U_V(r)\}\Phi(\vec{r}) = E\Phi(\vec{r}), \quad (3)$$

where

$$U_S(r) = U_S^{(0)} \frac{\rho(r)}{\rho(0)}, \quad (4)$$

$$U_V(r) = U_V^{(0)} \frac{\rho(r)}{\rho(0)}, \quad (5)$$

are scalar and vector potentials, respectively, the introduction of a mass shift $\Delta M(r)$ is equivalent to adding a scalar potential. Since $\Delta M(r)$ and $U_S(r)$ are in principle indistinguishable, we can simply replace them as

$$\Delta M(r) + U_S(r) \rightarrow U_S(r) \quad (6)$$

so that we may redefine a mass shift as being equivalent to the scalar potential:

$$\Delta M(r) = U_S(r). \quad (7)$$

The binding energy in the non-relativistic regime is a solution of the Schrödinger equation with a total potential energy

$$V(r) = U_S(r) + U_V(r). \quad (8)$$

Whereas $V(r)$ is constrained by observed binding energies, it is in general difficult to separate the scalar part of the potential. An example is seen in the most popular case, namely, of nucleons; the attractive potential of about 50 MeV deduced from a non-relativistic treatment does not necessarily mean that the mass shift of the nucleon is -50 MeV. In great contrast, the relativistic mean field theory [10] claims the presence of a much larger

negative mass shift (scalar potential ~ -300 MeV) which is compensated by an equally large positive vector potential to account for the small nuclear binding energies. A similar situation may also exist for a Λ particle, where an attractive potential of about 30 MeV was obtained from observed binding energies based on a non-relativistic analysis of Λ hypernuclei [11]. There has been no clear-cut way to extract the scalar part of a strong-interaction potential because of the presence of an unknown vector part.

Thus, alternative methods to deduce effective hadron masses from measurements of invariant masses have been proposed and experimentally pursued. Invariant mass spectroscopy is to determine the mass of a decaying particle which decays to two or more particles of which the masses ($m_1, m_2, \dots$) are known. From measured momenta of daughter particles ($\vec{p}_1, \vec{p}_2, \dots$) one can reconstruct the invariant mass M_{inv} of the parent hadron as follows

$$M_{inv}^2 = E^2 - \vec{P}^2 = (m_1^2 + p_1^2)^{1/2} + (m_2^2 + p_2^2)^{1/2} + \dots - (\vec{p}_1 + \vec{p}_2 + \dots)^2 \quad (9)$$

Obviously, the relation (9) holds only for an isolated particle, and as soon as the hadron is coupled to a medium, its validity is lost; the energy and momentum of the counter part (medium) is missing from observation. Thus, in principle, one *cannot* determine the modified hadron mass in this way. This is an inherent problem associated with the concept of the invariant mass, whereas there are trivial difficulties arising from multiple scattering and final state interactions of the decay particles, which may deteriorate the relation (9) furthermore.

Nevertheless, one can measure the momenta of the decay particles and reconstruct eq.(9) as if the decaying parent were an isolated particle. Hereafter, we call such a fictitious mass as "pseudo invariant mass" (PIM). Obviously, PIM is not a discrete but a broadened quantity, since the particle is confined in space. In the next sections we will examine the meaning of such pseudo invariant masses (M_{inv}^*) in comparison with free (M_0) and modified (M^*) masses .

3. Pseudo Invariant Masses of Bound Hadrons

3.1. Coulombic Atoms

Let us begin with a simple and trivial example, where a negatively charged particle without strong interaction (say, it is a negative muon) is bound by a point nucleus. By definition, the lepton mass M_0 is not subject to any strong interaction shift. The energy in this case is the energy eigenvalue of a bound state with a principal quantum number n and a binding energy B_n , namely,

$$E_n = M_0 - B_n. \quad (10)$$

The momentum P is the internal momentum which distributes. Its mean square value is given by the virial theorem as

$$\langle P^2 \rangle = 2M_0 \cdot B_n. \quad (11)$$

Thus, the rms value of PIM becomes

$$M_{inv}^* = M_0 - 2 B_n = M_0 + \langle V(r) \rangle. \quad (12)$$

This simple example shows that the PIM is not at all the particle mass M_0 , not even the bound state energy, but $M_0 - 2 B_n = M_0 + \langle V(r) \rangle$. Would one say from such an observation that the hadron mass is shifted by $-2 B_n$?! Of course, NO. It is obvious that the PIM is nothing to do with the mass shift problem, but is related to the average potential. The distribution of PIM in this case is shown in Fig.1(a).

3.2. Hypernuclei

Next, let us take a Λ particle bound in a hypernuclear state. The ground state of a light Λ hypernucleus decays as $\Lambda \rightarrow p + \pi^-$, and thus pseudo invariant mass spectroscopy is conceivable. Even if we assume no intrinsic mass shift, the PIM is shifted more than the binding energy B_n . The PIM distribution calculated is shown in Fig.1(b).

Now, the following problem is to be raised. If the hadron mass is increased by $U_S(r)$ inside the nucleus, can this be significantly reflected in a PIM spectrum? The answer is *almost NO*, since PIM depends primarily on the total potential $V(r) = U_S(r) + U_V(r)$ which is constrained by an observed binding energy. In Fig.1(b) we show PIM spectra for various values of U_S . The shapes change only slightly. Since $U_S(r)$ and $U_V(r)$ cannot be separately determined, PIM does not provide any information on the mass shift. For example, a large negative mass shift ($U_S^{(0)} = -240$ MeV), which was suggested by Bouyssy [12], is far from $M_{inv}^* - M_0$, as shown in Fig.2, but there is no experimental way to prove it. This difficult situation holds for other baryons as well. However, if one could precisely measure the momentum dependence of PIM, as shown in Fig.1(b), one might in principle obtain some indication of $U_S(r)$, though it is practically extremely difficult.

3.3. Deeply Bound π^- States

Very recently, deeply bound pionic states (1s and 2p) in ^{207}Pb , which had been predicted by Toki *et al.* [13], were discovered in the $^{208}\text{Pb}(d, ^3\text{He})$ reaction [14], and from the deduced π^- binding energies the local potential strength in the Ericson-Ericson potential [16], or equivalently, the pion mass shift, was uniquely determined to be 19 ± 3 MeV [15]. Here, the hadronic mass shift is related to the $q \rightarrow 0$ limit of the real part of the s-wave local potential.

This type of "in-medium mass spectroscopy" to determine the hadronic mass through the scalar part of the potential which is deduced from measured binding energies can be extended to other mesons. In the future, a similar idea of identifying "Coulomb-assisted bound states" [17] may be applied to the case of K^- mesons. Furthermore, search for nuclear bound states of the η meson for the same purpose will be carried out [18].

In the pionic case, where the strong interaction is repulsive, deeply-bound states are accommodated with the aid of an attractive Coulomb interaction [13]. Using the energies and wavefunctions we calculate PIM's in this case, which are shown in Fig.1(c). The PIM deduced from the 1s π^- state is less than the free pion mass by about 10 MeV because of the Coulomb attraction, while the genuine mass shift is opposite, +19 MeV.

All these examples of bound particles show that PIM depends on the average potential energy:

$$M_{inv}^* \sim M_0 + \langle V(r) \rangle = M_0 + \langle U_S(r) + U_V(r) \rangle = M^* + \langle U_V(r) \rangle, \quad (13)$$

and thus is totally different from the modified mass M^* which we are seeking for.

4. Pseudo Invariant Masses of Unbound Hadrons in Nuclei

4.1. Linear Motion into and from a Medium

Now, we consider unbound hadrons which are created and decay in nuclear medium. Let us begin with a simple case, where a hadron traverses in a nuclear medium, where the hadron feels potentials (4,5). The energy and momentum of the hadron satisfy

$$(E - U_V^{(0)})^2 = M^{*2} + P^2, \quad (14)$$

where

$$M^* = M_0 + U_S^{(0)} \quad (15)$$

is the modified mass. Thus,

$$M_{inv}^{*2} = E^2 - P^2 = M^{*2} + 2U_V^{(0)}\sqrt{M^{*2} + P^2} + U_V^{(0)2}, \quad (16)$$

which yields in the limit $P \ll M^*$

$$M_{inv}^* \sim M^* + U_V^{(0)} = M_0 + U_S^{(0)} + U_V^{(0)}. \quad (17)$$

Namely, PIM in this case is approximately equal to the free mass plus the total potential $U_S^{(0)} + U_V^{(0)}$, whereas the mass shift is given by $U_S^{(0)}$. In general, there is a large difference between M^* and M_{inv}^* , and there is no experimental way to deduce M^* . It was suggested by Yazaki [19] that a precise measurement of M_{inv}^* as a function of P may yield $U_S^{(0)}$ and $U_V^{(0)}$ separately through eq.(16).

It is clear that a theoretically predicted M^* ($= M_0 + U_S^{(0)}$) cannot be identified as M_{inv}^* , though most of theoreticians incorrectly compare their theoretical values with experimental PIM spectra. Furthermore, the above simple relation (17) does not hold for a particle which faces rapid collisions in the nuclear medium, as will be discussed below.

4.2. Decaying Hadrons in Collisions

The cause for a mass modification in the nuclear medium is the interaction of the hadron with nuclear medium. The fact that a mass shift is believed to be proportional to the nuclear density assumes intrinsically that the origin of the mass shift is the encounter of the hadron with nucleons inside the nucleus. Even in the normal nuclear medium each nucleon undergoes rapid collisions with a collision length

$$l_c \sim c \times \frac{p_F}{M} \times \frac{\hbar}{E_F} \sim 1.2 \text{ fm} \quad (18)$$

where $p_F = 250 \text{ MeV}/c$ and $E_F = 40 \text{ MeV}$. Not only is the collision length as short as the internucleon distance, but also the wave packet size,

$$l_{\text{packet}} \sim \frac{\hbar}{p_F} \sim 0.8 \text{ fm}, \quad (19)$$

is as large as the nucleon radius. This means that *any in-medium hadron cannot be an isolated particle at any instance*; its decay kinematics should be influenced by continuously occurring collisions. This situation is somewhat similar to the Mössbauer effect [20] in the sense that the decay kinematics of an essentially two-body decay is influenced by surrounding medium, namely, the original "free-decay" recoil momentum is counter balanced by a momentum transfer from the medium. If we construct a pseudo invariant mass in the two-body decay process $M_0 + E_{ex} \rightarrow M_0$, PIM is equal to $M_0 + E_{ex}$ in the free decay with a recoil energy, but is lowered by $E_{ex}^2/2M_0$ in the case of a recoilless decay.

To consider this effect in a simple way we propose a binary encounter model in which the hadron constitutes a binary system together with a "quasi-nucleon" with a mass M_N^* . This binary system, with a total mass $M_{\text{total}} = M^* + M_N^*$, now decays to two particles 1 and 2 and one quasi-nucleon, as shown in Fig.3(upper). Let us assume that the colliding quasi-nucleon receives a momentum transfer q in the decay process. Then, the PIM composed of the decay particles 1 and 2 with masses m_1 and m_2 , respectively, is

$$M_{\text{inv}}^{*2} = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 \quad (20)$$

$$= (E - E_N)^2 - (\vec{P} - \vec{q})^2 \quad (21)$$

$$= M^{*2} - 2(E_N E - \vec{q} \cdot \vec{P} - M_N^* M^*) \quad (22)$$

where E and $\vec{P}$ are the total energy and momentum of the binary system of the mass $M^* + M_N^*$. In the rest frame of $\vec{P} = 0$ and for a small q , the PIM is given by

$$M_{\text{inv}}^* \rightarrow \sqrt{M^{*2} - \frac{M^* + M_N^*}{M_N^*} q^2}. \quad (23)$$

So, the PIM is expected to be shifted toward a lower value. In the limit $M_N^* \gg M^*$ it is

$$\sim M^* - \frac{q^2}{2M^*}. \quad (24)$$

Let us consider the cases of ρ and ϕ mesons. Their PIM's are shown versus the momentum transfer q in Fig.3. Even at a moderate momentum transfer, say, $q = p_F = 250 \text{ MeV}/c$, the following shifts are expected:

$$M_{\text{inv}}^*(\rho) = M_\rho - 76 \text{ MeV}, \quad (25)$$

$$M_{\text{inv}}^*(\phi) = M_\phi - 65 \text{ MeV}. \quad (26)$$

In Fig.3 we assumed free values for M_ρ and M_ϕ (770 MeV for ρ and 1020 MeV for ϕ). In a short-range correlation $q \gg p_F$ the M_{inv}^* is further reduced.

From this consideration we conclude that the PIM of short-lived hadrons such as ρ and ϕ mesons is subject to a considerable negative shift due to the collisional effect, even if the hadron mass is unchanged. The PIM provides information on the collision kinetics as well as the collision dynamics (genuine mass shift), but the latter cannot be distinguished from the former.

4.3. Interpretation of Experimental Data

The above consideration leads to further warning on a direct connection of a pseudo invariant mass to the genuine mass shift. PIM spectra are generally subject to collisional shifts and broadening merely as a kinematical effect. It is interesting to interpret some experimental data from this viewpoint.

First, we expect that any PIM spectrum for ρ meson decays (e^+e^- and $\mu^+\mu^-$) in nuclei may reveal broadened and shifted components, which should not be confused with a dynamical mass shift. Recent experiments of CERES [4] and HELIOS-3 [5] on the dilepton invariant mass showed anomalous excess components lying below the ρ meson mass, which are interpreted in terms of a modified in-medium mass by Li *et al.* [3], Cassing *et al.* [8] and Saito *et al.* [9], who predicted certain negative mass shifts of ρ meson in the nuclear medium. The decreased theoretical mass M^* can, however, not be directly connected to the enhancement of PIM spectra in the same mass region, because i) PIM reflects the total potential $M_{inv}^* = M_0 + U_S^{(0)} + U_V^{(0)} = M^* + U_V^{(0)}$ and ii) furthermore it is subject to an additional kinematical effect due to collisions. One has to examine the kinematical broadening and shift as well as the QCD mass and energy shifts on equal footing before connecting M_{inv}^* to M^* , as performed by Klingl and Weise [21], because they all come from the same collisions in the nuclear medium. It is an interesting question to see how the effect studied by Klingl and Weise is related to the phenomenological vector potential $U_V(r)$ which gives rise to PIM spectra.

Recently, Akiba *et al.* [6] reported an invariant mass spectrum of K^+ and K^- measured in high energy heavy ion reactions, which shows a clear sharp peak at the free mass of the ϕ meson (1020 MeV). Any broadened and shifted component is difficult to be isolated in the lower side of the PIM spectrum because there is a substantial background underneath the peak, though its possibility is not excluded. In the light of the above consideration the observed peak must correspond to the free-decay component of peripherally produced ϕ mesons decaying outside the colliding nuclei. The ϕ mesons decaying inside is expected to contribute only to an indistinguishable background below the peak.

When a hadron decays to two or more long-lived hadrons inside a nucleus, as in the case of $\rho \rightarrow \pi^+ + \pi^-$, the momentum of the decay hadron changes when it goes out of the boundary. Therefore, the PIM spectrum is deteriorated furthermore. Not only does the change of a decay particle momentum p_i depend on the potential height $V_0^{(i)}$ of the particle i (not of the parent hadron), but also the total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \dots$ depends on the geometry of the relevant momenta. In addition, the decay hadrons are also subject to collisional effects. All these trivial kinematical effects tend to make it difficult to deduce the genuine mass shift from PIM. We need comprehensive and unified treatments of all the effects that contribute to PIM.

Dilepton invariant mass spectra in the J/Ψ region have also been measured in nucleus-

nucleus collisions [22]. Since the lifetime of this particle is so long that they all decay outside the colliding nuclei, the invariant mass in this case is a genuine invariant mass. Neither collisional deterioration nor dynamical shift is expected.

5. Conclusion

We have clarified that the invariant mass spectroscopy applied to short-lived hadrons in nuclear systems provides "pseudo invariant masses" which are related to average potential energies both in bound and unbound systems. We have shown in various examples that they are not connected to hadron masses modified in the nuclear medium which are the scalar part of the potential. Furthermore, the pseudo invariant masses are lowered and broadened by the influence of rapid collisions with surrounding nucleons. Careful analyses are thus required to extract a correct implication of pseudo invariant mass spectra. Any theoretical approach to modified hadron masses, if they are to be compared with experiments, is thus required to calculate pseudo invariant mass spectra under the same footing .

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FIGURES

FIG. 1. Potential energies and pseudo invariant mass (PIM) distributions in typical bound systems. M_{inv}^* stands for a rms value of PIM. (a) Pure Coulombic atom. (b) A Λ hypernucleus $^{12}_\Lambda\text{C}$, in which different values of $U_S(0)/M_0$ are assumed. (c) The $1s$ π^- state in ^{207}Pb , where the mass shift $m_{\pi}^* - m_{\pi}$ is experimentally determined to be 19 MeV.

FIG. 2. A total non-relativistic potential $V(r)$ which is composed of a scalar part $U_S(r)$ and a vector part $U_V(r)$ in the relativistic framework. The PIM and the modified mass are indicated.

FIG. 3. (Upper) Binary encounter model for a hadron in rapid collisions with surrounding nucleons which are represented by an "effective nucleon" (N^*) with a mass M_N^* . The binary system with a total mass $M^* + M_N^*$ decays as $\rightarrow m_1 + m_2 + M_N^*$ with a momentum transfer to the effective nucleon. (Lower) Dependence of the PIM of the particles (m_1, m_2) on the momentum transfer of N^* .