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FRACTAL SUBSTRUCTURES OF THE UNIVERSE

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Abstract

It is pointed out that the Universe has fractal substructures such as clusters of galaxies, galaxies, clusters of stellar systems, and stellar systems due to the scaling properties in these gravitational subsystems with respect to the numbers and masses of constituents, and the size-scales and time-scales of substructures. Also suggested is a clue to explain both the “less-large-number hypothetical relations” of $N_G \sim N_S (\sim N_C \sim 10^{11})$ where N_G and N_S (N_C) are the “total number of galaxies” in the Universe and the average total number of stars in a galaxy (the average total number of comets in a solar system), respectively, and the “even-less-large-number hypothetical relations” of $n_1 \sim n_2 \sim n_3 (\sim 10^3)$ for $n_1 \equiv R_U/D_G$, $n_2 \equiv D_G/R_G$, and $n_3 \equiv R_G/D_S$ where R_U (R_G) is the “radius of the Universe” (the average radius of an ordinary galaxy) and D_G (D_S) is the average distance between two neighboring galaxies (that between two neighboring stars).

The large number hypothesis (LNH) by Dirac [1] states that the Eddington large numbers [2] $N_1 (\equiv \alpha/Gm_em_p \cong 10^{39})$, $N_2 (\equiv m_e/\alpha H \cong 10^{40})$, and $N_3 (\cong 4\pi\rho/3m_p H^3 \cong 10^{79})$ are not independent but related with each other. Recently, I have shown that not only the “dynamical” LNH relation of $N_3 \sim N_1 N_2$ [3] but also the “geometrical” LNH relation of $N_3 \sim (N_2)^2$ [4] holds so that the LNH may not be taken as a hypothesis but become the large number rule (LNR). In the analogy of the LNH, I have proposed the “less large number hypothesis (LLNH)” of $N_G \sim N_S$ where N_G and N_S are the total number of galaxies in the Universe and the average total number of stars in a galaxy since both of them are of the order of 10^{11} [5]. I have also suggested the two “geometrical LLNH relations” of $N_G \sim (R_U/R_G)^2$ and $N_S \sim R_G/R_S$, where R_U , R_G , and R_S are the “radius of the Universe”, the average radius of a galaxy, and that of a star, and pointed out that the “even less large number hypothesis (ELLNH)” of $n_1 \sim n_2 \sim n_3$ may also work for $n_1 \equiv R_G/D_G$, $n_2 \equiv D_G/R_G$, and $n_3 \equiv R_G/D_S$, where D_G and D_S are the average distance between two neighboring galaxies and that between two neighboring stars, since all of them are of the order of 10^3 [5]. We should note that the first geometrical LLNH relation of $N_G \sim (R_U/R_G)^2$ seems to have been visually confirmed by the pictures of Hubble Deep Field [6].

More recently, I have discussed possible effects of the non-vanishing sizes of particles (atoms, nuclei, nucleons, quarks, and leptons) in the early Universe (the temperature T) in an extended Friedmann model of the Universe (the scale a) and especially pointed out the following possibilities: 1) if $r_q > (2N_B/\pi)^{-1/3}a$, most of the proposed scenarios for $T > 10^3$ TeV including the inflationary Universe one are unrealistic, 2)

$r_q < (2N_B/\pi)^{-1/3}a$ due to the smallness of r_q ($\lesssim 10^{-27}$ cm), 3) $r_q < (2N_B/\pi)^{-1/3}a$ due to the smallness of N_B in which case the baryon number (or quark number) must be generated at $T \lesssim 10^3$ TeV if $r_q \gtrsim 10^{-17}$ cm (where r_q and N_B are the effective radius of quarks and the baryon number in the Universe), and 4) for $T \gtrsim 10^3$ TeV, the Universe was filled not with quark-gluon plasma but with “subquark plasma” [7]. Furthermore, I have suggested and discussed in some detail that the Einstein-Friedmann field equations should be extended in order not only to accommodate the LNR, LLNH, and ELLNH but also to discuss squeezed states in the early Universe by taking into account the non-vanishing sizes of particles, stars, and galaxies [7]. We should note that the unreality of inflationary Universe scenario has very recently been suggested both by Supernova Cosmology Project and by High-Z Supernova Search Team [8] from their direct observations of the Hubble parameter “at half the age of the Universe”.

Very lately, I have made one more speculation of extending the LLNH into a stronger form of $N_G \sim N_S \sim N_C$, where N_C is the average total number of comets (whose sizes are of the order of 10 km) in a stellar system, since N_C in our solar system is of the order of 10^{11} [9]. This “strong LLNH relation” strongly and almost uniquely suggests the following possible origin of the double coincidence: Since the big-bang of the Universe, either one of galaxies in the Universe, stars in a galaxy, and comets in a stellar system has been created simultaneously by itself due to fluctuations of matter, the numbers of fluctuation in these subsystems of the Universe which form a kind of entropy are of the same order of magnitude, and the Universe has a kind of fractal substructures [9]. We should note that the very early but complete formation of galaxies at a redshift

of $Z = 4.92$ has more lately reported by Franx *et al.* [10]. The purpose of this letter is to try to explain this simultaneous creation of galaxies, stars, and comets with the same order of magnitude for their total numbers in the substructures of the Universe. Especially, I shall point out that the Universe has fractal substructures such as clusters of galaxies, galaxies, clusters of stellar systems, and stellar systems due to the scaling properties in these gravitational subsystems with respect to the numbers and masses of constituents, and the size-scales and time-scales of substructures.

Suppose that the Universe or a substructure of the Universe (a cluster of galaxies, a galaxy, or a cluster of stellar systems) consists of N constituents (clusters of galaxies, galaxies, clusters of stellar systems, or stellar systems). For simplicity, take these constituents as point particles with the masses m_i and the space-time coordinates x_i^μ ($i = 1, 2, 3, \dots, N$). For further simplicity, let us assume that the space-time is flat and the motion of the constituents is non-relativistic. (A possible extension of this classical Newtonian cosmological model to a general-relativistic Einstein-Friedmann model will be discussed later.) Then, the motion of the constituents in a gravitational system can be described by the following Newtonian equations of motion:

$$\frac{d^2 \vec{x}_i}{dt^2} = -G \sum_{\substack{j=1 \\ (j \neq i)}}^N \frac{m_j (\vec{x}_i - \vec{x}_j)}{|\vec{x}_i - \vec{x}_j|^3} \quad (i = 1, 2, 3, \dots, N), \quad (1)$$

where G is the Newtonian gravitational constant. As is well known, no one can solve this set of equations analytically except for the case of $N = 2$. I have no intention to solve this set of equations. Instead, I would like to imagine what the motion of the constituents described by this set of equations would look like. To this end, let us

introduce the following “generalized Kepler’s scaling transformation”:

$$(\vec{x}_i, t) \equiv (L\vec{\xi}_i, T\tau) \text{ and } m_i = M\mu_i \quad (i = 1, 2, 3, \dots, N), \quad (2)$$

where L , T , and M are arbitrary constants with the length, time, and mass dimensions, respectively. Then, the set of equations (1) will be transformed into

$$\frac{d^2 \vec{\xi}_i}{d\tau^2} = -\frac{GMT^2}{L^3} \sum_{\substack{j=1 \\ (j \neq i)}}^N \frac{\mu_j(\vec{\xi}_i - \vec{\xi}_j)}{|\vec{\xi}_i - \vec{\xi}_j|^3} \quad (i = 1, 2, 3, \dots, N). \quad (3)$$

This form of equations seems very suggestive and even inspiring: it not only indicates Kepler’s law in the Kepler motion for $N = 2$ (which means “Kepler’s scaling” with the equality of T^2/L^3) but also suggests many other hidden properties in the motion of constituents of substructures of the Universe. In order to find some of such properties more explicitly, let us rewrite the set of equations (3) in the form of

$$\frac{d^2 \vec{\xi}_i}{d\tau^2} = -\frac{\omega}{2} \vec{f}_i(\vec{\xi}_j) \quad (i, j = 1, 2, 3, \dots, N), \quad (4)$$

where ω and $\vec{f}_i(\vec{\xi}_j)$ are defined by

$$\frac{\omega}{2} \equiv \frac{GNMT^2}{L^3} \quad (5)$$

and

$$\vec{f}_i(\vec{\xi}_j) \equiv \frac{1}{N} \sum_{\substack{j=1 \\ (j \neq i)}}^N \frac{\mu_j(\vec{\xi}_i - \vec{\xi}_j)}{|\vec{\xi}_i - \vec{\xi}_j|^3} \quad (i, j = 1, 2, 3, \dots, N). \quad (6)$$

Then, one can find the following two idealistic cases: 1) If the total numbers of constituents in two gravitational systems (A and B) are the same, i.e. $N_A = N_B (= N)$, and if the masses of constituents in each system are all identical, i.e. $\mu_i^A = \mu_A$ and $\mu_i^B = \mu_B (i = 1, 2, 3, \dots, N)$, these two systems are dynamically “equivalent” since

$\vec{f}_i^A(\vec{\xi}_j^A) = \vec{f}_i^B(\vec{\xi}_j^B)$ for $\vec{\xi}_j^A = \vec{\xi}_j^B$ and $\omega_A = \omega_B$ for $\mu_A T_A^2/L_A^3 = \mu_B T_B^2/L_B^3$; 2) If the total number of constituents in a gravitational system (A) is given by the ratio of the cubic length-scale for the system to that for its lower subsystem (B), i.e.

$$N_A = (L_A/L_B)^3, \quad (7)$$

and if the masses of constituents in each system are all identical, these two systems has the same time-scale since $\mu_A = N_B \mu_B$ and, therefore, $\omega_A = \omega_B$ for $T_A = T_B$. We should note that the same time-scale in a system and its lower subsystem may guarantee dynamical stability of both systems without suffering from “frictions” among subsystems. It may remind us of the equality of “day” and “year” not only for Mercury and Venus but also for the Moon.

Although any gravitational systems in the Universe are far from these two idealistic cases, I have found that the systems of galaxies in the Universe and stars in a galaxy (and comets in a stellar system) with the same order of magnitude for $N_G \sim N_S (\sim N_C \sim 10^{11})$, those of clusters of galaxies in the Universe and clusters of stars in a galaxy both with $N \sim 10^8 - 10^{10}$, or those of galaxies in a cluster of galaxies and stars in a stellar cluster with $N \sim 10^1 - 10^3$ simulate the case 1) while the systems of clusters of galaxies in the Universe and galaxies in a cluster of galaxies with N (clusters of galaxies) $\sim 10^8 - 10^{10}$, L (size of the Universe) $\sim 10^{10}$ yr, and L (size of cluster of galaxies) $\sim 10^7$ yr or those of stellar clusters in a galaxy and stars in a stellar cluster with N (stellar clusters) $\sim 10^8 - 10^{10}$, L (size of a galaxy) $\sim 10^4$ yr and L (size of a stellar cluster) $\sim 10^1$ yr simulate the case 2), satisfying the relation (7). We should note that the former case is closely related to the LLNH of $N_G \sim N_S (\sim N_C)$ while the

latter to the ELLNH of $n_1 \sim n_3$.

From the above considerations, I can reach the following gross picture on substructures of the Universe: The Universe has 10^{8-10} clusters of galaxies. The Universe and these clusters of galaxies have time-scales of the same order. A galaxy has 10^{8-10} stellar clusters. A galaxy and these stellar clusters have much smaller time-scales of the same order. (A stellar system would have 10^{8-10} comets. A star and these comets would have even much smaller time-scales of the same order.) As a whole, the Universe has a kind of fractal structure of substructures. The origin of this fractal structure in the Universe is definitely the scaling properties in these gravitational subsystems with respect to the numbers and masses of constituents, and the size-scales and time-scales of substructures.

A possible extension of the classical Newtonian cosmological model to a general-relativistic Einstein-Friedmann model can be made by changing the set of Newtonian equations of motion (1) into the Einstein-Friedmann field equations of

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G \sum_{i=1}^N m_i u_\mu^i u_\nu^i \delta^3(\vec{x} - \vec{x}_i(\lambda)) \quad (8)$$

and the set of geodesic equations of

$$\frac{d^2 x_i^\mu}{d\lambda^2} + \Gamma_{\alpha\beta}^\mu \frac{dx_i^\alpha}{d\lambda} \frac{dx_i^\beta}{d\lambda} = 0 \quad (i = 1, 2, 3, \dots, N). \quad (9)$$

Let us introduce the “general-relativistic Kepler’s scaling transformation” of

$$x_i^\mu = L\xi_i^\mu \text{ and } m_i = M\mu_i \quad (i = 1, 2, 3, \dots, N). \quad (10)$$

Then, Eqs. (8) and (9) will be transformed into

$$r_{\mu\nu} - \frac{1}{2}g_{\mu\nu}r = \frac{8\pi GM}{L} \sum_{i=1}^N \mu_i u_\mu^i u_\nu^i \delta^3(\vec{\xi} - \vec{\xi}_i(\lambda)) \quad (11)$$

and

$$\frac{d^2 \xi_i^\mu}{d\lambda^2} + \gamma_{\alpha\beta}^\mu \frac{d\xi_i^\alpha}{d\lambda} \frac{d\xi_i^\beta}{d\lambda} = 0 \quad (i = 1, 2, 3, \dots, N) \quad (12)$$

where $r_{\mu\nu} = L^2 R_{\mu\nu}$ and $\gamma_{\alpha\beta}^\mu = L\Gamma_{\alpha\beta}^\mu$. From these equations, one can reach the same conclusions as in the classical Newtonian cosmological model. We should note that neither whether the Universe is closed, flat, or open nor whether there exists the cosmological term or not would affect our conclusions although the latest observations have indicated the open and eternally expanding Universe due to the low matter density [8].

Let us now present a possible clue to explain both the LLNH and ELLNH relations. The LLNH seems to be closely related to similarity between the dynamics of galaxies in the Universe and that of stars in a galaxy while the ELLNH to similarity between the dynamics of galaxies in a cluster of galaxies and that of stars in a stellar cluster. As is well known [11], the masses of ordinary stars must be of the order of $G^{-3/2}m_p^{-2}(\sim 10^{57}m_p)$ or, in other words, an ordinary star consists of about $(G^{1/2}m_p)^{-3}(\sim 10^{57})$ baryons. Therefore, by accepting the similarity between the Universe and a galaxy and by knowing $N_3(\equiv 4\pi\rho/3m_p H^3 \cong 10^{79}) \sim N_1 N_2(\equiv 1/Gm_p H \cong 10^{79})$, we can obtain the LLNH relation of

$$N_G \sim N_S \sim [N_3/(G^{1/2}m_p)^{-3}]^{1/2}(\cong G^{1/4}m_p H^{-1/2}) \sim 10^{11}. \quad (13)$$

Furthermore, by accepting the similarity between a cluster of galaxies and a stellar cluster and by applying the stability condition (7) both to clusters of 10^{1-3} galaxies in the Universe, i.e. N (clusters of galaxies) $\sim (n_1)^3$, and to clusters of 10^{1-3} stars in a

galaxy, i.e. N (stellar clusters) $\sim (n_3)^3$, we can obtain the ELLNH relation of

$$n_1 \sim n_3 \sim (N_G \text{ or } N_S/10^{1-3})^{1/3} \sim 10^3. \quad (14)$$

In a sense, these large number hypotheses including the original Dirac's LNH can be taken as laws of scaling of scalings in the Universe.

In conclusion, let us point out that very very lately Sylos-Labini *et al.* [12] have found the fractal dimension of $D = 2$ in their analysis of the redshift surveys. We are looking forward to seeing the results from more detailed future analyses of their type. Let us also point out that it seems extremely interesting to see whether computer simulations of time-dependent solutions for the set of equations of motion (1) would exhibit any fractal structures. It seems worth doing them even in the lower dimensions of $D = 1$ and 2. It is to see how fluctuations in the initial conditions would evolve in gravitational systems. After all, all that happened, are happening, and will happen in the Universe (including even the Creation of life) be due to fluctuations of the space-time and matter or the chaotic motion of over 10^{79} particles governed by strong, electroweak, and gravitational forces.

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