Semileptonic $B$ decays into excited charmed mesons from QCD sum rules

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Abstract

Exclusive semileptonic $B$ decays into excited charmed mesons are studied with QCD sum rules in the leading order of heavy quark effective theory. Two universal Isgur-Wise functions $\tau$ and $\zeta$ for semileptonic $B$ decays into four lowest lying excited $D$ mesons ($D_1$, $D_2^*$, $D_0'$, and $D_1'$) are determined. The decay rates and branching ratios for these processes are calculated.

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I. INTRODUCTION

The heavy quark effective theory (HQET) [1,2] is a useful tool to describe the spectroscopy and matrix elements of mesons containing a heavy quark. In the infinite mass limit, the spin and parity of the heavy quark and that of the light degrees of freedom are separately conserved. Coupling the spin of light degrees of freedom $j_\ell$ with the spin of heavy quark $s_Q = 1/2$ yields a doublet of meson states with total spin $j = j_\ell \pm 1/2$. The ground state mesons with $Q \bar{q}$ flavor quantum numbers contain light degrees of freedom with spin-parity $j_\ell P = 1/2$, giving a doublet $(0^-, 1^-)$. For $Q = c$ these mesons are the doublet $(D, D^*)$, while $Q = b$ gives the doublet $(B, B^*)$. The excited charmed mesons with $j_\ell P = 1^+, 3^+$ are two spin symmetry doublets $(0^+, 1^+)$ and $(1^+, 2^+)$. These mesons are doublets $(D'_0, D'_1)$ and $(D_1, D_2^*)$. The $D_1$ and $D_2^*$ mesons have been observed with rather small widths, while $D'_0$ and $D'_1$ have not been observed for the reason of being too broad. The properties of these lowest lying excited mesons have attracted attention in recent years. The mass and decay widths have been studied with potential model [3,4], relativistic Bethe-Salpeter equation [5] and QCD sum rules [6–8].

Progress has been achieved recently in the studies of semileptonic $B$ decays into excited charmed mesons $(D'_0, D'_1, D_1$ and $D_2^*)$. Semileptonic $B$ decay into an excited heavy meson has been observed in experiments [9–12]. With some assumptions, CLEO and ALEPH collaborations have reported respectively the branching ratios $\mathcal{B}(B \to D_1 e \bar{\nu}_e) = (0.56 \pm 0.13 \pm 0.08 \pm 0.04)\%$ and $\mathcal{B}(B \to D_1 e \bar{\nu}_e) = (0.74 \pm 0.16)\%$, as well as the limits $\mathcal{B}(B \to D_2^* e \bar{\nu}_e) < 0.8\% (90\%$ C.L) and $\mathcal{B}(B \to D_2^* e \bar{\nu}_e) < 0.2\%$ [10,12]. Theoretically, HQET provides a systematic method for investigating such processes. The semileptonic $B$ decay rate to an excited charmed meson is determined by the corresponding matrix elements of the weak axial-vector and vector currents. Heavy quark symmetries can be used to reduce the form factors parameterizing the matrix elements. In the infinite quark mass limit these matrix elements are described respectively by one universal Isgur-Wise function and vanish at zero recoil [13,14]. Extensive investigation in [14] shows that the leading $1/m_Q$ correction at zero recoil can be calculated in a model independent way in terms of the masses of charmed meson states.

The universal function embodies details of low energy strong interactions and cannot be calculated from first principles. It must be calculated in some nonperturbative approaches. For that purpose, there are many viable approaches, including different quark models [15–21], relativistic Bethe-Salpeter equation [22] and QCD sum rules [23–26]. In the present work we study the $B$ semileptonic decays to excited charmed meson states $(D'_0, D'_1)$ and $(D_1, D_2^*)$ with QCD sum rule in the leading order of HQET. In particular we compute the relevant universal Isgur-Wise functions that describe such decays in the $m_Q \to \infty$ limit.

The remainder of this paper is organized as follows. In Section II we present the
II. ANALYTIC FORMULAE FOR SEMILEPTONIC DECAY AMPLITUDES

\( B \to (D_1, D_2')\ell \bar{\nu} \) AND \( B \to (D_1', D_2)\ell \bar{\nu} \)

The theoretical description of semileptonic decays involves the matrix elements of vector and axial vector currents \((V^\mu = \bar{c}\gamma^\mu b\) and \(A^\mu = \bar{c}\gamma^\mu\gamma_5 b\)) between \(B\) mesons and excited \(D\) mesons. For the processes \(B \to D_1\ell \bar{\nu}\) and \(B \to D_2'\ell \bar{\nu}\), these matrix elements can be parameterized as

\[
\begin{align*}
\langle D_1(v', \epsilon) | V^\mu | B(v) \rangle &= \sqrt{m_{D_1}m_B} [f_{V_1} \epsilon^\mu + (f_{V_2} v^\mu + f_{V_3} v'^\mu) \epsilon^* \cdot v], \quad (1a) \\
\langle D_1(v', \epsilon) | A^\mu | B(v) \rangle &= i \sqrt{m_{D_1}m_B} f_A \epsilon^{\alpha\beta\gamma} v_\beta v'_\gamma, \quad (1b) \\
\langle D_2'(v', \epsilon) | A^\mu | B(v) \rangle &= \sqrt{m_{D_2'}} m_B [k_{A_1} \epsilon^{\alpha\mu} v_\alpha + (k_{A_2} v^\mu + k_{A_3} v'^\mu) \epsilon^* \cdot v], \quad (1c) \\
\langle D_2'(v', \epsilon) | V^\mu | B(v) \rangle &= i \sqrt{m_{D_2'} m_B} k_V \epsilon^{\alpha\beta\gamma} \epsilon^*_{\alpha\sigma} v^\sigma v'_\gamma. \quad (1d)
\end{align*}
\]

The form factors \(f_i\) and \(k_i\) are functions of \(y = v \cdot v'\). In the limit \(m_Q \to \infty\) they can be expressed by a universal dimensionless function \(\tau(y)\) [13,14].

\[
\begin{align*}
f_{V_1}(y) &= \frac{1}{\sqrt{6}} (1 - y^2) \tau(y), & f_{V_2}(y) &= -\frac{3}{\sqrt{6}} \tau(y),
\end{align*}
\]

\[
\begin{align*}
f_{V_3}(y) &= \frac{1}{\sqrt{6}} (y - 2) \tau(y), & f_A(y) &= -\frac{1}{\sqrt{6}} (1 + y) \tau(y),
\end{align*}
\]

\[
\begin{align*}
k_{A_1}(y) &= -(1 + y) \tau(y), & k_{A_2}(y) &= 0, & k_{A_3}(y) &= \tau(y), & k_V(y) &= -\tau(w).
\end{align*}
\]

Since the polarisation vector of the spin-one state \(D_1\) and the polarization tensor of the spin-two state \(D_2'\) satisfy \(\epsilon^* \cdot v' = 0\) and \(\epsilon^{\alpha\mu} v'_\alpha = 0\) respectively, only the form factor \(f_{V_1}\) contributes at zero recoil \((v = v')\).

The form factors that parameterize the \(B \to D_0'\ell \bar{\nu}\) and \(B \to D_1'\ell \bar{\nu}\) matrix elements of the weak currents are defined by

\[
\begin{align*}
\langle D_0'(v') | V^\mu | B(v) \rangle &= 0, & \langle D_0'(v') | A^\mu | B(v) \rangle &= \sqrt{m_{D_0} m_B} [g_+ (v^\mu + v'^\mu) + g_- (v^\mu - v'^\mu)], \quad (3a) \\
\langle D_1'(v', \epsilon) | V^\mu | B(v) \rangle &= \sqrt{m_{D_1} m_B} [g_{V_1} \epsilon^\mu + (g_{V_2} v^\mu + g_{V_3} v'^\mu) \epsilon^* \cdot v], \quad (3b) \\
\langle D_1'(v', \epsilon) | A^\mu | B(v) \rangle &= \sqrt{m_{D_1} m_B} i g_A \epsilon^{\alpha\beta\gamma} \epsilon^*_{\alpha\sigma} v^\sigma v'_\gamma, \quad (3c)
\end{align*}
\]
where \( g_i \) are functions of \( y \). At zero recoil the matrix elements are determined by \( g_+ (1) \) and \( g_{V_1} (1) \). In the infinite mass limit form factors \( g_i \) can be written in terms of a single function \( \zeta (y) \) [13,14],

\[
\begin{align*}
g_+ (y) &= 0 , \\
g_-(y) &= \zeta (y) , \\
g_A (y) &= \zeta (y) , \\
g_{V_1} (y) &= (y - 1) \zeta (y) , \\
g_{V_2} (y) &= 0 , \\
g_{V_3} (y) &= - \zeta (w) .
\end{align*}
\]

Note that the notations of Ref. [14] have been used here. \( \tau \) is \( \sqrt{3} \) times the function \( \tau_{3/2} \) of Ref. [13], while \( \zeta \) is two times the function of \( \tau_{1/2} \). All of these matrix elements of the weak currents vanish at zero recoil \( y = 1 \), since the \( B \) meson and the \((D_1, D_2) \) or \((D'_0, D'_1) \) mesons are in different heavy quark spin symmetry multiplets, and the current at zero recoil is related to the conserved charges of heavy quark spin-flavor symmetry.

The differential decay rates are given by (taking the mass of the final lepton to zero)

\[
\begin{align*}
\frac{d \Gamma_{D_1}}{dy} &= \frac{G_F^2 |V_{cb}|^2 m_B^5}{72 \pi^3} r_1^3 (y + 1) (y^2 - 1)^{3/2} \left[ (y - 1)(1 + r_1^2) + y(1 - 2r_1 y + r_1^2) \right] |\tau (y)|^2 , \\
\frac{d \Gamma_{D_2}}{dy} &= \frac{G_F^2 |V_{cb}|^2 m_B^5}{72 \pi^3} r_2^3 (y + 1) (y^2 - 1)^{3/2} \left[ (y + 1)(1 - r_2^2) + 3y(1 - 2r_2 y + r_2^2) \right] |\tau (y)|^2 , \\
\frac{d \Gamma_{D_0}}{dy} &= \frac{G_F^2 |V_{cb}|^2 m_B^5}{48 \pi^3} r_0^3 (y^2 - 1)^{3/2} |1 - r_0^2|^2 |\zeta (y)|^2 , \\
\frac{d \Gamma_{D_1'}}{dy} &= \frac{G_F^2 |V_{cb}|^2 m_B^5}{48 \pi^3} r_1' (y - 1) \sqrt{y^2 - 1} \left[ (y - 1)(1 + r_1')^2 + 4y(1 - 2r_1' y + r_1'^2) \right] |\zeta (y)|^2 ,
\end{align*}
\]

where \( r_1 = m_{D_1} / m_B , \ r_2 = m_{D_2} / m_B , \ r_0 = m_{D_0} / m_B \) and \( r_1' = m_{D_1'} / m_B \).

### III. SUM RULES FOR ISGUR-WISE FUNCTIONS \( \tau \) AND \( \zeta \)

#### A. Interpolating currents for heavy mesons of arbitrary spin and parity and two-point correlation function

A basic element in the application of QCD sum rules to excited heavy mesons is to choose a set of appropriate interpolating currents in terms of quark fields each of which creates (annihilate) a definite excited state of the heavy mesons. The proper interpolating current \( J_{a_1 a_2}^{\alpha \beta} \) for the state with the quantum number \( j, P, j_\ell \) in HQET was given in [7]. These currents have nice properties. They were proved to satisfy the following conditions

\[
\begin{align*}
\langle 0 | J_{a_1 a_2}^{\alpha \beta} (0) | j', P', j_\ell' \rangle &= i \int d \delta \delta_{P P'} \delta_{j j'} \eta_{a_1 a_2}^{j j'}, \\
i \langle 0 | T \left( J_{a_1 a_2}^{\alpha \beta} (x) J_{b_1 b_2}^{\gamma \delta} (0) \right) | 0 \rangle &= \delta_{BB'} \delta_{JJ'} \delta_{j j_\ell} (-1)^j S g_1^{\alpha \gamma} \cdots g_1^{\alpha \gamma} \cdots, \\
&\times \int dt \delta (x - vt) \Pi_{P j_\ell} (x)
\end{align*}
\]
in the \( m_Q \to \infty \) limit. Where \( \eta^{\alpha_1 \cdots \alpha_j} \) is the polarization tensor for the spin \( j \) state, \( v \) is the velocity of the heavy quark, \( g_{1}^{\alpha \beta} = g^{\alpha \beta} - v^{\alpha} v^{\beta} \) is the transverse metric tensor, \( S \) denotes symmetrizng the indices and subtracting the trace terms separately in the sets \( (\alpha_1 \cdots \alpha_j) \) and \( (\beta_1 \cdots \beta_j) \), \( f_{P,j} \) and \( \Pi_{P,j} \) are a constant and a function of \( x \) respectively which depend only on \( P \) and \( j \). Because of equations (9) and (10), the sum rules in HQET for decay amplitudes derived from a correlator containing such currents receive contribution only from one of the two states with the same spin-parity \((j,P)\) in the \( m_Q \to \infty \). Starting from the calculations in the leading order, the decay amplitudes for finite \( m_Q \) can be calculated unambiguously order by order in the \( 1/m_Q \) expansion in HQET.

In the following we focus our attention on the semileptonic decays, \( B \to D_1, D_2 \) and \( B \to D_0, D'_1 \). The relevant doublets to be considered are ground states and lowest lying positive parity states, namely, doublets \((0^-,1^-)\), \((0^+,1^+)\) and \((1^+,2^+)\). The currents for creating \( 0^- \) and \( 1^- \) are usual pseudoscalar and vector currents

\[
J_{0,-}^{\dagger} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_5 q , \\
J_{1,-}^{\dagger} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma^\alpha q .
\] (11)

As pointed out in [7], there are two possible choices for currents creating \( 0^+ \) and \( 1^+ \) of the doublet \((0^+,1^+)\), either

\[
J_{0,+}^{\dagger} = \frac{1}{\sqrt{2}} \bar{h}_v q , \\
J_{1,+}^{\dagger} = \frac{1}{\sqrt{2}} \bar{h}_v \gamma^\alpha \gamma_\ell q ,
\] (12) (13)
or

\[
J_{0,+}^{\dagger} = \frac{1}{\sqrt{2}} \bar{h}_v (-i) \mathcal{D}_l q , \\
J_{1,+}^{\dagger} = \frac{1}{\sqrt{2}} \bar{h}_v \gamma^\alpha \gamma_\ell (-i) \mathcal{D}_l q ,
\] (14) (15)

where \( \mathcal{D} \) is the covariant derivative. Similarly, there are two possible choices for the currents creating \( 1^+ \) and \( 2^+ \) of the doublet \((1^+,2^+)\). One is

\[
J_{1,+}^{\dagger} = \sqrt{\frac{3}{4}} \bar{h}_v \gamma^5 (-i) \left( \mathcal{D}_l^\alpha - \frac{1}{3} \gamma_\ell^\alpha \mathcal{D}_l \right) q ,
\] (16)

\[
J_{2,+}^{\dagger} = \sqrt{\frac{1}{2}} \bar{h}_v \left( \gamma^\alpha_1 \mathcal{D}_l^\alpha_2 + \gamma^\alpha_2 \mathcal{D}_l^\alpha_1 - \frac{2}{3} g^{\alpha_1 \alpha_2 \ell} \mathcal{D}_l \right) q .
\] (17)

Another choice is obtained by adding a factor \(-i \mathcal{D}_l\) to (16) and (17). Note that, without the last term in the bracket in (16) the current would couple also to the \( 1^+ \) state in the doublet \((0^+,1^+)\) even in the limit of infinite \( m_Q \).
Using QCD sum rules, one studies the analytic properties of the three-point correlators \( \zeta \). Here \( m \) to be \([7]\). When the currents (16) and (17) are used the sum rule for the \((1^+,1^+)\) doublet is well-known. It is: \([2,25]\)

\[
f_+^{\frac{1}{2}} e^{-2\frac{\Lambda}{T}/T} = \frac{3}{16\pi^2} \int_0^{\omega_{\gamma}} \omega^2 e^{-\omega/T} d\omega - \frac{1}{2} \langle \bar{q}q \rangle \left( 1 - \frac{m_0^2}{4T^2} \right). \tag{18}\]

For the doublet \((0^+,1^+)\), when the currents \( J'_{0,+2} \) and \( J'_{1,+2} \) in (14), (15) are used the sum rule (same for the two states) after the Borel transformation is found to be \([7]\)

\[
f_+^{\frac{1}{2}} e^{-2\frac{\Lambda}{T}/T} = \frac{3}{2\pi^2} \int_0^{\omega_{\gamma}} \omega^2 e^{-\omega/T} d\omega - \frac{1}{2} \langle \bar{q}q \rangle. \tag{19}\]

The corresponding formula when the current \( J_{0,+2} \) and \( J_{1,+2} \) in (12) and (13) are used instead of \( J'_{0,+2} \) and \( J'_{1,+2} \) is the following \([7]\)

\[
f_+^{\frac{1}{2}} e^{-2\frac{\Lambda}{T}/T} = \frac{3}{16\pi^2} \int_0^{\omega_{\gamma}} \omega^2 e^{-\omega/T} d\omega + \frac{1}{2} \langle \bar{q}q \rangle - \frac{1}{8T^2} m_0^2 \langle \bar{q}q \rangle. \tag{20}\]

When the currents (16) and (17) are used the sum rule for the \((1^+,2^+)\) doublet is found to be \([7]\)

\[
f_+^{\frac{1}{2}} e^{-2\frac{\Lambda}{T}/T} = \frac{1}{2\pi^2} \int_0^{\omega_{\gamma}} \omega^2 e^{-\omega/T} d\omega - \frac{1}{12} m_0^2 \langle \bar{q}q \rangle - \frac{1}{25} \left( \frac{1}{\pi} G^2 \right) T. \tag{21}\]

Here \( m_0^2 \langle \bar{q}q \rangle = \langle \bar{q}g\sigma_{\mu\nu}G^{\mu\nu}q \rangle \). Above results will be used in the following sections.

### B. sum rules for \( \tau \) and \( \zeta \)

For the amplitudes of the semileptonic decays to excited states in the infinite mass limit, the only unknown quantities in the expressions (5)-(8) are the universal functions \( \tau(y) \) and \( \zeta(y) \). Let us first consider the Isgur-Wise function \( \tau \). In order to calculate this form factor using QCD sum rules, one studies the analytic properties of the three-point correlators

\[
i^2 \int d^4x d^4z e^{i(k'-x-k-z)} \langle 0 | T \left( J^\nu_{1,+\frac{1}{2}}(x) J^{\mu\nu}_{V,A}(0) J^\dagger_{0,-\frac{1}{2}}(z) \right) | 0 \rangle = \Xi(\omega,\omega',y) L_{V,A}^{\nu\mu}, \tag{22a}\]

\[
i^2 \int d^4x d^4z e^{i(k'-x-k-z)} \langle 0 | T \left( J^\alpha_{2,+\frac{1}{2}}(x) J^{\mu\alpha\nu}_{V,A}(0) J^\dagger_{0,-\frac{1}{2}}(z) \right) | 0 \rangle = \Xi(\omega,\omega',y) L_{V,A}^{\nu\mu\alpha\beta}, \tag{22b}\]

where \( J^{\nu}_{V,A} = \bar{h}(v')\gamma^\nu h(v) \), \( J^{\mu\nu}_{A} = \bar{h}(v')\gamma^\mu\gamma_5 h(v) \). The variables \( k, k' \) denote residual “off-shell” momenta which are related to the momenta \( P \) of the heavy quark in the initial
state and \( P' \) in the final state by \( k = P - m_Q v, \ k' = P' - m_Q v' \) respectively. For heavy quarks in bound states they are typically of order \( \Lambda_{QCD} \) and remain finite in the heavy quark limit. \( \mathcal{L}_{V,A} \) are Lorentz structures associated with the vector and axial vector currents (see Appendix).

The coefficient \( \Xi(\omega, \omega', y) \) in (22) is an analytic function in the “off-shell energies” \( \omega = 2v \cdot k \) and \( \omega' = 2v' \cdot k' \) with discontinuities for positive values of these variables. It furthermore depends on the velocity transfer \( y = v \cdot v' \), which is fixed at its physical region for the process under consideration. By saturating (22) with physical intermediate states in HQET, one finds the hadronic representation of the correlator as following

\[
\Xi_{\text{hadro}}(\omega, \omega', y) = \frac{ f_{-\frac{1}{2}} f_{+\frac{1}{2}} \tau(y)}{(2\Lambda_{-\frac{1}{2}} - \omega - i\epsilon)(2\Lambda_{+\frac{1}{2}} - \omega' - i\epsilon)} + \text{higher resonances} ,
\]

where \( f_{P,j} \) are constants defined in (9), \( \Lambda_{P,j} = m_{P,j} - m_Q \). As the result of equation (9), only one state with \( j^P = 1^+ \) or \( j^P = 2^+ \) contributes to (23), the other resonance with the same quantum number \( j^P \) and different \( j_l \) does not contribute. This would not be true for \( j^P = 1^+ \) if the last term in (16) is absent.

Using the expression for the heavy quark propagator \( \int dt \delta(x - vt)(1 + \not{v})/2 \) in HQET the correlators in (22) have the following form

\[
i^2 \int dt_1 dt_2 \, e^{i(\omega t_1 + \omega' t_2)/2} \, \text{Tr} \left\{ \frac{1 + \not{\gamma}}{2} \gamma_{V,A} \frac{1 + \not{\gamma}'}{2} \langle 0 | \Gamma' q(v't_2) \bar{q}(-v t_1) | 0 \rangle \right\} ,
\]

where \( \Gamma \) and \( \Gamma' \) are functions of Dirac matrices and covariant derivatives appearing in the definitions of the currents. For \( \omega \) and \( \omega' \) in the deep Euclidian region the integral is dominated by small values of \( t_1 \) and \( t_2 \). As usually done in QCD sum rule approach, \( \omega \) and \( \omega' \) are analytically continued to the deep Euclidian region. Therefore OPE in the short distance can be applied to the above expression for any value of \( y \) not much larger than 1. This is because, for small \( t_1, t_2 \) and \( y \) of the order \( \mathcal{O}(1) \), all components of \( vt_1 \) and \( v't_2 \) can be made small by choosing the rest frame of \( B \). In our case the maxima value of \( y \) is less than 1.5. Therefore \( \Xi(\omega, \omega', y) \) in (22) can be approximated by a perturbative calculation supplemented by nonperturbative power corrections proportional to the vacuum condensates which are treated as phenomenological parameters. The perturbative contribution can be represented by a double dispersion integral in \( \omega \) and \( \omega' \) plus possible subtraction terms. We find that the theoretical expression for the correlator has the form:

\[
\Xi_{\text{theo}}(\omega, \omega', y) \simeq \int d\nu d\nu' \frac{\rho^{pert}(\nu, \nu', y)}{(\nu - \omega - i\epsilon)(\nu' - \omega' - i\epsilon)} + \text{subtractions} + \Xi_{\text{cond}}(\omega, \omega', y).
\]

Following the standard QCD sum rule procedure the calculations of \( \Xi(\omega, \omega', y) \) are straightforward. Confining us to the leading order of perturbation and the operators with dimension \( D \leq 5 \) in OPE, the relevant Feynman diagrams are shown in Fig 1.
The perturbative part of the spectral density is
\[ \rho_{\text{pert}}(\tilde{\omega}, \tilde{\omega}', y) = \frac{3}{27 \pi^2} \frac{1}{(1 + y)(y^2 - 1)^{3/2}} \frac{1}{2} \left( -3 \tilde{\omega}^2 + (\tilde{\omega}'^2 + 2 \tilde{\omega} \tilde{\omega}') (2y - 1) \right) \times \Theta(\tilde{\omega}) \Theta(\tilde{\omega}') \Theta(2y \tilde{\omega} - \tilde{\omega}^2 - \tilde{\omega}'^2). \] (25)

The QCD sum rule is obtained by equating the phenomenological and theoretical expressions for \( \Xi \). In doing this the quark-hadron duality needs to be assumed to model the contributions of higher resonance part of Eq. (23). Generally speaking, the duality is to simulate the resonance contribution by the perturbative part above some threshold energies. In the QCD sum rule analysis for \( B \) semileptonic decays into ground state \( D \) mesons, it is argued by Neubert and Shifman in [2,24,25] that the perturbative and the hadronic spectral densities cannot be locally dual to each other, the necessary way to restore duality is to integrate the spectral densities over the “off-diagonal” variable \( \tilde{\omega}' = (\tilde{\omega} - \tilde{\omega}')/2 \), keeping the “diagonal” variable \( \tilde{\omega} = (\tilde{\omega} + \tilde{\omega}')/2 \) fixed. It is in \( \tilde{\omega} \) that the quark-hadron duality is assumed for the integrated spectral densities. We shall use the same prescription in the case of \( B \) semileptonic decays into excited state \( D \) mesons.

The \( \Theta \) functions in (25) imply that in terms of \( \tilde{\omega}_+ \) and \( \tilde{\omega}_- \) the double discontinuities of the correlator are confined to the region \( -\sqrt{y^2 - 1}/(1 + y) \leq \tilde{\omega}_- \leq \sqrt{y^2 - 1}/(1 + y) \) \( \tilde{\omega}_+ \) and \( \tilde{\omega}_+ \geq 0 \). According to our prescription an isosceles triangle with the base \( \tilde{\omega}_+ = \tilde{\omega}_c \) is retained in the integration domain of the perturbative term in the sum rule.

In view of the asymmetry of the problem at hand with respect to the initial and final states one may attempt to use an asymmetric triangle in the perturbative integral. However, in that case the factor \((y^2 - 1)^{3/2}\) in the denominator of (25) is not canceled after the integration so that the Isgur-Wise function or its derivative will be divergent at \( y = 1 \). Similar situation occurs for the sum rule of the Isgur-Wise function for transition between ground states if a different domain is taken in the perturbative integral [25].

In order to suppress the contributions of higher resonance states a double Borel transformation in \( \omega \) and \( \omega' \) is performed to both sides of the sum rule, which introduces two Borel parameters \( T_1 \) and \( T_2 \). For simplicity we shall take the two Borel parameters equal: \( T_1 = T_2 = 2T \). In the following section we shall estimate the changes in the sum rules in the case of \( T_1 \neq T_2 \).

After adding the non-perturbative part and making the double Borel transformation one obtains the sum rule for \( \tau \) as follows
\[ \tau(y) f_{-\frac{1}{2}} f_{+\frac{1}{2}} e^{-(\tilde{\lambda}_{-\frac{1}{2}} + \tilde{\lambda}_{+\frac{1}{2}})/T} = \frac{1}{2 \pi^2} \frac{1}{(y + 1)^3} \int_0^{\omega_c} d\omega_+ \omega_+^3 e^{-\omega_+/T} - \frac{1}{12} m_0^2 \langle \bar{q}q \rangle \frac{1}{T} - \frac{1}{3} \frac{\alpha_s}{\pi} \langle GG \rangle \frac{y + 5}{(y + 1)^2} \] (26)

We now turn to the study of \( \zeta(y) \). To obtain QCD sum rules for this Isgur-Wise function one starts from three-point correlators.
\[ \mathcal{L} \int d^4x d^4z \, e^{i(k' \cdot x - k \cdot z)} \langle 0 | T \left( J_{0,+, \frac{1}{2}}(x) \, \mathcal{J}_A^{\mu(v,v')}(0) \, J_{0,+, \frac{1}{2}}^\dagger(z) \right) | 0 \rangle = \Xi(\omega, \omega', y) \mathcal{D}_A^\mu, \] 
\[ \mathcal{L} \int d^4x d^4z \, e^{i(k' \cdot x - k \cdot z)} \langle 0 | T \left( J_{1,+, \frac{1}{2}}^\nu(x) \, \mathcal{J}_A^{\mu(v,v')}(0) \, J_{0,+, \frac{1}{2}}^\dagger(z) \right) | 0 \rangle = \Xi(\omega, \omega', y) \mathcal{D}_V^{\mu , A}, \] 

the inserting of vector current in (27a) vanishes.

As mentioned above there are two possible choices for the currents creating \( 0^+ \) and \( 1^+ \) of the doublet \( (0^+, 1^+) \). Let us first consider the sum rule by using the interpolating currents (14) and (15) with derivative. The perturbative part of the spectra density is found to be

\[ \rho_{\text{pert}}(\tilde{\omega}, \tilde{\omega}', y) = \frac{3}{2^6 \pi^2} \left( \frac{y + 1}{(y^2 - 1)^{3/2}} \right) (\tilde{\omega}'^2 - \tilde{\omega}^2) \Theta(\tilde{\omega}) \Theta(\tilde{\omega}') \Theta(2y\tilde{\omega}^2 - \tilde{\omega}'^2) , \]

(28)

With the same procedure the resulting sum rule for \( \zeta \) takes the form

\[ \zeta(y) \int_{-\frac{1}{2}}^{\frac{1}{2}} f_{-, \frac{1}{2}} f_{+, \frac{1}{2}} e^{-(\tilde{\lambda}_-, \frac{1}{2} + \tilde{\lambda}_+, \frac{1}{2})/T} = \frac{1}{8\pi^2} \frac{1}{(y + 1)^2} \int_0^{\omega_+} d\omega_+ \omega_+^3 e^{-\omega_+/T} - \frac{1}{12} m_0^2(\bar{q}q) \frac{1 + y}{T} \]

\[ - \frac{1}{3 \times 2^6} \frac{\alpha_s}{\pi} \langle GG \rangle \frac{7y + 1}{y + 1} . \]

(29)

When the currents (12) and (13) without derivative are used, there is no diagram shown in Fig (c). The evaluation of the perturbative graph gives the spectral density

\[ \rho_{\text{pert}}(\tilde{\omega}, \tilde{\omega}', y) = \frac{3}{2^5 \pi^2} \frac{1 + y}{(y^2 - 1)^{3/2}} (\tilde{\omega}' - \tilde{\omega}) \Theta(\tilde{\omega}) \Theta(\tilde{\omega}') \Theta(2y\tilde{\omega}^2 - \tilde{\omega}'^2) . \]

(30)

After rewriting the spectral function in terms of \( \tilde{\omega}_\pm \), performing the double Borel transformation and the integral over \( \tilde{\omega}_- \) in the confined region, we find that the perturbative contribution vanishes for \( T_1 = T_2 \). Therefore, the sum rule in this case is not a good one. We shall not use it in the numerical analysis in the following.

We end this subsection by noting that the QCD \( O(\alpha_s) \) corrections have not been included in the sum rule calculations. However, the Isgur-Wise function obtained from the QCD sum rule actually is a ratio of the three-point correlator to the two-point correlator results. While both of these correlators subject to large perturbative QCD corrections, it is expected that their ratio is not much by these corrections significantly because of cancelation. This has been proved to be true in the analysis for \( B \) semileptonic decay to ground state heavy mesons [25].

**IV. NUMERICAL RESULTS AND IMPLICATIONS**

We now turn to the numerical evaluation of these sum rules and implications. For the QCD parameters entering the theoretical expressions, we take the standard values
\[ \langle \bar{q}q \rangle = -(0.23)^3 \text{ GeV}^3, \]
\[ \langle \alpha_s GG \rangle = (0.04) \text{ GeV}^4, \]
\[ m_0^2 = (0.8) \text{ GeV}^2, \]
\[ \langle \bar{q}q \rangle = -(0.23)^3 \text{ GeV}^3, \]
\[ \langle \alpha_s GG \rangle = (0.04) \text{ GeV}^4, \]
\[ m_0^2 = (0.8) \text{ GeV}^2, \]
\[ (31) \]

In the numerical calculations we use the physical masses for \( B, D_1 \) and \( D_2^* \) [27]. Whereas for \( D_0' \) and \( D_1' \) we take the theoretical mass values of the doublet in the leading order obtained in [7]. Therefore we use
\[ m_B = 5.279, \quad m_{D_1} = 2.422, \quad m_{D_2} = 2.459, \quad m_{D_0'} = m_{D_1'} = 2.4 , \]
\[ (32) \]
as well as \( V_{cb} = 0.04. \)

In order to obtain information for \( \tau(y) \) and \( \zeta(y) \) from the sum rules which is independent of specific input values of \( f' \)’s and \( \bar{\Lambda}' \)’s, we adopt the strategy to evaluate the sum rule by eliminating the explicit dependence on the parameters \( f' \)’s and \( \bar{\Lambda}' \)’s by using the sum rules for the correlator of two heavy-light currents. Dividing the three point sum rules in (26) and (29) by the square roots of relevant two point sum rules in (18), (19) and (21), we obtain expressions for the \( \tau \) and \( \zeta \) as functions of the Borel parameter \( T \) and the continuum thresholds. This procedure may reduce the systematic uncertainties in the calculation.

Let us evaluate numerically the sum rules for \( \tau(y) \) and \( \zeta(y) \). Imposing usual criterium for the upper and lower bounds of the Borel parameter, we found they have common sum rule “window”: \( 0.7 < T < 1.1 \), which overlaps with those of two-point sum rules [25,7]. Notice that in the case of transition between ground states the normalization of the Isgur-Wise function at zero recoil leads to the requirement that the Borel parameter \( T_1 = T_2 \equiv 2T \). That is, the Borel parameter in the sum rule for three-point correlators is twice the Borel parameter in the sum rule for the two-point correlators. In Fig. 2, we show the range of predictions for \( \tau(y) \) and \( \zeta(y) \) obtained by varying the continuum thresholds \( \omega_c, \omega_{c0}, \omega_{c1} \) and \( \omega_{c2} \), and fixing the Borel parameter at \( T = 0.9 \). In the evaluation we have taken \( 2.0 < \omega_c < 2.5, \ 1.9 < \omega_{c0} < 2.4, \ 2.3 < \omega_{c1} < 2.8 \) and \( 2.5 < \omega_{c2} < 3.0 \). Where \( \omega_{c0}, \omega_{c1} \) and \( \omega_{c2} \) are relevant two-point sum rule continuum thresholds defined in (18), (19) and (21), their ranges are determined by two-point sum rule analyses [25,7]. The results are stable against reasonable variations of these parameters.

The resulting curves for \( \tau(y) \) and \( \zeta(y) \) may be well parameterized by the linear approximations
\[ \tau(y) = \tau(1) \left( 1 - \rho_\tau^2 (y - 1) \right) , \quad \tau(1) = 0.74 \pm 0.15 , \quad \rho_\tau^2 = 0.90 \pm 0.05 , \]
\[ \zeta(y) = \zeta(1) \left( 1 - \rho_\zeta^2 (y - 1) \right) , \quad \zeta(1) = 0.26 \pm 0.08 , \quad \rho_\zeta^2 = 0.50 \pm 0.05 . \]
\[ (33) \]
\[ (34) \]
The errors reflect the uncertainty due to \( \omega \)’s and \( T \).

We have estimated the variations of Isgur-Wise functions at zero recoil in the case of \( T_1 \neq T_2 \). After Borel transformation the weight function in the dispersion integrals is
\[ \exp\{- (1/T_1 + 1/T_2) \bar{\omega}_+ - (1/T_1 - 1/T_2) \bar{\omega}_- \}. \]

We found that for \( T_2 = 3T_1/2 \) the value of \( \tau(1) \) increases about 10%, while the value of \( \zeta(1) \) increases about 30%. The variations are slower for \( T_1 \) larger than \( T_2 \). This gives us the uncertainties of the results within reasonable region of \( T_1/T_2 \).

The numerical values of \( \tau \) and \( \zeta \) at zero recoil are compared with other approaches in Table I. For \( \tau \) we find a broad agreement with some of the constituent quark model results, whereas for \( \zeta \) we only agree with [21].

Using the forms of linear approximations for \( \tau(y) \) and \( \zeta(y) \) we can compute the total semileptonic rates and branching ratios. The maximal values of \( y \) in the four cases are
\[
\begin{align*}
y_{D_1}^{D_1} &= (1 + r_1^2)/2r_1 \approx 1.32, \\
y_{D_2}^{D_2} &= (1 + r_2^2)/2r_2 \approx 1.31, \\
y_{D_0}^{D_0} &= y_{D_0}^{D_0} = (1 + r_0^2)/2r_0 \approx 1.33, \\
y_{D_1}^{D_1} &= (1 + r_1^2)/2r_1 \approx 1.33,
\end{align*}
\]

We found that the semileptonic decay into \( j_\ell = 1^+ / 2 \) is strongly suppressed. The decay widths to it are about one order of magnitude smaller than ones into \( j_\ell = 3^+ / 2 \). This predicted suppression needs to be checked in the future experiment.

In Table II we present our results for the branching ratios of B semileptonic decays to lowest lying positive parity charmed mesons. We have taken \( \tau_B = 1.62 \) ps. The ratio of the two semileptonic rates for \( B \) decays into \( D_1 \) and \( D_2^* \) mesons is
\[
R = B(B \to D_2^* \ell \bar{\nu})/B(B \to D_1 \ell \bar{\nu}) = 1.55. \tag{36}
\]

We are now at the position to compare the theoretical predictions with the available experimental data. Recently the ALEPH and CLEO collaborations reported the measurement of the branching ratios for semileptonic \( B \) decay to excited \( D \) mesons. For \( B \to D_1 \ell \bar{\nu} \) the average value of their results is
\[
B(B \to D_1 \ell \bar{\nu}) = (6.5 \pm 2.0) \times 10^{-3}. \tag{37}
\]

The \( B \to D_2^* \ell \bar{\nu} \) branching ratio has not yet been determined. CLEO set the limit \( B(B \to D_2^* \ell \bar{\nu}) < 0.8\% \) [10], while ALEPH found \( B(B \to D_2^* \ell \bar{\nu}) < 0.2\% \) [12].

Our prediction for \( R \) is larger than available experimental data though we agree with the upper CLEO limit for \( D_2^* \). This leads one to consider \( O(1/m_{c,b}) \) corrections. One may consider the mixing of the \( D_1 \) with the \( j_\ell = 1^+ / 2 \) meson, but it would worsen our prediction since, due to the very small decay amplitudes into the \( j_\ell = 1^+ / 2 \) states, it would lessen our
prediction for $\mathcal{B}(B \to D_1 \ell \bar{\nu})$. We are thus lead to consider direct $O(1/m_{c,b})$ corrections in the decay amplitudes. A detailed discussion of $O(1/m_{c,b})$ corrections can be found in Refs. [14], which indeed enhance the rate to $B \to D_1 \ell \bar{\nu}$ and lead to the expection that its branching ratio is greater than that for $B \to D_2^* \ell \bar{\nu}$.

**V. CONCLUSION**

In this work we have presented the investigation for semileptonic $B$ decays into excited charmed mesons. Within the framework of HQET we have evaluated the universal Isgur-Wise functions $\tau(y)$ and $\zeta(y)$ by using QCD sum rules in the leading order of $\alpha_s$. The $\tau(y)$ and $\zeta(y)$ functions can be well fitted by linear approximations $\tau(y)(\zeta(y)) = \tau(1)(\zeta(1))(1 - \rho_\tau(\zeta)(y - 1))$. The values of $\tau$ and $\zeta$ at zero recoil have been given. From a comparison with the results of other approaches we found a broad agreement with some of the quark model results for $\tau(1)$, whereas a smaller value for $\zeta(1)$.

We have computed, for the decays $B \to (D_1, D_2^*) \ell \bar{\nu}$ and $B \to (D_0', D_1') \ell \bar{\nu}$, the differential decay widths and the branching ratios. Our predictions for $D_1$ are smaller than experimental data, even take into account the wide spreading and large uncertainty of experimental results. Our predictions for $D_2^*$ are below the experimental upper bounds of CLEO's results. We also predict the relation $\mathcal{B}(B \to D_2^* \ell \bar{\nu})/\mathcal{B}(B \to D_1 \ell \bar{\nu})$, which might be larger than available experimental limit. The discrepancy may be attributed to a large $1/m_c$ correction which enhances the transition to $D_1$. We predict tiny branching ratios for $D_0'$ and $D_1'$, but there is no direct experimental check yet.

After finishing this paper we have learned of a paper by Colangelo et al. [28], in which the semileptonic decays $B \to (D_0', D_1')$ are studied by using a similar approach. In particular, the radiative corrections are included in the calculations of Isgur-Wise function $\xi$ in their work.

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**APPENDIX A**

We list here the lorentz structures used in the paper.
\[ \mathcal{L}_V^{\mu\nu} = \frac{1}{\sqrt{6}} \left[ (y^2 - 1)g^{\mu\nu} + 3v^\mu v^\nu + (1 - 2y)v'^\mu v'^\nu - (3yv^\mu v'^\nu + (y - 2)v'^\mu v^\nu) \right], \quad \text{(A1)} \]

\[ \mathcal{L}_A^{\mu\nu} = i\frac{1}{\sqrt{6}} (1 + y)\epsilon^{\mu\alpha\beta} v_\alpha v'_\beta, \quad \text{(A2)} \]

\[ \mathcal{L}_{2V}^{\alpha\beta} = -\frac{i}{2} \left( \epsilon^{\mu\alpha\sigma\rho}(v_\beta - v'_\beta y) + \epsilon^{\mu\beta\sigma\rho}(v_\alpha - v'_\alpha y) \right) v_\alpha v'_\beta, \quad \text{(A3)} \]

\[ \mathcal{L}_A^{\alpha\beta} = \frac{1}{3}(1 + y)g^{\alpha\beta}(v^\mu - v'^\mu) - \frac{1}{2}(1 + y)\epsilon^{\alpha\beta}(v^\mu - v'^\mu) - \frac{1}{2}(1 + y)g^{\alpha\mu}v'^\beta y - \frac{1}{3}(1 + y)g^{\alpha\mu}v'^\beta y \\
+ \frac{1}{2}(1 - y)v'^\beta v'^\mu + \frac{1}{2}(1 - y)v'^\alpha v'^\beta v'^\mu - \frac{1}{3}(1 + y)v'^\alpha v'^\beta v'^\mu \\
- \frac{2}{3}(1 + y)v'^\beta v'^\mu (v'^\alpha v'^\beta + v'^\alpha v'^\beta) v'^\mu, \quad \text{(A4)} \]

\[ \mathcal{D}_A^{\mu} = v^\mu - v'^\mu, \quad \text{(A5)} \]

\[ \mathcal{D}_V^{\mu
u} = (y - 1)g^{\mu\nu} + v'^\mu v'^\nu - v'^\mu v^\nu, \quad \text{(A6)} \]

\[ \mathcal{D}_A^{\mu\nu} = -i\epsilon^{\mu\alpha\beta} v_\alpha v'_\beta. \quad \text{(A7)} \]
REFERENCES


Figure Captions

Fig. 1. Feynman diagrams contributing to the sum rules for the Isgur-Wise form factors.

Fig. 2. Prediction for the Isgur-Wise form factors. The upper band corresponds to the $\tau$, the lower one to the $\xi$. 
TABLE I. Parameters of the form factors $\tau$, $\zeta$

<table>
<thead>
<tr>
<th>$\tau(1)$</th>
<th>$\rho_{\tau}^2$</th>
<th>$\zeta(1)$</th>
<th>$\rho_{\zeta}^2$</th>
<th>Ref.</th>
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<tr>
<td>0.74</td>
<td>0.9</td>
<td>0.26</td>
<td>0.5</td>
<td>This work</td>
</tr>
<tr>
<td>0.97</td>
<td>2.3</td>
<td>0.2</td>
<td>1.1</td>
<td>[21]</td>
</tr>
<tr>
<td>0.71</td>
<td>1.5</td>
<td>0.8</td>
<td>1.0</td>
<td>[14]</td>
</tr>
<tr>
<td>0.76</td>
<td>0.97</td>
<td>0.42</td>
<td>0.97</td>
<td>[22]</td>
</tr>
<tr>
<td>0.49</td>
<td>0.9</td>
<td>0.5</td>
<td>0.4</td>
<td>[26]</td>
</tr>
<tr>
<td>0.54</td>
<td>2.8</td>
<td>0.62</td>
<td>2.8</td>
<td>[16]</td>
</tr>
<tr>
<td>1.14</td>
<td>1.9</td>
<td>0.82</td>
<td>1.4</td>
<td>[18]</td>
</tr>
<tr>
<td>0.89</td>
<td>1.45</td>
<td>0.12</td>
<td>0.73</td>
<td>[20], [17]</td>
</tr>
<tr>
<td>0.94</td>
<td>1.50</td>
<td>0.45</td>
<td>0.83</td>
<td>[20], [15]</td>
</tr>
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</table>

TABLE II. Branching ratios for semileptonic $B$ decays to P-wave charmed mesons

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Branching ratio</th>
<th>Exp. result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \to D_1 \ell\bar{\nu}$</td>
<td>0.34</td>
<td>$0.56 \pm 0.13 \pm 0.08 \pm 0.04$ [10]</td>
</tr>
<tr>
<td>$B \to D_2^* \ell\bar{\nu}$</td>
<td>0.52</td>
<td>&lt; 0.8(90% C.L.) [10]</td>
</tr>
<tr>
<td>$B \to D_0^0 \ell\bar{\nu}$</td>
<td>0.019</td>
<td>–</td>
</tr>
<tr>
<td>$B \to D_1^0 \ell\bar{\nu}$</td>
<td>0.025</td>
<td>–</td>
</tr>
</tbody>
</table>
Figures

Fig. 1

Fig. 2