MASS PROTECTION VIA TRANSLATIONAL INVARIANCE

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We propose a way of protecting a Dirac fermion interacting with a scalar field from acquiring a mass from the vacuum. It is obtained through an implementation of translational symmetry when the theory is formulated with a momentum cutoff, which forbids the usual Yukawa term. We consider that this mechanism can help to understand the smallness of neutrino masses without a tuning of the Yukawa coupling. The prohibition of the Yukawa term for the neutrino forbids at the same time a gauge coupling between the right-handed electron and neutrino. We prove that this mechanism can be implemented on the lattice.

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1. Introduction

In the minimal standard model a right-handed chirality for the neutrino is absent, left-handed leptons are in weak SU(2) doublets and right-handed leptons in weak SU(2) singlets. If a $\nu_R$ does exist, then the neutrino could get a Dirac mass

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and a fundamental problem is to understand why $m_{\nu_e}/m_e$ is such a small number ($< 10^{-5}$). The standard answer to this problem is the see-saw mechanism, which is based on the introduction of a Majorana mass term for the right-handed neutrino with a high enough new mass scale.\(^1\)

We explore in this letter another possible cause for the smallness of neutrino masses, which is offered by a new implementation of translational symmetry in the context of the Standard Model (SM) as an effective theory.

To be more specific, the SM is a theory where first, the symmetries are postulated, and second, the representations in which the elementary particles appear are chosen. This method has been explicitly implemented for the gauge symmetries and the little group of the Lorentz group, classifying particles into different representations of these groups. The different possible representations of the group of translations have not been relevant up to now (to our knowledge) in the formulation of the SM. It is in this context that we make our proposal. We will show that the choice of a different representation for the left and right-handed fermion fields offers the possibility to forbid the usual Yukawa mass term. We remark that the choice of the representations will be justified (at least for the moment) simply by the phenomenological results of the theory.

2. Mass protection mechanism

To illustrate how this mechanism works, we will consider a chiral model with a left and a right fermion coupled to a complex scalar field.

Setting
\[
\phi(x) = \phi_1(x) + i\phi_2(x),
\]
we will consider that the scalar field $\phi$ gets a VEV $\langle \phi_1(x) \rangle = v, \langle \phi_2(x) \rangle = 0$. Both the scalar action and the vacuum are translationally invariant when $\phi$ transforms in the usual way under a translation by a vector $r$,
\[
\phi'(x) = \phi(x + r).
\]

In order to identify the new representation for translations it is necessary to introduce a momentum cutoff ($-\Lambda \leq p_\mu \leq \Lambda$), which is a natural way of incorporating the limitations on the domain of validity of the model, which we consider as a low energy effective theory. As a consequence of the introduction of the cutoff, the group $O(4)$ (Euclidean version of the homogeneous Lorentz group) is replaced by the discrete subgroup generated by rotations of angle $\pi/2$ in each plane, and the group of translations is replaced by the discrete subgroup generated by translations of $\pi/\Lambda$ in each direction.

Now when one considers the possible linear representations of the discretized translations ($r^\mu = n^\mu \pi/\Lambda, n^\mu \in \mathbb{Z}$) for the fermion field one has only two different choices which are compatible with the usual representations of rotations,
\[
\psi'(p) = e^{i\mathbf{r} \cdot \mathbf{p}}\psi(p),
\]
which is the momentum space version of the usual transformation law (2) and a new representation,

$$\psi'(p) = e^{i\tilde{p} \cdot \tilde{p}} \psi(p)$$

where $\tilde{p}_\mu = p_\mu - \Lambda \epsilon(p_\mu)$, and $\epsilon(p_\mu) = \text{sign}(p_\mu)$.

In coordinate space, the representation (4) reads simply

$$\psi'(x) = e^{i\Lambda \sum_\nu \tau_\nu \psi(x + r)}.$$  

Note that for an elementary translation, the difference between Eqs. (3) and (4) is just a minus sign, though this sign will lead to very relevant consequences as we will see.

In the context of effective theories, the new representation of translations could offer the possibility to describe remnants at low energy of high energy phenomena which cannot be described in terms of higher dimensional operators, like inhomogeneous vacua for example. At the end of this section we comment a little more on a possible interpretation of this new representation of translations.

In order to get any consequences from the use of the new representation in an action build out of bilinears in the fermion field one has to consider a different representation for each chirality. In this case, the usual Yukawa term in momentum space,

$$y \bar{\psi}_L([p + k]) \phi(k) \psi_R(p),$$  

is forbidden by translational invariance. In Eq. (6), $[p + k]$ is the momentum compatible with the cutoff obtained by adding or subtracting if necessary $2\Lambda$ to the components of $p + k$. The interaction term compatible with the new implementation of translations is

$$y \bar{\psi}_L(\tilde{[p + k]}) \phi(k) \psi_R(p),$$

where the tilde symbol was already introduced in Eq. (4).

Now one can compare the results for the fermion propagator in the two cases: either with the same transformation law under translations for both chiralities (which implies the usual Yukawa interaction) or with different representations for each chirality (in this case, the appropriate Yukawa term is Eq. (7)).

In the approximation where the fluctuations of the scalar field are neglected, one has, in the usual case, the propagator of a free fermion with mass $m = yv$. In the case of the new translation representation, one finds the propagator of a massless fermion up to corrections proportional to inverse powers of the cutoff $\Lambda$. So we can say that translational invariance can protect a Dirac fermion from acquiring a mass from the vacuum.

However, as the term (7) couples momentum modes that differ in $\Lambda$, a non-perturbative implementation of this mechanism could be problematic owing to the well-known fermion doubling phenomenon. We will later see that this is not the case.
Coming back to possible physical interpretations of the new representation of translations, one can note that the very existence of different representations for the left- and right-handed parts of a field comes from the presence of a cutoff. One could interpret this by saying that the chirality transforming in the new representation will have a different coupling to the physics beyond the cutoff from that of the chirality that transforms in the usual way under translations.

3. Application to the SM

This mechanism could be applied to understand the mass difference between the electron and the neutrino in the SM (or, in general, between a charged lepton and its corresponding neutrino) by assuming that the right-handed neutrino transforms differently from the left-handed neutrino under translations. We remark that we do not introduce a new symmetry, but only use a new representation of an existing symmetry, translational invariance.

To apply this mechanism in the case of the electron and the neutrino, we first have to remember that the left-handed electron and the left-handed neutrino form an SU(2) doublet, so they are coupled by the gauge field in terms such as $\bar{e}_L \gamma^\mu W_\mu \nu_L$. In order to leave these terms translationally invariant with the gauge field transforming in the trivial representation of translations, we need the same representation of translations for both the left-handed electron field and the left-handed neutrino field. Besides, we want to give mass to the electron with a usual Yukawa term through the Higgs mechanism, so we have to choose the same representation for the two chiral components of the electron field. For the neutrino field, we take different representations for the two chiral components to forbid the usual Yukawa term. Then the right-handed electron field and the right-handed neutrino field are in different representations and they cannot be in the same weak isospin multiplet. This situation is in fact assumed in the SM.

Majorana terms such as $\psi^T_L(x)C\psi_L(x)\phi(x)\phi(x)$, $\psi^T_R(x)C\psi_R(x)$ (where $C$ is the charge-conjugation operator) are compatible with translational invariance independently of the choice of representations and should be included in a more detailed analysis of neutrino masses.

4. Lattice implementation

As we have previously said, the term (7) couples momentum modes that differ in $\Lambda$ so that one could be afraid of possible problems in a nonperturbative implementation of our proposal. In fact, a lattice field theory is a good starting point to derive properties of the theory in a rigorous way, but it suffers from the illness of the fermion doubling phenomenon. As it is well known, this illness is more serious when the two chiralities of a fermion are not treated on the same footing. Therefore, it is pertinent to control that the mechanism we have proposed is compatible with
a lattice formulation, in the sense that the lattice doublers still decouple, while the unconventional choice of representation of the translational symmetry for the lattice fermion field protects the generation of a fermion mass to all orders in perturbation theory.

The lattice action for the scalar field is (we will take the lattice spacing $a = 1$):

$$S_B = -\kappa \sum_{x,\mu} (\phi_x^* \phi_{x+\hat{\mu}} + \phi_{x+\hat{\mu}}^* \phi_x) + \sum_x \{\phi_x^* \phi_x + \lambda (\phi_x^* \phi_x - 1)^2\}. \quad (9)$$

On the lattice, we take for the representation of translations for the fermion field:

$$\psi_{Lx}^i = e^{-i\alpha_L} \psi_{Lx+\hat{\mu}}, \quad \psi_{Rx}^i = e^{-i\alpha_R} \psi_{Rx+\hat{\mu}}, \quad (10)$$

under a translation of one lattice spacing in the $\hat{\mu}$ direction. As in the previous discussion, in order to have compatibility with the usual representations of rotations, only 0 and $\pi$ will be the allowed values for $\alpha_L, \alpha_R$ in Eq. (10). Then the lattice naïve kinetic term:

$$S_F = \sum_{x,\mu} \frac{1}{2} (\bar{\psi}_x \gamma_\mu \psi_{x+\hat{\mu}} - \bar{\psi}_{x+\hat{\mu}} \gamma_\mu \psi_x). \quad (11)$$

is rotational and translational invariant for any of the two allowed values for $\alpha_L$ and $\alpha_R$ and the Yukawa term is forbidden when $\alpha_L \neq \alpha_R$. But with the new behaviour under translations, we have to admit coupling constants which depend on the space coordinates. With the choice

$$\alpha_R = \pi, \quad \alpha_L = 0, \quad (12)$$

a term of the following type:

$$y \sum_x (-1)^{x_\mu} (\bar{\psi}_{Lx} \phi_x \psi_{Rx} + \bar{\psi}_{Rx} \phi_x^* \psi_{Lx}) \quad (13)$$

is allowed by the symmetry.

A difference between the continuum and the lattice discussion is that the naïve lattice kinetic term (11) suffers from the doubling phenomenon. To avoid this last problem, we will follow the methods of Refs. 3 and 4 and write the boson-fermion interaction as:

$$S_{FB} = y \sum_x (-1)^{x_\mu} (\bar{\psi}_{Lx}^{(1)} \phi_x \psi_{Rx}^{(1)} + \bar{\psi}_{Rx}^{(1)} \phi_x^* \psi_{Lx}^{(1)}), \quad (14)$$

where

$$\psi^{(1)}(p) = F(p) \psi(p). \quad (15)$$
$F(p)$ is a form factor required to be 1 for $p = 0$ and to vanish when $p$ equals any of the doubler momenta. With this method, we have a theory with 16 fermions, 15 of which do not interact with physical particles and decouple from the real world.

We will show now that the translationally invariant action including the new term $S_{FB}$ (14),

$$S = S_B + S_F + S_{FB},$$

(16)

gives a zero mass for the fermion.

Let us first note that the presence in the action of the term $S_{FB}$ with such an unusual coupling does not modify the vacuum $\langle \phi_{1x} \rangle = v$. This is a consequence of both analytical and numerical studies of the antiferromagnetic (AFM) phase of the chiral Yukawa model. Under the change of variables $\phi_x' = \varepsilon_x \phi_x$, where $\varepsilon_x = (-1)^{\sum_{\nu} x_{\nu}}$, the action is invariant if the couplings are mapped according to

$$(\kappa, y\varepsilon_x) \longrightarrow (-\kappa, y).$$

(17)

With these couplings, a stable AFM phase exists where the scalar gets a staggered mean value $\langle \phi_{1x}' \rangle = \varepsilon_x v_{st}$. We can then conclude that the original vacuum $\langle \phi_{1x} \rangle = v$ is also a stable vacuum for the action (16).

In order to do perturbation theory, let us rewrite $S_{FB}$ in the form

$$S_{FB} = y \sum_x \varepsilon_x (\bar{\psi}_x^{(1)} \{ \phi_x P_R + \phi_x^* P_L \} \psi_x^{(1)}),$$

(18)

where $P_L = \frac{1}{2}(1 - \gamma_5)$, $P_R = \frac{1}{2}(1 + \gamma_5)$.

We write

$$\phi_{1x} = v + \eta_{1x}, \quad \phi_{2x} = \eta_{2x},$$

(19)

where $\eta_{1,2}$ represent the small perturbations. In momentum space, the fermionic matrix at tree-level order is

$$i\delta(p-p') \not{\delta}(p) + yvF(p)F_{\pi}(p)\delta(p-p'+\pi),$$

(20)

where $\not{\delta}(p) = \sum_\mu \gamma_\mu \sin p_\mu$, $F_\pi(p) \equiv F(p + \pi)$, and $\pi \equiv (\pi, \pi, \pi, \pi)$. We have $F_\pi(0) = 0$. This matrix is not diagonal in momentum space, as it connects every momentum $p$ with $p + \pi$ in a box of the form

$$G^{-1}(p) = \begin{pmatrix} i \not{\delta}(p) & yvF(p)F_\pi(p) \\ yvF(p)F_\pi(p) & i \not{\delta}(p+\pi) \end{pmatrix}.$$  (21)

It can be diagonalized with the non-unitary transformation

$$G_D^{-1}(p) = TG^{-1}(p)T, \quad T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix},$$

(22)

to give

$$G_D = \frac{1}{i \not{\delta}(p) + im(p)},$$

(23)
where \( m(p) = y v F(p) F_\pi(p) = m(p + \pi) \). This propagator has 16 poles at momenta \((0, 0, 0, 0), (\pi, \pi, \pi, \pi), (\pi, 0, 0, 0), (\pi, \pi, 0, 0), \) etc., which implies zero mass at tree level for the physical fermion and all the doublers. A straightforward mean field calculation\(^4\) leads to the same conclusion but with a contribution to \( v \) coming from the fermionic condensate besides the contribution of the boson VEV.

Now we want to know if this masslessness is maintained when loop corrections are added. From the interaction term with the bosons \( \eta_1 \) and \( \eta_2 \) defined in Eq. (19),

\[
y \int_{-\pi}^{\pi} \frac{d^4 p}{(2\pi)^4} \frac{d^4 p'}{(2\pi)^4} \bar{\psi}(p) F(p) \{ \eta_1 (p - p' + \pi) + \eta_2 (p - p' + \pi) i \gamma_5 \} F(p') \psi(p'),
\]

we get the Feynman rules of interest: a fermion-boson-fermion vertex which inserts a momentum \( \pi \), and contributions coming from the diagonal and nondiagonal (connecting momenta \( k \) and \( k + \pi \)) elements of the propagator matrix. One can then calculate the 1-loop fermion self-energy diagram, which is then added to the inverse propagator to give

\[
G_{1\text{-loop}}^{-1}(p) = \left( \begin{array}{cc}
(\eta(p) + \Sigma^A(p)) & m(p) + \Sigma^B(p) \\
-\eta(p) + \Sigma^A(p) & m(p) + \Sigma^B(p)
\end{array} \right),
\]

where \( \Sigma^{A,B}(p) \equiv \Sigma^{A,B}(p + \pi) \), and

\[
\Sigma^A(p) = y^2 F^2(p) \int_{-\pi}^{\pi} \frac{d^4 k}{(2\pi)^4} F^2(k) \frac{-i \eta(k)}{s^2(k) - m^2(k)}
\times \{ G_1(p - k + \pi) + G_2(p - k + \pi) \},
\]

\[
\Sigma^B(p) = y^2 F(p) F_\pi(p) \int_{-\pi}^{\pi} \frac{d^4 k}{(2\pi)^4} F(k) F_\pi(k) \frac{-m(k)}{s^2(k) - m^2(k)}
\times \{ G_1(p - k + \pi) - G_2(p - k + \pi) \},
\]

\( G_j \) being the \( \eta_j \) boson propagator \((j = 1, 2)\):

\[
G_j(q) = \frac{(2\kappa)^{-1}}{2 \sum_{\rho} (1 - \cos q_\rho) + m^2_j},
\]

with \( m_1 = (4\lambda/\kappa)v^2 \) and \( m_2 = 0 \). It is easy to see that \( \Sigma_\pi^B(p) = \Sigma^B(p) \).

The new propagator (25) cannot be diagonalized with the transformation (22). But in fact there is a general diagonalization that can be used for both the tree-level and the one-loop inverse propagators, (21) and (25) respectively. The transformation is

\[
UG^{-1}V, \quad U = \begin{pmatrix} P_L & iP_R \\ iP_R & P_L \end{pmatrix}, \quad V = \begin{pmatrix} P_R & iP_L \\ iP_R & P_L \end{pmatrix}.
\]
Actually, it is easy to convince oneself that the matrices $U$ and $V$ will diagonalize
the propagator at every loop order: notice that the change of field variables given
by these two matrices, which in coordinate space is written as

$$
\psi_{Rx} = \chi_{Rx}, \quad \bar{\psi}_{Rx} = \bar{\chi}_{Rx},
$$

and

$$
\psi_{Lx} = i\varepsilon_x \chi_{Lx}, \quad \bar{\psi}_{Lx} = i\varepsilon_x \bar{\chi}_{Lx},
$$

leaves the kinetic term invariant and changes the term (14) into

$$
iy \sum_x (\bar{\chi}^{(16)}_{Lx}\phi_x\chi^{(1)}_{Rx} + \bar{\chi}^{(1)}_{Rx}\phi^*_x\chi^{(16)}_{Lx}),
$$

selects the fermion centered around the doubler momentum $\pi$. In terms of the
new variables $\chi$ we have a coupling that does not depend on the space coordi-
nates, so that in momentum space the propagator will be diagonal at every order
in perturbation theory.

$\Sigma^B$ is proportional to $F(p)F_{\pi}(p)$, so it vanishes at $p = 0$ and at all doubler
momenta. $\Sigma^A$ is also zero for the same values because it is proportional to $F(p)$
and for $p = 0$ it is the integral of an odd function. In conclusion, the position of
the poles is not modified by the one-loop corrections, therefore both the physical
fermion and the doublers have zero masses.

This is also true when one investigates two-loop diagrams. In fact, there is
a non-perturbative argument that shows that the chirally nondiagonal part of the
proper vertex with two external legs (i.e. the inverse of the propagator) is zero when
evaluated at any doubler momentum, including $p = 0, \pi$. As discussed in Ref. 3,
there is a symmetry of the action under appropriate (“shift”) transformations of the
fermion field, which, when applied to the term (32), leads to Ward identities
which express that

$$
\Gamma^{(2)}_{LR}(p) = 0
$$

when evaluated at $p$ equal to one of the doubler momenta. This is a consequence of
the decoupling method together with the form of the term (32), and it is responsible
for the cancellation of $\Sigma^B$.

5. Conclusion

We have shown that the freedom in the choice of the representations of the trans-
lational symmetry can provide a mechanism which imposes a great restriction on
the generation of the mass for a fermion. In particular, one could understand the
absence of a Dirac mass contributions for the neutrino. With this mechanism, the
prohibition of the Yukawa term for the neutrino forbids at the same time a gauge
coupling between the right-handed electron and neutrino.
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