Spin effects in high-energy proton-proton scattering within a diquark model

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Abstract

We study \( pp \) scattering at high energies and moderately large momentum transfer, using a model in which the proton is viewed as being composed of a quark and a diquark. We show that this model leads to single and double spin transverse asymmetries which are neither small nor vanish at high energies.

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1 Introduction

The adequate theoretical description of spin effects in high-energy exclusive processes at moderately large momentum transfer is one of the unsolved problems in QCD. As is well known, massless QCD leads to hadronic helicity conservation and, hence, to zero single-spin asymmetries. Mass and higher order perturbative QCD corrections lead to a non-vanishing single-spin transverse asymmetries:

$$A_N \propto \frac{m \alpha_s}{\sqrt{-t}}.$$  \hspace{1cm} (1)

A QCD analysis reveals that the mass parameter $m$ appearing in (1) is of order of the hadron mass [1] and should not be interpreted as a current quark mass. So, one may expect a substantial single-spin asymmetry for momentum transfer, $-t$, of the order of a few GeV$^2$. Actual estimates within QCD inspired models provide only values of the order of a few per cents for single-spin asymmetries, indeed much smaller than the experimental results.

Experimentally, there are many observations of large spin effects at high energies and moderately large momentum transfer [2]. Sizeable differences between the cross sections for different spin orientations of the initial state protons as well as large double-spin, $A_{NN}$, and single-spin, $A_N$, transverse asymmetries have been observed in the BNL experiment [3] for beam momenta $p_B$ less than 28 GeV. The FNAL experiment [4] finds values for $A_N$ of about 10-20% at $p_B = 200$ GeV and momentum transfers $|t| \geq 2$ GeV$^2$. This result is of the same order of magnitude than the BNL asymmetry at $p_B = 28$ GeV and similar values of $t$. Combining these observations with corresponding ones made at small momentum transfer [5], one is lead to the conclusion that spin effects in high-energy reactions exhibit a weak energy dependence.

Elastic scattering at high energies and fixed momentum transfer ($|t|/s$ small) is customarily believed to be under control of the $t$-channel colour-singlet Pomeron (and, eventually Odderon) exchange that has a dominant non-flip coupling. The observed spin effects thus seem to require the existence of an additional Pomeron-like exchange in the helicity-flip amplitudes that has - up to eventual $\ln s$ factors - the same energy dependence as the standard Pomeron but is not in phase with it. Within QCD the Pomeron is interpreted as $t$ channel exchange of gluons with total charge conjugation of unity ($C = +1$). Present attempts to understand it theoretically are based on the simple two-gluon picture for this object [6]. It is important to note that in such models the Pomeron couples to quarks and not directly to the hadrons. According to [7], the two gluons representing the Pomeron preferentially interact with the same quark within a given hadron. As a consequence of this property,
the Pomeron effectively couples to the hadron like an $C = +1$ isoscalar photon \cite{7, 8} and approximately reproduces the salient features of the additive quark model. However, neither the non-perturbative two-gluon model \cite{8} nor the BFKL model \cite{9} provides a spin-dependent Pomeron coupling. The question of gauge invariance for the Landshoff-Nachtmann Pomeron \cite{8} has been investigated by Diehl \cite{10}.

In several models high energy spin effects have been investigated. Thus, for instance, in \cite{11} the spin-dependent quark-Pomeron coupling was constructed from a gluon-loop contribution. It was shown that this quark-Pomeron coupling leads to fairly large spin asymmetries in diffractive quark-antiquark pair production and exhibits only a weak energy dependence \cite{12}. In \cite{13} rotating matter inside the proton was claimed to be the origin of spin effects. The authors of \cite{14} considered the Pomeron interaction with the light quark-antiquark cloud of the proton. While these models provide spin effects at high energies in fair agreement with experiment they suffer from the large number of adjustable parameters they depend on. Moreover, the applicability of these models is restricted to small momentum transfer.

Here, in this work, we are interested in spin effects at high energies and moderately large momentum transfer ($3 \text{ GeV}^2 < |t| << s$). In view of the polarization physics programs proposed for the future proton accelerators \cite{15} this kinematical region is of topical interest. Our approach is based on the diquark picture \cite{16} where the proton is viewed as being composed of a quark and a diquark in the dominant valence Fock state instead of three quarks. The diquarks represent an effective description of non-perturbative effects; their composite nature is taken into account by diquark form factors. The diquark picture of the proton simplifies our calculations drastically due to the reduced number of constituents. The combination of the quark-diquark picture of the proton and the hard scattering approach developed by Brodsky and Lepage \cite{17} leads to successful descriptions of electromagnetic form factors and other exclusive reactions \cite{18, 19} at fairly large momentum transfer. Spin effects are generated from spin 1 (vector) diquarks in that model. The model also provides phase differences between different helicity amplitudes in some cases and can therefore account for single-spin asymmetries in principle. However, even within the diquark model which is much simpler to handle than the three-quark picture of the proton, a full hard scattering analysis of elastic proton-proton scattering is beyond feasibility at present (see, for instance, \cite{20}). Therefore, in order to simplify and in regard to the fact that we are not interested in the real hard scattering region for which the diquark model was originally designed, we use that model in combination with the two-gluon exchange picture as a representative of the Pomeron. We calculate the helicity-flip amplitude explicitly in that framework while,
at the end, the non-flip amplitudes are described by a standard phenomenological Pomeron exchange. We note that Ramsey and Sivers [21] also proposed a hard scattering model that produces substantial spin effects. This model is based on quark-exchange and the Landshoff pinch contribution [22] to the pp helicity amplitudes.

In Sect. 2 we begin with a few kinematical preliminaries. A brief description of the diquark model is presented in Sect. 3. The general structure of the various diquark contributions to elastic pp scattering is discussed in Sect. 4. In Sect. 5 we present our numerical results for spin asymmetries in elastic pp scattering and compare them to experimental data. Concluding remarks are given in Sect. 6.

2 Proton-proton scattering at high energies

The momenta and the Mandelstam variables of elastic proton–proton scattering are defined by

\[ p(p_1) + p(p_2) \rightarrow p(p_3) + p(p_4) \]  

and

\[ s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2. \]

Elastic pp scattering can be described in terms of helicity amplitudes

\[ T_{\lambda_4 \lambda_3; \lambda_2 \lambda_1} = \bar{u}(p_4, \lambda_4)\bar{u}(p_3, \lambda_3)\hat{T}(s, t)u(p_2, \lambda_2)u(p_1, \lambda_1). \]  

of which only five are independent. In (3) \( u \) denotes the spinor of a proton with momentum \( p_i \) and helicity \( \lambda_i \). In the kinematical region of interest the double helicity–flip amplitudes are believed to be much smaller than the helicity non–flip ones and the two non-flip amplitudes are of equal magnitude approximately. These properties hold in most of models (see, for instance, [13, 14]) and we will assume that they also hold in our approach. Therefore, we have to calculate or to model a non-flip and a flip amplitude only. In this situation we can, for convenience and without loss of generality, fix the helicities of the protons 1 and 3 at \(+1/2\). The covariant structure of (3) then simplifies to

\[ T_{\lambda_4 +; \lambda_2 +} = \bar{u}(p_4, \lambda_4)\bar{u}(p_3, +)\hat{T}(s, t)u(p_2, \lambda_2)u(p_1, +) \]

\[ = \bar{u}(p_4, \lambda_4)[sA(t, s) + p_1 B(t, s)]u(p_2, \lambda_2) \]  

The invariant function \( A \) is related to the helicity flip amplitude while \( B \) controls the non-flip amplitude up to a small correction of order \( m^2/t \) (\( m \) being the proton mass) from \( A \).
which we omit in our approach throughout. There is no need for antisymmetrization of the amplitudes since the $p_3 \leftrightarrow p_4$ interchanged contribution is suppressed by inverse powers of $s$ in the kinematical region of interest ($t \leftrightarrow u \simeq s$).

In terms of the invariant functions $A$ and $B$ the differential cross sections is given by

$$\frac{d\sigma}{dt} = \frac{1}{64\pi} |B|^2.$$  \hfill (5)

The single-spin asymmetry reads

$$A_N = -2\sqrt{-t} \frac{\text{Im}[BA^*]}{|B|^2}$$ \hfill (6)

while the double spin transverse asymmetry is given by

$$A_{NN} = -2t \frac{|A|^2}{|B|^2}.$$ \hfill (7)

The $A_{NN}$ asymmetry is related to the differential cross-sections in parallel and anti-parallel spin states by

$$\frac{d\sigma(\uparrow\downarrow)/dt}{d\sigma(\uparrow\uparrow)/dt} = \frac{1 + A_{NN}}{1 - A_{NN}}.$$ \hfill (8)

In the following we are going to calculate the leading contribution to the invariant function $A$ within the diquark model, omitting corrections like $m^2/t$. The invariant function $B$, on the other hand, is modelled by a phenomenological Pomeron. We will make use of two alternative parametrizations valid for $|t|$ larger than 3 GeV$^2$ (after the dip region of the differential cross section): Following, for instance, the authors of [14], we parametrize $B$ as an exponential

$$B(s, t) = isb \exp (-a\sqrt{|t|}).$$ \hfill (9)

This ansatz is understood as being a consequence of multiple pomeron exchange (MPE). Alternatively, we use the parametrization

$$B(s, t) = is \frac{c}{t^3}.$$ \hfill (10)

which may be viewed as a phenomenological version of the Landshoff pinch contribution (LP) [22] to $pp$ scattering. In our numerical estimations we shall use the MPE fit for $b = 45.967$ GeV$^{-2}, a = 3.745$ GeV$^{-1}$ and the LP fit for $c = 6.284$ GeV$^6$. Both the parametrizations, (9) and (10), describe rather well the $pp$ differential cross section data at ISR energies [23]. An eventual residual energy dependence of the experimental data (perhaps of $\ln s$ type) will be ignored here. It is irrelevant for our purpose of investigating spin effects.
3 The diquark model

As we said in the introduction we will make use of the diquark model of the proton advocated for in \cite{16, 18, 19}. Here we give a brief description of that model. In the hard scattering approach proposed by Brodsky and Lepage \cite{17} the process $pp \rightarrow pp$ is expressed by a convolution of distribution amplitudes (DA) with hard-scattering amplitudes calculated in collinear approximation within perturbative QCD. In a collinear situation in which intrinsic transverse momenta are neglected and all constituents of a hadron have momenta parallel to each other and parallel to the momentum of the parent hadron, one may write the valence Fock state of the proton in a covariant fashion (omitting colour indices for convenience)

$$|p, \lambda\rangle = f_S \varphi_S(x_1) B_S u(p, \lambda) + f_V \varphi_V(x_1) B_V (\gamma^\alpha + p^\alpha/m) \gamma_5 u(p, \lambda)/\sqrt{3}. \quad (11)$$

The two terms in (11) represent configurations consisting of a quark and either a spin-isospin zero ($S$) or a spin-isospin one ($V$) diquark, respectively. The couplings of the diquark with the quarks in a proton lead to the flavour functions

$$B_S = u S_{[u,d]}, \quad B_V = [u V_{[u,d]} - \sqrt{2} d V_{[u,u]}]/\sqrt{3}. \quad (12)$$

The DA $\varphi_{S(V)}(x_1)$, where $x_1$ is the momentum fraction carried by the quark, represents the light-cone wave function integrated over transverse momentum and is defined in such way that

$$\int_0^1 dx_1 \varphi_{S(V)}(x_1) = 1. \quad (13)$$

The constant $f_{S(V)}$ acts as the value of the configuration space wave function at the origin.

The invariant function $A$ will be calculated in the spirit of the hard scattering approach \cite{17} where the quarks and diquarks are connected by the minimal number of gluons, i.e. by three. We also will employ several kinematical simplifications since we only consider the region $m^2 << |t| << s$. Colour neutralization requires the $t$-channel exchange of two gluons. The third one is exchanged within one of the proton-proton vertices. In so far our model for the flip amplitude bears resemblance to the Landshoff-Nachtmann \cite{8} two-gluon model of the Pomeron. The helicity-flip amplitude can be expressed as a product of a helicity non-flip vertex (HNF) and flip vertex (HF). The structure of the HNF vertex is shown in Fig. 1. For this vertex we only consider scalar diquarks in order to keep the model simple. The graphs contributing to the product of the HNF and the HF are shown in Figs. 2–5. To the HF vertex only vector diquarks contribute since, obviously, from scalar diquarks a helicity flip cannot be generated. The graphs shown in Figs. 2 and 3 contain 3-point diquark vertex functions while
those shown in Figs. 4 (three-gluon interactions) and 5 (without three-gluon interactions) contain 4-point functions. In principle there is also a graph with a quartic gluon coupling. However, its contribution is suppressed at large $s$. It has been shown in [20] that this set of graphs leads to gauge-invariant scattering amplitudes. The n-point functions, indicated by blobs in Figs. 2–5, are given by a product of the relevant graphs for point-like diquarks (see, for instance, Fig. 6) and appropriate phenomenological diquark form factors. These form factors take into account the composite nature of the diquarks. Since the 5-point functions provide only small corrections to the final results we omit them in our analysis.

The perturbative part of the diquark model, i.e. the coupling of gluons to diquarks follows standard prescriptions (for notations refer to [19]):

$$SgS : i g_s t_{ij}^a (p_1 + p_2)_\mu$$

$$VgV : -ig_s t_{ij}^a \left\{ g_{\alpha\beta} (p_1 + p_2)_\mu - g_{\mu\alpha} [(1 + \kappa) p_1 - \kappa p_2]_\beta - g_{\mu\beta} [(1 + \kappa) p_2 - \kappa p_1]_\alpha \right\} \tag{14}$$

where $g_s = \sqrt{4\pi\alpha_s}$ is the QCD coupling constant. $\kappa$ is the anomalous magnetic moment of the vector diquark and $t^a = \lambda^a/2$ the Gell-Mann colour matrix. The couplings $DgD$ are supplemented by appropriate contact terms required by gauge invariance, e.g.

$$gSgS : -ig_s^2 \{t^a t^b\}_{ij} g_{\mu\nu} \tag{15}$$

The phenomenological diquark form factors are taken from [16, 18]:

$$F_S^{(3)}(Q^2) = Q_S^2 / (Q_S^2 + Q^2); \quad F_V^{(3)}(Q^2) = \left( Q_V^2 / (Q_V^2 + Q^2) \right)^2; \tag{16}$$

$$F_S^{(4)}(Q^2) = a_S F_S^{(3)}(Q^2); \quad F_V^{(4)}(Q^2) = a_V \left( Q_V^2 / (Q_V^2 + Q^2) \right)^3. \tag{17}$$

The constants $a_S$ and $a_V$ are strength parameters introduced in order to take care of diquark excitation and break-up. These parametrizations are constrained by the requirement that asymptotically the diquark models evolves into the standard Brodsky-Lepage hard scattering model [17].

4 The structure of the model amplitude

According to our discussion in Sect. 3 the invariant function $A$ can be expressed as a product of the helicity non-flip vertex to which only scalar diquarks contribute and the flip vertex...
that, in our model, is controlled by vector diquarks:

\[
A(s, t) = \frac{(4\pi)^3}{3t^2} f_S^2 f_V^2 \times \int d\alpha_1 d\beta_1 \frac{\phi_S(\alpha_1)\phi_S(\beta_1)}{\alpha_1\alpha_2\beta_1\beta_2} \alpha_s(-\alpha_1\beta_1 t)\alpha_s(-\alpha_2\beta_2 t) F_S^{(3)}(-\alpha_2\beta_2 t)
\]

\[
\times \int dx_1 dy_1 \phi_V(x_1)\phi_V(y_1) \sum_i C_i \hat{A}_i
\]

(18)

\(\alpha_1\) and \(\beta_1\) denote the fractions of the baryon momentum carried by the quarks in the initial and final baryons entering the HNF-vertex, respectively. \(\alpha_2 = 1 - \alpha_1\) and \(\beta_2 = 1 - \beta_1\) are the momentum fractions the diquarks carry. \(x_1, (x_2), y_1(y_2)\) are the analogue quantities for the HF-vertex. \(C_i\) is the color factor. To facilitate the discussion we split \(A\) into contributions from various groups of Feynman graphs. The \(\hat{A}_i\) are written as a contraction of the two tensors representing the HNF and HF vertices

\[
\hat{A}_i = H_{\mu\nu}^{n.f.} \cdot H_{fi}^{\mu\nu}
\]

(19)

The HNF tensor has the simple form

\[
H_{\mu\nu}^{n.f.} = \bar{u}(p_3+)\left[\gamma_\mu(p_1 + p_3)_\mu + \gamma_\nu(p_1 + p_3)_\nu\right]u(p_1, +)
\]

(20)

The HF tensors are to be calculated from the Feynman graphs shown in Figs. 2-6. They contain a factor of \(\alpha_s\) with an appropriate argument (representing the virtuality of the internal gluon) and the vector diquark form factor besides the characteristics of the relevant Feynman graphs. We refrain from quoting the \(H_{fi}^{\mu\nu}\) explicitly but discuss the functions the functions \(\hat{A}_i\) directly.

The graph 2a includes a propagator (marked by a cross) whose denominator contains a term proportional to \(s\). Neglecting in this denominator terms proportional to \(t\) and \(m^2\) in accordance with the condition \(m^2, |t| \ll s\), we have

\[
\hat{A}_{(2a)} = \hat{a}_{(2a)}(\alpha_1, \beta_1) \left[\frac{1}{s y_1(\alpha_1 - \beta_1) + i \epsilon} + \frac{1}{-s y_1(\alpha_1 - \beta_1) + i \epsilon}\right]
\]

\[
= -\frac{2i\pi}{s y_1} \hat{a}_{(2a)}(\alpha_1, \alpha_1) \delta(\alpha_1 - \beta_1).
\]

(21)

where the regular function \(\hat{a}_{2a}(\alpha_1, \alpha_1)\) is given in Tab. 1. The contribution from graph 2b is given by \(\hat{A}_{(2a)}(x_1, y_1) = \hat{A}_{(2b)}(y_1, x_1)\). There is a group of graphs in which the large variable \(s\) appears in two propagators denominators \((i = 2c, 3a, 3b, 4a, 4b)\):

\[
\hat{A}_i = \hat{a}_i(\alpha_1, \beta_1) \frac{1}{s(\alpha_1 - \beta_1)f_{i1} + d_{i1} + i \epsilon_1} \cdot \frac{1}{s(\alpha_1 - \beta_1)f_{i2} + d_{i2} + i \epsilon_2},
\]

(22)
Table 1: Color factors and of the functions $d_{ij}$, $f_{ij}$ and $\hat{a}_i$ at $\beta_1 = \alpha_1$ for sample graphs (for definitions see text). The contribution from graphs 4a and 5a is actually given for subgraph 6a.

<table>
<thead>
<tr>
<th>Graph</th>
<th>$C_i$</th>
<th>2a $\frac{8}{27}$</th>
<th>$\hat{a}_{(2a)} = \frac{-2s^2x_3x_2y_1\alpha_3(-x_3x_2y_1x_2y_3-x_3-y_1)}{\text{mx}_2xy} F_v^{(3)}(-x_2y_2t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3a $\frac{i}{3}$</td>
<td>$\hat{a}_{(3a)} = \frac{-2s^2x_3x_2y_1\alpha_3(-x_3x_2y_1x_2y_3-x_3-y_1)}{\text{mx}_2xy} [2(x_2 + y_2)(2y_1^2 - y_1\alpha_2 - 2x_1y_1 + 2x_1\alpha_2 + \alpha_1^2 - 1) -\kappa(5x_1^2 - 4y_1^3 - 10y_1 - 5x_1y_1 + 3x_1y_1^2 - 4 - 3x_1y_1 + 4\alpha_1^2) -5x_1\alpha_1 - 2x_1\alpha_2^2 + 2x_1\alpha_1 + 6y_1^2\alpha_1 - 2y_1\alpha_1^2 - 4y_1\alpha_1 + x_1y_1\alpha_1] F_v^{(3)}(-x_2y_2t)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4a $\frac{i}{3}$</td>
<td>$\hat{a}_{(4a)} = \frac{s^2x_3x_2y_1\alpha_3(-x_3x_2y_1x_2y_3-x_3-y_1)}{\text{mx}_2xy} \frac{2y_2\alpha_1}{x_2\alpha_2} + \kappa(3y_1\alpha_1 - y_1 - 6\alpha_1^2 + 8\alpha_1 + 5x_1\alpha_1 - 4 - 5x_1) F_v^{(4)}(-x_2y_2t)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5a $\frac{8}{27}$</td>
<td>$\hat{a}_{(5a)} = \frac{2s^2x_3x_2y_1\alpha_3(-x_3x_2y_1x_2y_3-x_3-y_1)}{\text{mx}_2xy} \frac{2x_2\alpha_1 - \kappa(\alpha_1 + y_2\alpha_2 - 3x_2\alpha_1 + 4\alpha_1^2) F_v^{(4)}(-x_2y_2t)}{mx_2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{(3a)1}(\alpha_1 - x_2)(\alpha_1 - y_2) t + (x_2 - y_2)^2m^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{(3a)2} = -(\alpha_1 - y_2)^2 + y_1^2m^2$, $f_{(3a)1} = x_2 - y_2$, $f_{(3a)2} = y_1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{(4a)1}(\alpha_1 - x_1)(\alpha_1 - y_1) t + (x_1 - y_1)^2m^2$</td>
<td></td>
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</tr>
<tr>
<td>$d_{(4a)2} = -(\alpha_1 - y_1)^2 + y_1^2m^2$, $f_{(4a)1} = x_1 - y_1$, $f_{(4a)2} = y_2$</td>
<td></td>
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</tr>
<tr>
<td>$d_{(5a)1} = (\alpha_1 - y_2)^2 + y_2^2m^2$, $d_{(5a)2} = (\alpha_1 - x_2)(\alpha_1 - y_2) t + (x_2 - y_2)^2m^2$</td>
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<tr>
<td>$d_{(5a)2} = (\alpha_1 - y_2)^2 + y_2^2m^2$, $f_{(5a)1} = -y_2$, $f_{(5a)2} = y_1$, $f_{(5a)3} = x_2 - y_2$</td>
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</tr>
</tbody>
</table>
where \( f_{ij} \) and \( d_{ij} \) are functions of the momentum fractions \( \alpha_1, \beta_1, x_1, y_1 \). Moreover, the \( d_{ij} \) depend on \( t \) and \( m^2 \) too. Obviously, these terms in the \( d_{ij} \) have to be kept now. Otherwise the integrals in (18) would not exist. \( \hat{A}_i \) can easily be integrated over \( \beta_1 \) by using partial fractioning and the standard formula

\[
\frac{1}{z + i\epsilon} = \mathcal{P} \frac{1}{z} - i\pi \delta(z)
\]

(23)

where \( \mathcal{P} \) denotes the principal value integral. In the kinematical region of interest, namely \( m^2, |t| \ll s \), the principal value part can be shown to be suppressed by \( 1/s \) as compared to the \( \delta \) function part. The \( \delta \) function provides the condition \( \beta_1 = \alpha_1 + \mathcal{O}(1/s) \) in this case. Hence, to leading order in \( s \), we approximate (22) by

\[
\hat{A}_i \approx -\frac{i\pi}{s} \hat{a}_i(\alpha_1, \alpha_1) \delta(\beta_1 - \alpha_1)
\]

(24)

Representative examples of the functions \( d_{ij} \) and \( f_{ij} \) as well as of the \( \hat{a}_i \) are quoted in the table.

The other integrations appearing in (18) have to be done numerically using (23) again. Since in general \( \text{signum}(f_{ii}) \) is not equal to \( \text{signum}(f_{i\bar{i}}) \) the \( \hat{A}_i \) have both real and imaginary parts. An exception is the graph 2c where \( f_{(2c)1} = x_1 \) and \( f_{(2c)2} = y_1 \). In this case the two principal value integrals cancel and the leading contribution to \( \hat{A}_{2c} \) therefore simplifies to

\[
\hat{A}_{(2c)} \approx -\frac{2\pi^2}{s} \hat{a}_{(2c)}(\alpha_1, \alpha_1) \delta(\alpha_1 - \beta_1) \delta(d_{(2c)1}f_{(2c)1} - d_{(2c)2}f_{(2c)2})
\]

(25)

With the help of this new \( \delta \) function a second integration in (18) can be immediately carried out.

The graphs 5a and 5b, comprising 4-point diquark vertex functions, have \( s \) in three propagators. The contribution of these graphs can be written in the form

\[
\hat{A}_i = \hat{a}_i(\alpha_1, \beta_1) \prod_{j=1}^{3} \frac{1}{s(\alpha_1 - \beta_1)f_{ij} + d_{ij} + i\epsilon_j}
\]

(26)

As an example we quote the functions \( \hat{a}_{5a} \) for the graph 5a together with the \( d_{(5a)ij} \) and \( f_{(5a)ij} \) in the table. To leading order in \( s \) these contributions are also dominated by the imaginary parts of the propagator poles at \(-d_{ij}/(sf_{ij})\). Up to corrections of order \( 1/s \) this again implies \( \beta_1 = \alpha_1 \). Thus, we find for \( i = 5a, 5b \)

\[
\hat{A}_i \approx -\frac{i\pi}{s} \hat{a}_i(\alpha_1, \alpha_1) \delta(\alpha_1 - \beta_1)
\]
\[
\left[ \frac{\text{signum}(f_{i1})}{d_{i2}f_{i1} - d_{i1}f_{i2} + i\text{signum}(f_{i1})\epsilon_2} \quad \frac{1}{d_{i3}f_{i1} - d_{i1}f_{i3} + i\text{signum}(f_{i1})\epsilon_3} \right] + (1, 2, 3) \text{ cyclic} \tag{27}
\]

How to proceed from here should be obvious.

Finally let us discuss the graph 3c. A pole only appears in the \(s\)-channel propagator and \(\hat{A}_{(3c)}\) is of the form

\[
\hat{A}_{(3c)} = \hat{a}_{(3c)} \frac{1}{s(\alpha_1 - \beta_1)(y_1 - x_1) + d_{(3c)} + i\epsilon}. \tag{28}
\]

It can be shown that the leading log contribution from this graph to the integral over \(y_1, \beta_1\) in (18) is dominated by the region near \(\alpha_1 = \beta_1\) and \(y_1 = x_1\):

\[
\int_0^1 dy_1 \int_0^1 d\beta_1 \frac{F(s, t, \beta_1, y_1, \ldots)}{s(\alpha_1 - \beta_1)(y_1 - x_1) + d_{(3c)} + i\epsilon} \sim F(s, t, \beta_1 = \alpha_1, y_1 = x_1 \ldots) I(s), \tag{29}
\]

where \(F\) absorbs all terms appearing in (18) including \(\hat{a}_{(3c)}\) and

\[
I(s) = \int_0^1 dy_1 \int_0^1 d\beta_1 \frac{1}{s(\alpha_1 - \beta_1)(y_1 - x_1) + d_{(3c)} + i\epsilon}. \tag{30}
\]

Approximately this integral is given by

\[
I(s) \sim \int_{-1/2}^{1/2} du \int_{-1/2}^{1/2} dv \frac{1}{suv + d_{(3c)} + i\epsilon} + \mathcal{O}(1/s) = \frac{2}{s} \left[ \text{dilog} \left( -\frac{s}{4d_{(3c)}} \right) - \text{dilog} \left( \frac{s}{4d_{(3c)}} \right) \right] \sim -\frac{2i\pi}{s} \ln s.
\]

Note, that \(\hat{a}_{(3c)} \propto s^2\) as the contributions from the other graphs (see the table). Thus, the dominant contribution from graph 3c is

\[
A_{(3c)}^{LL} \propto i s \ln (s) f(t). \tag{32}
\]

We see that we are here dealing with a ladder gluon graph. As is well-known [9, 24] the higher ladder graphs cannot be ignored at large \(s\). They behave as \((\alpha_s \ln s)^n\) asymptotically and the full set of ladder graphs exponentiates and represents the Pomeron

\[
A_{(3c)}^{LL}(s, t) \propto i s \sum_n C_n(t)(\alpha_s \ln(s))^n \propto i s f(t) \exp(\alpha_s \ln(s)\phi(t)) = i f(t) s^\alpha r_{\text{rem}}(t) \ll i s f(t)
\]

Hence, the term (32) should be regarded as part of the Pomeron contribution to the scattering amplitude and is therefore to be subtracted from the full contribution of graph 3c. We actually do this by simply subtracting (32) from (28). This subtraction prescription does not spoil gauge invariance, see [24]. After subtraction the remainder behaves \(\propto s\) and has both real and imaginary parts.
5 Numerical results for spin-dependent pp scattering

In our numerical studies of proton-proton scattering we use the following form of the scalar and vector diquark DA

$$\varphi_S(x_1) = N_S x_1 x_2^3 \exp \left[ -b^2 \left( \frac{m_q^2}{x_1} + \frac{m_S^2}{x_2} \right) \right]$$

$$\varphi_V(x_1) = N_V x_1 x_2^3 (1 + 5.8 x_1 - 12.5 x_2^3) \exp \left[ -b^2 \left( \frac{m_q^2}{x_1} + \frac{m_V^2}{x_2} \right) \right]$$

and the set of parameters

$$f_S = 73.85 \text{ MeV}, \quad Q_S^2 = 3.22 \text{ GeV}^2, \quad a_S = 0.15,$$

$$f_V = 127.7 \text{ MeV}, \quad Q_V^2 = 1.50 \text{ GeV}^2, \quad a_V = 0.05, \quad \kappa = 1.39$$

as proposed in [18, 19]. The values of the masses in the exponentials are taken as 330 MeV (for the quarks) and 580 MeV (for the diquarks). The transverse size parameter $b$ is taken to be $0.498 \text{ GeV}^{-1}$. The normalization constants $N_S$ and $N_V$ have the values 25.97 and 22.29, respectively. As we mentioned in the preceding section the $\beta_1$ integration is trivial. The other three integrations over the hard amplitude and the proton DAs are carried out numerically. Since we neglect $1/s$ corrections throughout we find an energy independent ratio of the helicity-flip and non-flip amplitudes.

Let us discuss the role of the contributions from the individual graphs briefly. The contributions from the graphs 2a and 2b to $A$ are purely imaginary. Thus, although these contributions lead to helicity flips they do not produce a phase difference between the $A$ and $B$ and, hence, do not contribute to the single spin asymmetry. The graph 2c yields a real contribution that is quite small, about a few percent of $\text{Im } A$ at $|t| \lesssim 10 \text{ GeV}^2$. The contributions to the real part of $A$ provided by the graphs 3a and 3b though substantial are compensated by the contribution from graph 3c to a large extent. The contributions of the graphs 4a, 4b, 5a and 5b to the real part of $A$ are very small as the numerical evaluation reveals. Their imaginary parts, however, are not small as is that from graph 3c. These imaginary contributions play an important role for the double spin asymmetry parameter $A_{NN}$.

The results of our calculations for the invariant function $A$ are shown in Fig. 7. As can be seen from that figure the imaginary part of $A$ is much larger than its real part. The real part of $A$ changes sign at $|t| \sim 3.5 \text{ GeV}^2$. The absolute value of the ratio of $A$ and $B$ represents the ratio of helicity-flip and non-flip amplitudes. This ratio is fairly large $\sqrt{-t} |A|/|B| \sim 0.2 - 0.3$ at $|t| \gtrsim 3 \text{ GeV}^2$ indicating the substantial amount of helicity flips generated through the vector diquarks in our model.
The interference of the real part of $A$ with the purely imaginary ansatz for the function $B$ yields the single-spin asymmetry $A_N$ (6). Our prediction for $A_N$ is shown in Fig. 8 and compared to the high-energy experimental data at $|t| \geq 3\text{GeV}^2$ ($\sqrt{s} = 19\text{GeV}$) [4]. The quality of the present data is poor and prevents any severe test of our predictions. The predicted asymmetry amounts to about 20–30% for $|t| > 6\text{GeV}^2$; it is of the same order of magnitude as has been observed in the low-energy BNL experiment [3]. The decrease of the asymmetry at smaller momentum transfer is connected with the observed zero of Re$A$ (see Fig. 7).

The predictions for the double spin asymmetry $A_{NN}$ are shown in Fig. 9. $A_{NN}$ turns out to be of the order of $10^{-20\%}$. Our results for the spin asymmetries are rather close to those obtained in [14, 25] although the latter are valid in the momentum transfer region $2\text{GeV}^2 < |t| < 4\text{GeV}^2$. The spin observables obtained within the model are essentially independent on the parameterizations (9,10) used for the invariant function $B$.

6 Summary

On the basis of the diquark model we have calculated spin effects in high-energy proton-proton scattering at moderately large momentum transfer. The two-gluon graphs for the colour–singlet $t$-channel exchange have been considered for the invariant function $A$ while the invariant function $B$, dominating the helicity non-flip amplitudes, is parametrized by a standard phenomenological Pomeron. It describes qualitatively the differential cross section of the elastic $pp$ scattering. The function $A$ is calculated under the assumption that the $t$-channel gluons couple to one constituent, quark or diquark, each in the helicity non-flip vertex. In the helicity flip vertex we include the perturbative $\alpha_s$ correction. Hence, we consider minimally connected graphs which allow to keep all constituents collinear. In our model the helicity flips are generated by vector diquarks. It turns out that the invariant function $A$ is of substantial magnitude and not in phase with the Pomeron contribution. Our model, therefore, provides a single-spin asymmetry that is rather large for momentum transfer $|t| \geq 3\text{GeV}^2$. The double spin transverse asymmetry in this kinematical region are rather large in our model. The important feature of the spin effects obtained in our model is their approximate energy independence. On the other hand, they decrease with increasing momentum transfer. Our results are valid at large $s$ and moderately large momentum transfer (>few GeV$^2$). This kinematical region can be investigated for instance in the proposed HERA-$\bar{N}$ experiment [26].
Finally we want to stress that our predictions for $A_N$ should not be taken literally since phase differences are hard to calculate, they depend on many subtle details which are not well under control in a model. The diquark model on which our model is based was designed for a different kinematical region. In so far, a failure of our prediction for $A_N$ would not necessarily imply a failure of the diquark model in general but would rather indicate that the phase differences are not well under control and/or that the diquark model is applied beyond its range of applicability.

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References


**Figures**

FIG.1. Structure of the spin-non-flip proton vertex.

FIG.2. Feynman graphs containing the 3-point diquark function (without 3-gluon coupling).

FIG.3. Feynman graphs containing the 3-point diquark function (with 3-gluon coupling).

FIG.4. Feynman graphs containing the 4-point diquark function (with 3-gluon coupling).

FIG.5. Feynman graphs containing the 4-point diquark function (without 3-gluon coupling).

FIG.6. Structure of the 4-point diquark function.

FIG.7. t-dependence of the amplitude $A$, solid line: imaginary part; dot-dashed line: real part.

FIG.8. Model predictions for single-spin asymmetry (solid line: for the MPE model (9); dashed line: for the LP model (10)).

FIG.9. Model predictions for double-spin asymmetry (solid line: for the MPE model (9); dashed line: for LP model (10)).
Fig. 9
\[ \begin{align*} 
\text{Diagram} & = \text{Diagram} + \text{Diagram} \\
& = a + b 
\end{align*} \]
a

b
\[
\begin{align*}
\text{Diagram} & = \text{Diagram}_a + \text{Diagram}_b + \text{Diagram}_c
\end{align*}
\]