Consistent Sets and Contrary Inferences: 
Reply to Griffiths and Hartle

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Abstract

It was pointed out recently [A. Kent, Phys. Rev. Lett. 78 (1997) 2874] that the consistent histories approach allows contrary inferences to be made from the same data, corresponding to commuting orthogonal projections in different consistent sets. To many, this seems undesirable in a theory of physical inferences. It also raises a specific problem for the consistent histories formalism, since that formalism is set up so as to eliminate contradictory inferences, yet there seems to be no sensible physical distinction between contradictory and contrary inferences. It seems particularly hard to defend this asymmetry, since (i) there is a well-defined quantum histories formalisms which admits both contradictory and contrary inferences, and (ii) there is also a well-defined formalism, based on ordered consistent sets of histories, which excludes both.

In a recent comment, Griffiths and Hartle, while accepting the validity of the examples given in the above paper, restate their own preference for the consistent histories formalism. As this brief reply explains, in so doing, they fail to address the arguments actually made against their approach to quantum theory.

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Griffiths and Hartle[1] correctly reiterate that logical contradictions in the consistent histories formalism can be avoided by restricting one’s reasoning to a single consistent set. This is, of course, generally understood, and clearly stated in the original Letter.[2] But it is no great virtue. Any formalism which ascribes probability distributions to sets of histories in quantum theory can be made free from contradiction by such a restriction, whether or not any consistency criterion is imposed on the sets, whatever rule is used to define history probabilities. There are infinitely many such formalisms, and a priori the consistent histories formalism has no special status. The questions we should ask of it, or any other quantum history formalism, are whether its rules are natural and lead to physically plausible and scientifically useful conclusions. It is the naturality of the consistency criterion and the plausibility of its physical implications which the Letter addresses. (Criticisms of the consistent histories approach as a scientific theory can be found elsewhere.[e.g.3,4,5,6])

To examine any history-based formalism properly, it is necessary to stand outside that formalism, to ask: Why these rules, rather than others? What are their physical consequences? How do they compare with those of alternative rules? To attempt to defend the consistent histories formalism simply by restating the internal consistency of its own rules, and to argue that a particular asymmetry is acceptable simply because it is a feature of the formalism, as Griffiths and Hartle in effect do, is to fail to address these key questions.

Our understanding of what it means for two physical propositions to be contradictory or contrary is — historically and, unless we are already irrevocably committed to the formalism under discussion, logically — prior to our understanding of the features of a given formalism. Our views on whether the formalism is natural, plausible or useful are, inevitably, framed in terms of that prior understanding. The standard contradictory/contrary terminology thus seems to me most appropriate, though of course the argument is invariant under translation.

To recap: the Letter points out that the consistent histories formalism allows contrary inferences, although one of the arguments used in its justification is that it prevents contradictory inferences. As Ref. [1] notes, “There are no initial and final states for which projectors $P$ and $(1 - P)$ can have probability one in different consistent families, because the probability assigned to any event on the basis of given data is independent of the consistent family”. But — it is vital to be clear on this — this is a deliberately chosen
feature, not a logically necessary requirement. In many history-based formalisms of quantum theory — for example, that based on unrestricted sets of histories — a proposition \( P \) can have probability one in one set and its complement \( (1 - P) \) non-zero probability in another, conditioned on the same data. These formalisms too produce no inconsistency if reasoning is carried out only within one set.[4,6] The term “consistent histories” is in this respect misleading.

No argument for the asymmetry between contradictory and contrary inferences has yet been produced. To say[comment] “a pair \( P \) and \( 1 - P \) can always be associated with the logical notion of ‘contradictory’, while perpendicular projectors need not be associated with the logical notion of ‘contrary’ ” is, again, to report a feature of the consistent histories formalism, not a general truth. In attempting to defend the asymmetry, Griffiths and Hartle simply restate it.

Consistent historians must also accept that contrary statements about past events — “the electron went through slit A with probability one” and “the electron went through slit B with probability one”, say — can be equally valid, and hence that standard ordering inferences based on our observations can fail. For example, the statement that a particle was observed in one region need not imply that it was observed in a region containing the first, even when each statement can consistently be considered together with our observations.[7]

I should stress again that these facts do not amount to a logical refutation of the consistent histories approach. Readers will no doubt form their own views of their implications for its naturality and plausibility.

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**References**