Conformal Symmetry and Unification

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Abstract.
The Weyl-Weinberg-Salam model is presented. It is based on the local conformal gauge symmetry. The model identifies the Higgs scalar field in SM with the Penrose-Chernikov-Tagirov scalar field of the conformal theory of gravity. Higgs mechanism for generation of particle masses is replaced by the originated in Weyl's ideas conformal gauge scale fixing. Scalar field is no longer a dynamical field of the model and does not lead to quantum particle-like excitations that could be observed in HE experiments. Cosmological constant is naturally generated by the scalar quadric term. The model admits Weyl vector bosons that can mix with photon and weak bosons.

INTRODUCTION

In 1918, Herman Weyl presented the idea and notion of gauge invariance [1]. It was a consequence of natural generalization of Riemannian geometry used in Einstein’s General Relativity theory (GR). Weyl assumed that Einstein’s metricity condition

\[ \nabla g = 0 \] (1)

could be replaced by a less restrictive conformal condition

\[ \nabla g_{\mu\nu} \sim g_{\mu\nu}. \] (2)

Thus he supposed that for a vector transported around a closed loop by parallel displacement not only the direction but also the length can change, but the angle between two parallelly transported vectors has to be conserved. Weyl observed that if the Einstein’s torsion free condition

\[ \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} = T^\lambda_{\mu\nu} = 0. \] (3)

\[ ^1 \] Talk given at the International Conference Particles, Field and Gravitation, Lodz, April 1998
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is kept, then - similarly as in the case of GR - there is a relation between the metric and the affine structure of tangent bundle $TM$. In contrary to GR case, the Weyl connection is not given uniquely by the Christoffel symbol: it could depend on an arbitrary vector field in principle. This vector field is a compensating potential for a local conformal group of scalar multiplicative transformations conserving the conformal condition (2). Weyl called this group the gauge group as it sets a reference scale from point to point in the space-time. Initially he interpreted the new vector field as the electromagnetic potential and has proposed a dynamics for the model that was based on the bilinear in the generalized curvature Lagrangian.

The dynamics of original Weyl’s theory turned out to be much more complicated than the dynamics of Einstein GR. The idea of gauge conformal invariance of the theory was also a subject of intensive criticism. Weyl’s conformal theory leaves the freedom for the space-time dependent choice of length standards. It seamed that this gauge freedom clashes with quantum phenomena that provide an absolute standard of length. The point is however, that the freedom to set arbitrary length standards along an atomic path does not mean that atomic frequencies will depend on atomic histories (what was the most popular argument in early literature). In Weyl’s theory, an atomic frequency depends on the length standard at a given point but not on a history of the atom. Simultaneously, all other dimensional quantities measured at this point depend on this standard in the same way. Consequently dimensionless ratios are standard independent and experimental predictions do not depend on a particular conformal gauge fixing.

The more fundamental arguments raised against conformal theory were based on the reasonable claim that an acceptable theory should not introduce needles objects and notions. If atomic clocks measure time in an absolute way and velocity of light is an absolute physical quantity (or is definite at least) then the relativism of length is unnatural and redundant. However, we should point out a very essential assumption concerning atomic clocks that is hidden in the above. This is an extrapolation of our flat and first order experience that all atomic clocks are proportional always and everywhere. One assumes – roughly speaking – that the ratios of electron mass to proton mass and to other quantum standards are always and everywhere the same. One can believe that this statement is true but one should remember (especially when such effects like red shift or other distant signals are interpreted) that at the large scale this statement is only an assumption. It should be (and it could be! [2]) a subject of experimental verification. Conformally invariant gauge theory apparently makes a room to relax from such a priori suppositions [4], but in fact it does not predicts itself a dynamics for evolution of fundamental physical ”constants”

Weyl conformal theory is a gauge theory of length. It was proposed as a geometrical theory of electromagnetism. Soon after, Dirac proposed his theory
of quantum relativistic electron in the flat space \[3\]. It was a gauge theory of complex electron’s phase and it turned out that it provides more adequate framework for description of electromagnetic phenomena. Weyl’s proposal was abandoned by the author himself (but still in 1973 Weyl’s gauge theory of scale was considered by ... Dirac as a candidate for description of electromagnetism \[4\]).

The original Dirac’s theory of electron was extended to the curved space case \[5–7\]. Taking a four-dimensional manifold \(M\), a copy of two dimensional complex vector field \(F_p M\) can be attached to every point \(p\) of \(M\). Then two, in principle independent pairs of affine and metric structures can be implemented on the manifold. The natural tangent boundle \(TM\) can be equipped with an affine connection \(\Gamma\) and the field of metric \(g\). Independently a connection \(\gamma\) can be defined in the boundle \(FM\) and an arbitrary field \(\varepsilon\) of Levi-Civita metric can be chosen (for generic two-dimensional complex vector space there is a natural class of antisymmetric Levi-Civita metrics that differ by a complex factor).

The two structures \(\{\Gamma, g\}\) and \(\{\gamma, \varepsilon\}\) can be naturally correlated. The important observation is that the Levi-Civita metric \(\varepsilon\) induce Lotenz metric \(\varepsilon \otimes \varepsilon\) at every fiber of the tensor product boundle \(FM \otimes FM\) (see e.g. \[8\] for further details). Thus the real part of \(FM \otimes FM\) (which is a four dimensional real vector boundle) can be related with the tangent vector boundle \(TM\).

It was found by Infeld and van der Waerden \[7\] that such correlation of boundles correlates also their metrics and affine structures. Keeping the restrictions of GR (metricity and torsion-free) they have shown that metric structure \(\varepsilon\) of \(FM\) is given by metric structure \(g\) of \(TM\) up to the arbitrary phase factor. Simultaneously the affine structure \(\gamma\) of \(FM\) is given by the affine structure \(\Gamma\) of \(TM\) up to an arbitrary vector field. This new vector field is a compensating potential for the \(U(1)\) local symmetry group of phase transformations of all Dirac fields in the theory. The authors have identified this new field with electromagnetic potential. Such identification was a subject of criticism as the new vector potential has been coupled universally to all fermions including chargeless neutrino. The modern Weinberg-Salam theory (WS) predicts that all fermions couple to \(U(1)\) gauge field. There is a second nonabelian gauge group \(SU(2)\) in the theory acting only on the left components of Dirac bispinors. Due to the structure of couplings and the effective mass matrix for gauge bosons the massless field - naturally identified with photon - is a combination of original \(U(1)\) and \(SU(2)\) bosons. It does not couple to neutrinos despite the fact that the original abelian vector potential does. Thus the Infeld - van der Waerden vector potential can be naturally identified with \(U(1)\) gauge group potential of the WS model without any conflict with theory and experiment.

The rest of the present paper is devoted to the description of the version of Weinberg-Salam theory conformally coupled with Weyl’s theory of gravity. The first version of the model was proposed in \[9\] (see also \[10\]). Similar ideas
were also presented in [11]. More comprehensive list of the bibliography of the subject can be found in [12].

Taking into account the roots of the theory it could be called the Weyl-Weinberg-Salam model (WWS).

WEYL-WEINBERG-SALAM MODEL

Let us fix the notation

Weyl’s potential will be denoted by $S_\mu$. Let us assume torsion free condition (3). Then the connection in $TM$ is given by

$$\Gamma^\rho_{\mu\nu} = \Omega^\rho_{\mu\nu} + f(S_{\mu\nu}g^\rho_\nu + S_{\nu\rho}g^\rho_\mu - S^\rho_{\mu\nu})$$

(4)

where $f$ is an arbitrary coupling constant (in principle it could be absorbed at this level by a redefinition of $S_\mu$ but it is convenient to keep it here and set its value later). Consequently Weyl’s conformal condition (2) gets the form

$$\nabla_\mu \hat{g} = -2fS_\mu \hat{g}$$

(5)

Equations (4) and (5) are invariant with respect to Weyl’s transformations

$$g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu} = e^{2\lambda} g_{\mu\nu}$$

(6)

$$S_\mu \rightarrow S_\mu - \frac{1}{f} \partial_\mu \lambda.$$  

(7)

Thus metric tensor is covariant with respect to Weyl’s transformations with degree 2. The Riemann and Ricci tensors constructed from (4) are conformally invariant objects but their contraction to scalar curvature $R$ is not. $R$ can enter linearly to a conformally invariant expression of dimension of action if it is combined with a scalar Penrose-Chernikov-Tagirov (PCT) field $\varphi_{\text{PCT}}$ [13] that transforms according to

$$\varphi_{\text{PCT}} \rightarrow e^{-\lambda} \varphi_{\text{PCT}}.$$  

(8)

Then the combination $\varphi^2_{\text{PCT}} R$ is conformally invariant. The conformal covariant derivative of $\varphi_{\text{PCT}}$ is given by

$$\nabla_\mu \varphi_{\text{PCT}} = (\partial_\mu - fS_\mu) \varphi_{\text{PCT}}$$

(9)

and it transforms according to (8).

The most general conformally invariant Lagrangian that leads to second order equations of motion for the metric-Weyl-scalar system reads [14]

$$L_g = -\frac{\alpha_1}{12} \varphi^2_{\text{PCT}} R + \frac{\alpha_2}{2} \nabla_\mu \varphi_{\text{PCT}} \nabla^\mu \varphi_{\text{PCT}} - \frac{\alpha_3}{4} H_{\mu\nu} H^{\mu\nu} - \frac{\lambda}{4!} \varphi^4_{\text{PCT}}$$

(10)
where

\[ H_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu. \]  

(11)

The coupling constants \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) are arbitrary but the last two constants can be absorbed in \( \varphi_{\text{PCT}} \) and \( S_\mu \) by a suitable redefinition of the fields. Observe however, that we are not able to absorb simultaneously \( \alpha_3 \) and \( f \). The last coupling remains arbitrary and has to be fixed by experiment.

We can also include the original Weyl Lagrangian being the square of Weyl tensor \( L_W = \rho C^2 \) where \( \rho \) is a coupling constant.

Now we can face the Weinberg-Salam part, or more generally, the full Standard Model of fundamental interactions [15].

First, we should recall [16] that Weyl’s vector potential \( S_\mu \) do not couple directly to Dirac fermions if they transforms according to the rule

\[ \Psi \rightarrow e^{-\frac{3}{2} \lambda} \Psi. \]  

(12)

The conformally invariant part of SM can be written in the following form:

\[ \mathcal{L}_c^{SM}[\varphi_H, n, V, \psi, g] = \mathcal{L}_c^{SM} + [-\varphi_H F + \varphi_H^2 B - \lambda \varphi_H^4]. \]  

(13)

\( \mathcal{L}_c^{SM} \) is the conventional SM Lagrangian without the “free” part for the modulus of the Higgs \( SU(2) \) doublet \( \varphi_H \) and without the Higgs mass term; \( B \) is the mass term of the vector fields generally denoted by \( V \) and \( F \) is the mass terms of the spinor fields generally denoted by \( \psi \)

\[ B = Dn(Dn)^* ; \quad F = (\bar{\psi}_L n) \psi_R + h.c.; \quad n = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} ; \quad n_1 n_1^* + n_2 n_2^* = 1 ; \]  

(14)

\( n \) is the angular component of the Higgs \( SU(2) \) doublet.

As there are two abelian gauge groups in the model also a mixed term

\[ \mathcal{L}_{SB} = \alpha_4 H_{\mu\nu} F^{\mu\nu} \]  

(15)

is admitted by all symmetries of the model in general.

The main idea of conformal unification consists in the identification of PCT scalar field \( \varphi_{\text{PCT}} \) with the modulus of Higgs doublet \( \varphi_H \) within the rescaling factor \( \chi \)

\[ \varphi_H = \chi \varphi_{\text{PCT}}. \]  

(16)

The total lagrangian of the conformally unified WWS model can be written as a sum of three terms described above

\[ L_T = L_g + \mathcal{L}_c^{SM} + \mathcal{L}_{SB} \]  

(17)

with the constraint (16) resolved.

The rescaling factor \( \chi \) of (16) is a new coupling constant, which coordinates weak and gravitational scales [17].
SCALE FIXING

The theory given by (17) does not contain any dimensional parameter. This is the necessary condition for it to be conformally invariant. As it was discussed in the Introduction in the context of the Weyl theory alone, dimensional quantities are observed in nature only indirectly. Measuring one of them, we always refer to some other dimensional quantity. We measure ratios of dimensional quantities and we are not able to measure anything more. Our statements express the ratios in the form that carries in the content of its measure an information on the denominator. Thus the dimensional quantities in the half seems to be nothing but only a product of human invention, a logical and a lingual abbreviation representing both the physical information and the chosen convention. There is no doubt that the abbreviation is convenient and useful in practice - in our “flat” surrounding at least (see however the Introduction again). The conformal theory reproduces this conventional abbreviation. It could be done with the help of the most natural mechanism for this purpose, the mechanism of scale fixing which is an example of the gauge fixing of the conformal gauge symmetry group (it is in fact the first historical example of the notion of gauge).

Gauge fixing freedom allows us to impose an additional condition on the theory variables. All lawful conditions (those that can be fulfilled by the fields obtained from a generic configuration by a gauge symmetry transformation) are classically equivalent but not all of them are equally convenient for a given practical purpose. In the case of our conformal theory, we are free to fix the dimensional scale. A natural choice is the one that fixes particle masses in our flat surrounding to their conventional space-time independent values. (In fact, nobody will admit in practice that the choice could be a different.) This could be achieved for the conformal symmetry gauge condition that fixes the scalar field in $L^c_{SM}$ (13) to a constant (space-time independent) value. Thus we can demand that

$$\varphi_H = \text{const} = v$$

and it is clear that a generic nonzero scalar field configuration can be conformally transformed to fulfill condition (18).

Choosing $v = 246\text{GeV}$ and choosing ordinary unitary gauge of weak group, we reproduce the whole structure of classical SM masses in WWS model.

It should be stressed here that no mechanism of spontaneous or dynamical symmetry breaking was used in order to produce particle masses. The conformal gauge fixing condition (18) was a sufficient tool. Let us also comment - but without further discussion - that however the condition (18) serves for easy identification the flat space particle content of the model, it needn’t be the best toll for other purposes. The condition leads to a massive sigma model that is not perturbatively renormalizable. (The fact does not prejudice
the renormalizability of the theory - if we can speak at all about a renormalizability of the theory including gravity. A convenient choice of gauge fixing condition is essential for perturbative analysis of the renormalizability problem. It is known, e.g. that the unitary gauge is not the best choice for this task in SM.

**TOWARD EXPERIMENT**

The properties of theory given by (17) depend on the value of coupling constants $\alpha_i$, $\rho$, $f$, $\lambda$ and $\chi$.

The striking feature of the conformal theory is the lack of ordinary Einstein term in (17). Observe however, that even in the simplest case $\chi = 1$ (the Higgs field identified with PCT scalar field), the condition (18) allows us to reproduce easily the Einstein term [11]. It is sufficient to demand that

$$\frac{\alpha_1}{12} v^2 = \frac{1}{8\pi G}$$

(19)

If the conformal gauge fixing condition (18) is chosen, a mass term for the Weyl’s vector field $S_\mu$ appears [11] and $S_\mu$ acquires mass

$$m_S^2 = \frac{1}{2} f (\alpha_2 - \alpha_1) v^2$$

(20)

The condition (19) leads to Weyl vector mass

$$m_S = 0.5 \cdot 10^{19} f \cdot GeV.$$  

(21)

In turn the Weyl’s mass equals zero only in the special case when $\alpha_2 = \alpha_1$. Then an additional symmetry is realized in the model. Without changing the action we can transform according to the rules of conformal transformations (6), (8) and (12) the metric, the scalar and the all fermion fields leaving the Weyl field unchanged. Similarly we can independently transform $S_\mu$ and (17) will not change. In that case the Weyl potential decouples from scalar field and if $\alpha_4 = 0$, it is coupled only to gravity. We get Penrose-Chernikov-Tagirov theory of scalar field conformally coupled with gravity [13]. In order to reproduce appropriate Newtonian limit already at the classical level, we have to demand that $\chi$ is very small [9,17]

$$\chi \sim \frac{v}{m_{PLANCK}}$$

(22)

In the flat limit approximation (the condition (18) is applied, dynamics of $g$ is frozen and $g$ is chosen to be the metric of Minkowski space) the conformally unified WWS theory leads to the SM-like $\sigma$-model. (It holds independently on the values of couplings $\alpha_i$, $\rho$, $f$ and $\lambda$ in (17)). There is
still $U(1) \times SU(2)_L \times SU(3)$ gauge symmetry but the feature of perturbative renormalizability is lost. Despite this fact the theory is still predictive. We can reproduce all SM 1-loop results for the processes without external Higgs lines. The SM Higgs mass is replaced in calculations by an effective cutoff that can be expressed (eliminated) by some measured quantity or a combination of observables. 1-loop predictions for 12 LEP observables were given in [18] in reasonable agreement with SM and experiment.

The flat limit of the presented unified WWS model can be a subject of experimental verification and discrimination. The direct verification will be provided (of course!) by LHC. This installation should produce data able to cover all admissible SM Higgs mass range. If no Higgs signal will be found (and we know from LEP that it should be found there if SM is valid) then conformal unified model predicting no dynamical scalar particle at all should be a serious alternative. In turn founding at LHC Higgs particle with the all its SM predicted properties will tell us that the minimal conformal unification is not good.

There was also proposed an indirect method for verification of the flat limit consequences of WWS model [19]. It is based on the observation that while the SM Higgs mass $m_H$ is an energy independent physical constant, the cutoff $\Lambda$ introduced in the 1-loop analysis of $\sigma$-model can depend in principle on the energy of the process considered and on its other parameters. The idea is to derive $m_H$ from two experiments performed at different energy scales. If it will happen that the derived masses disagree it will mean that SM fails while WWS accepts this phenomenon. This is a kind of negative test of SM. It was estimated that the proposed comparison could be made on the base of data given by LEP and CESR B and PEP II if some demanded but realistic luminosity will be achieved.

CONCLUSIONS

The Weyl-Weinberg-Salam model identifies the Higgs scalar field in SM with the Penrose-Chernikov-Tagirov scalar field of the conformally invariant theory of gravity. This identification is very natural and it leads to important physical consequences:

Higgs mechanism for generation of particle masses is replaced by the originated in Weyls ideas conformal gauge scale fixing. Scalar field is no longer a dynamical field of the model – it is rather a Goldsone direction in field space, the direction that is tangent to the conformal gauge group. Consequently it does not lead to quantum particle-like excitations that could be observed in HE experiments and it does not acquire quantum expectation value in the vacuum. Experimental flat limit consequences of the model could be tested in near future.

No cosmological consequences characteristic for the SM Higgs field can be
derived from the present model, but the scalar sector generates cosmological consequences in a different way. The quadric coupling constant $\lambda$ of the scalar PCT field which in WWS does not play any role in generating particle masses, has its effect in generation of cosmological constant. This constant is dimensional and consequently it is scale choice dependent. In the standard approach, its value is given by $\lambda$ and by the mass standards fixing gauge condition (18). Thus we get

$$\Lambda = \frac{\lambda}{4!} \left( \frac{v}{\chi} \right)^4. \quad (23)$$

The very new feature of the Lagrangian (17) is the mixed term (15) that leads to an interaction of Weyl and $U(1)$ Weinberg-Salam vector potentials. At quantum level it would result in a mixing of Weyl boson with photon and weak bosons - the effect in a sense similar to the known $\gamma - Z$ mixing. As the mass of $S_{\mu}$ and the coupling $\alpha_4$ is not predicted by the theory, the strength of the mixing effect could be small as well as very large. Also the mass $m_S$ cannot be easily estimated from the known data as there is no interaction of fermions with the Weyl potential. Thus definite answers concerning the presence and interactions of Weyl sector should be looked in experiments.

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