Comment on “Regge Trajectories for All Flavors”

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In a recent Letter [1] (and in ref. [2]), Filipponi, Pancheri and Srivastava report on the construction of a formula for linear Regge trajectories for all quark flavors:

\[ \alpha_{ji}(t) = 0.57 - \frac{(m_i + m_j)}{\text{GeV}} + \frac{0.9 \text{GeV}^2}{1 + 0.2 \left( \frac{m_i + m_j}{\text{GeV}} \right)^{3/2}} t, \] (1)

where \( m_i, m_j \) are the corresponding constituent quark masses for the \( j \bar{i} \) trajectory.

As the authors of [1, 2] remark, no unique quark mass can be extracted from (1), and each trajectory \( \alpha_{ji}(t) \) rather corresponds to its own set \( (m_i, m_j) \). The values of \( m_i \) can be extracted by using the vector meson masses with hidden flavor into Eq. (1): \( \alpha_{ii}(M_{j\bar{i}}^2) = 1 \). Such an extraction gives (in GeV, \( n = u, d \), and the superscript indicates the trajectory from which the corresponding value is extracted) \( m_n^{\rho} = 0.05 \), \( m_s^{\phi} = 0.23 \), \( m_c^{J/\psi} = 1.70 \), \( m_t = 5.12 \). Then, the values of \( m_i \)'s for the \( j \bar{i}, i \neq j \) trajectories should be related to the above hidden-flavor values by additivity of trajectory intercepts. This additivity is satisfied in two-dimensional QCD and many QCD-motivated models ([3] and references therein), and therefore should be considered as a firmly established theoretical constraint on Regge trajectories. It is easily seen that in the case of the trajectories (1), this constraint implies \( m_n^{\rho} + m_s^{J/\psi} = m_n^{D^*} + m_s^{D^*} \), \( m_s^{\phi} + m_c^{J/\psi} = m_s^{D^*} + m_c^{D^*} \), \( m_n^{\rho} = m_n^{B^*} + m_s^{B^*} \), \( m_s^{\phi} = m_s^{B^*} + m_b^{B^*} \). Thus, e.g., the parameters \( m_i \) of the \( D^* \) and \( D_s^* \) trajectories must be related to those of the \( \rho, \phi \) and \( J/\psi \) ones, even if no unique values of \( m_i \) can be extracted. Using now these parameters as given by the above relations for calculating the vector meson masses, through \( \alpha_{ii}(M_{jj}^2) = 1 \), one finds (in MeV) \( M(D^*) = 1882.5 \), \( M(D_s^*) = 2007.1 \), \( M(B^*) = 4566.3 \), \( M(B_s^*) = 4724.1 \), in contrast to the measured values [4] (in MeV) \( M(D^*) = 2008 \pm 2 \), \( M(D_s^*) = 2112.4 \pm 0.7 \), \( M(B^*) = 5324.8 \pm 1.8 \).

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In the last two cases, the discrepancy between the calculated and measured values is \( \sim 700 \text{ MeV} \) which is an unsatisfactorily large inaccuracy. Thus, the trajectories (1) cannot combine both meson spectroscopy and additivity of intercepts; fixing the parameters \( m_i \) to reproduce spectroscopy will necessarily result in violation of the intercept additivity constraint. We note that simple constituent quark model relations, e.g., \( M(B^*) = (M(\rho) + M(\Upsilon))/2 \), \( M(B^s) = (M(\phi) + M(\Upsilon))/2 \), give better values than Eq. (1): (in MeV) \( M(B^*) = 5114 \), \( M(B^s) = 5240 \). Moreover, the numerical values of intercepts given by (1) in the light quark sector contradict data. Indeed, Eq. (1) gives \( \alpha_{\rho} = 0.47 \), vs. \( \alpha_{\rho} = 0.55 \), as extracted by Donnachie and Landshoff from the analysis of \( pp \) and \( pp \) scattering data [5], and \( \alpha_{K^*} = 0.29 \), vs. \( \alpha_{K^*} \approx 0.40 \) as follows from the analysis of hypercharge exchange processes \( \pi^+ p \rightarrow K^+ \Sigma^+ \) and \( K^- p \rightarrow \pi^- \Sigma^+ \) [6]. Since the values of intercepts determine the \( s \)-dependence of the total cross-sections, \( \sigma_{\text{tot}} \propto s^{\alpha(0) - 1} \), and the differential cross-section profiles, \( d\sigma/dx_F \propto (1 - x_F)^{1 - 2\alpha(0)} \), it is among the requirements for the theory to predict the exact numerical values of intercepts.

In ref. [2], two of the authors notice that since the flavor dependent Regge slope \( \alpha' = \alpha'(0)/(1 + A\tilde{m}) \), \( \tilde{m} = m_i + m_j \) has a large negative derivative for small \( \tilde{m} \), it appears that the condition on all the slopes in the light quark sector \( \alpha' \sim 0.8 - 0.9 \text{ GeV}^{-2} \) can be satisfied only with almost exact mass degeneracy in this sector. This fact, as noticed in ref. [2], prevented the authors from constructing trajectories satisfying additivity of inverse slopes which is another constraint provided by the heavy quark limit [3], in addition to intercept additivity, which the trajectories (1) do not meet. Although their remark is correct, we disagree that \( \alpha' = \alpha'(0)/(1 + A\tilde{m}) \) is the only form that may be used in order to construct the trajectory. Indeed, as we discuss in [3], the form

\[
\alpha'_{ji} = \frac{4}{\pi} \frac{\alpha'}{1 + \sqrt{\alpha'(m_i + m_j)/2}},
\]

where \( \alpha' = 0.88 \text{ GeV}^{-2} \) is the standard Regge slope in the light quark sector, satisfies additivity of inverse slopes, and reproduces the values of the slopes in agreement with those extracted from data, for the following constituent quark masses (in GeV): \( m_n = 0.29 \), \( m_s = 0.46 \), \( m_c = 1.65 \), \( m_b = 4.80 \), which, in contrast to the above values given by (1), are not atypical of values used in phenomenological quark models.

We believe this analysis raises serious doubts as to the suitability of the formula (1) for the phenomenological description of quarkonia.

References


