Can thermal inflation solve the monopole problem?

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Abstract

It is shown that thermal inflation arises naturally in rank greater than five unified theories when non-renormalisable terms are introduced. Thermal inflation is driven by two Higgs fields $\Phi_{B-L}$ and $\Phi_{B-L}$ which also break $U(1)_{B-L}$ when acquiring vevs at the end of inflation. The inflationary period provides enough e-foldings to solve the monopole problem for $M_{B-L} \geq 10^{12}$ GeV. We point out that observations suggest that $M_{B-L} \approx 10^{14}$ GeV.

I. INTRODUCTION

Supersymmetric grand unified theories (GUTs) of the strong, weak and electromagnetic interactions have received lots of interests since it has been shown that the three gauge coupling constants of the minimal supersymmetric standard model (MSSM), when interpolated to high energies, meet in a single point at $2 \times 10^{16}$ GeV [1]. GUT theories can also be viewed as low energy limits of a more fundamental theory. If this more fundamental theory is superstrings theory, GUTs could explain the discrepancy which exists between the string scale and the scale at which all fundamental interactions other than gravity merge. Therefore the idea of GUT is an idea which stands on its own, whether we believe or not in the existence of an ultimate theory of nature; and all problems which GUT theories have to deal with, such as the monopole problem, still have to be solved. If we do believe in the existence of an ultimate theory, which is probably a theory of quantum gravity, then we may want to include Planck suppressed operators (or operators suppressed by some very high energy scale) when building a GUT model. These operators then underline the fact that the GUT emerges from a more fundamental theory, but without any assumption upon the nature of this more fundamental theory. A problem which all GUTs have to face, whether they are based on semi-simple or non-semi-simple gauge groups, is the monopole problem. We investigate in this paper the possibly of the GUT itself solving its own monopole problem, by the so-called thermal inflation [2] which can emerge when Planck suppressed operators are introduced. We call these types of models self-consistent.

Thermal inflation was originally invented to solve the moduli problem which arises in superstrings theories [2]. However, it has been recently shown that this does not always work, in particular in the case of very light moduli [3]. However, thermal inflation is a very interesting mechanism; we thus want to investigate the possibility of solving the monopole problem. We shall be looking at a particularly interesting set of GUT models based on rank greater than five theories such as SO(10), E(6), of GUT not based on semi-simple group such
as the trinification $SU(3)_c \times SU(3)_L \times SU(3)_R$ or the Pati-Salam unified group $SU(4)_c \times SU(2)_L \times SU(2)_R$. We shall assume that there is an intermediate left-right symmetry, whose energy scale scale will be constrained from the conditions for successful thermal inflation to solve the monopole problem. We will be mainly interested in $SO(10)$ GUT, the smallest GUT based on a semi-simple gauge group which unifies all fermions, including a right-handed neutrino. $SO(10)$ has many desirable features, since it can explain the small neutrino masses which have recently been observed at SuperKamiokande [4], it can predict fermion textures [5], it also gives natural solutions to the doublet-triplet splitting problem [6,7]. From a cosmological point of view, $SO(10)$ predicts the existence of superheavy magnetic monopoles which have to be diluted if the theory is to be viable. It also predicts the existence of $B-L$ cosmic strings which can explain the baryon asymmetry which is observed in our universe [8]. It has also been shown that it is a-priori possible to get an $SO(10)$ gauge symmetry at the GUT scale, with non-minimal Higgs structure, with arbitrary low left/right symmetry breaking scale [9].

In this paper we show that in GUT models based on rank greater than five gauge groups a period of the so-called thermal inflation [2] arises naturally if non-renormalisable term are introduced. If there is an intermediate left-right symmetry, the grand unified monopoles which are previously formed can be diluted by the thermal inflation, such as to be consistent with the observational bound. At the end of inflation, $B-L$ fat strings are produced.

In Sec.II we review the monopole problem.

In Sec.III A we define what we call self-consistent models. In Sec.III B, we review the phenomenology of GUT models which break down to the Standard Model via an intermediate Left-Right gauge symmetry. We show that such models can be made self-consistent.

In Sec.IV A, we show that thermal inflation arises naturally in rank greater than five GUTs if non renormalisable operators are included in the field sector which is used to lower the rank of the group by one unit, breaking $U(1)_{B-L}$. The inflationary period is driven by two inflaton fields. In Sec.IV B, we use the vacuum expectation value (vev) of the inflaton fields, which transform as $SU(5)$ singlets, to generate the $\mu$-term of the MSSM. In sec.IV C, we calculated the dilution factor which is obtained for various models. Cosmological constraints on the monopole abundance discussed in Sec.II in turn lead to constraints on the $B-L$ breaking scale.

Finally, in Sec.V, we conclude.

II. THE MONOPOLE PROBLEM

In this section, we wish to briefly review the GUT monopole problem. Recall that monopoles are point like topological objects which form at phase transitions when the vacuum manifold contains non-contractible 2-spheres. Monopoles are very heavy objects, and because they are too many, they would dominate the energy density of the universe soon after formation. Therefore some mechanism has to be invoked to dilute them.

The monopole problem does not only arise in grand unified theories based on semi-simple gauge group such as $SU(5)$, $SO(10)$ or $E(6)$, but it also arises in partial unification theories such as those based on the Pati-Salam group $SU(4)_c \times SU(2)_L \times SU(2)_R$ or the trinification based on the $SU(3)_c \times SU(3)_L \times SU(3)_R$ gauge symmetry. In fact it arises in all extensions of
the particle physics Standard Model as soon as the $U(1)_{Y}$ gauge symmetry of the Standard model is embedded in a non abelian gauge symmetry which breaks at high energy.

This can be understood as follows. Monopoles form according to the Kibble mechanism at phase transitions associated with the spontaneous symmetry breaking of a group $G$ down to a subgroup $H$ of $G$ if the second homotopy group of the vacuum manifold is non trivial $\pi_2(G/H) \neq I$. Now $\pi_2(G/H) \cong \frac{\pi_1(H)}{\pi_1(G)}$ where $H$ is the component of $H$ in $G$ connected to the identity. If we identify $H$ with the standard model gauge group, $\pi_1(H) = \pi_1(U(1)) = Z$ and hence topological monopoles form when $G$ breaks down to $H$ if $\pi_1(G) = I$. This is always possible if we choose to work with the universal covering groups [10]. To check whether the monopoles are topologically stable we must evaluate $\pi_2(G/K)$ where $K = SU(3)_c \times U(1)_Q$. If $\pi_1(G) = I$, $\pi_2(G/K) = \pi_1(K) = Z$ and hence the monopoles are topologically stable down to low energies.

Independently of the initial monopole density, monopole-antimonopole pairs annihilate until the monopole-to-entropy ratio reaches its final value [11,12]:

$$\frac{n_M}{s} \sim \frac{1}{\hbar^2 \beta \sqrt{g_s}} \frac{m}{M_{pl}}$$

where $n_M$ is the monopole density and $s$ is the entropy. $h$ is the monopole magnetic charge which is given by $h = -\frac{4\pi}{e}$, where $e$ is the gauge coupling constant, $\beta \sim (1-\delta)g_s$, where $g_s$ is the effective number of degrees of freedom and $g_{hs}$ is the effective number of helicity states for particles with mass $m+p < T$. Finally, $m$ is the monopole mass and $M_{pl}$ is the Planck mass. The monopole mass is bounded from below by [13]:

$$m \geq \frac{4\pi}{e} T_M$$

where $T_M$ is the temperature at which the monopoles form. For monopoles forming at the GUT scale $M_{GUT} \approx 2 \times 10^{16}$ GeV in a supersymmetric grand unified theory, we find:

$$\frac{n_M}{s} \sim 10^{-13}.$$  \hspace{1cm} (3)

However, observations show that the monopole density today should be much smaller than that predicted by Eq.(3). In fact, the strongest bound on the monopole density today comes from neutron stars. Indeed, neutron stars could in principle trap monopoles [12,14]. Since we know that monopoles can catalyse proton decay, these would then increase the star luminosity. Limits on the luminosity of neutron stars imply a bound on the monopole flux which translates into a bound on the monopole density given by [14]:

$$\frac{n_M}{s} \leq 10^{-31}.$$  \hspace{1cm} (4)

We therefore need a dilution factor given by:

$$\Delta_{NS} \sim 10^{18}$$  \hspace{1cm} (5)

for GUT scale monopoles.

The bound given in Eq.(4) can be improved by yet six orders of magnitude if one takes into account monopoles captured by the star when it was on the main sequence, before it became a neutron star [12,14]. We call is the strong neutron star bound:
In such a case, the dilution factor which is required to solve the monopole problem is:

\[ \frac{n_M}{s} \leq 10^{-37}. \]  

(6)

If one hopes monopoles to be observed, the monopole density should not be much smaller than that ones given in Eq.(6). If one wants to be conservative, it could be close to that given in Eq.(4).

### III. SOLVING THE MONOPOLE PROBLEM

#### A. Self-consistent unified theories

One would like a theory which is cosmologically consistent by itself. If a theory produces some cosmologically catastrophic objects such as superheavy monopoles, it is desirable that the solution to this problem comes from the theory itself. There are two such compelling mechanisms for the monopole problem to be solved. Firstly, there is the Langacker-Pi mechanism [15] which assumes that electromagnetism is broken for a while, such that cosmic strings form connecting monopole-antimonopole pairs; the whole system of monopoles connected by strings rapidly decays. This scenario relies on a specific choice of the spontaneous symmetry breaking pattern. Secondly, there is inflation which is a period of very rapid expansion of the early universe. Inflation makes the monopole density very low. Inflation must be driven by a scalar field with a flat potential: the inflaton. In supersymmetric extensions of unified theories, inflation may just be a consequence of model building [16] (with an appropriate choice for the initial conditions). The inflaton field can be identified with a Higgs field used to implement the spontaneous symmetry breaking pattern of the unified gauge group down to the standard model gauge group or with a scalar field singlet under this group; such singlets are sometimes needed to make some of the Higgs fields to get a vev. If there is no extra symmetry nor any extra field than those needed to implement the full spontaneous symmetry breaking pattern from the considered GUT gauge group down to $SU(3)_c \times U(1)_Q$, some set of appropriate initial conditions may be needed, we may say that inflation arises from the GUT itself; and if this period of inflation makes the monopole density below the observational bound, that the theory is self-consistent.

For example, GUT models based on rank greater than five gauge groups $G$ naturally lead to a period of hybrid inflation [16] when the superpotential used to lower the rank of the group is given by [17,18]:

\[ W = \alpha S \Phi \Phi - \mu^2 S \]  

(8)

where $S$ is a scalar field singlet under the considered gauge group, which we identify with the inflaton, $\Phi$ and $\bar{\Phi}$ and Higgs superfields in complex conjugate representations which lower the rank of $G$ when acquiring vev, $\alpha$ and $\mu$ are two positive constant and $\frac{\mu}{\sqrt{\alpha}}$ sets the symmetry breaking scale. The global minimum of the potential is at $S = 0$ and $\langle |\Phi| \rangle = \langle |\bar{\Phi}| \rangle = \frac{\mu}{\sqrt{\alpha}}$. Note that the above superpotential is consistent with the set of continuous R-symmetries.
In this scenario, the $\Phi$ and $\Phi$ fields are kept at the origin during inflation, see Refs. [18,16], and pick up a vev at the end of inflation. Therefore the symmetry breaking induced by the $\Phi$ and $\Phi$ field vevs takes place at the end of inflation. Hence it should not produce monopoles. Monopoles can be produced at a previous phase transition. Studying the formation of topological defects in all possible spontaneous symmetry breaking patterns from $G$ down to the Standard Model, with the assumption that the rank of the group is lowered with a superpotential given in Eq.(8), allows us to select the only models consistent with observation (see Ref. [19] for an SO(10) example).

Now consider a general hybrid inflationary scenario where the inflaton field is identified with a scalar field singlet under $G$ which is used to make some GUT Higgs field $\Phi$ (not necessarily the one which lowers the rank of the group) to acquire a vev. Assume also that the phase transition induced by the vev of $\Phi$ leads to the formation of monopoles. By a suitable choice of initial conditions, the Higgs field $\Phi$ can be non zero during inflation (the symmetry is already broken during inflation) and hence no monopole form at the end of inflation. Such a model based on SU(5) GUT has been constructed [20], where $\Phi$ is a 24-dimensional Higgs superfield whose vev breaks SU(5) down to the standard model gauge group. This theory is self-consistent.

In general, several Higgs fields in various representations of the considered unified gauge group $G$ are needed to implement the full spontaneous symmetry breaking pattern from $G$ down to the Standard Model. One could use one (some) of these Higgs fields to play the role of the inflaton(s) (we can have more than one inflaton field). If the role of the inflaton is played by a (or some) Higgs field(s) which do(es) not lead to the formation of monopoles (or domain walls) when acquiring vev, it can have zero value during inflation and only acquire a vev at the end of inflation. In that case, topological defects (if any) associated with the spontaneous symmetry breaking induced by the vev of the inflaton form at the end of inflation. The monopoles must then be formed by other Higgs field(s) which must acquire a vev before the time needed for the inflation to dilute the monopoles. Now, if the inflaton is (one of) the Higgs field(s) which lead(s) to the formation of monopoles, it must be non vanishing during the inflationary period; this is possible with a suitable choice of the initial condition.

In the thermal inflationary scenario [2], the scalar field which drives inflation is hold at the origin by finite temperature corrections to the effective potential during inflation, and only acquires a vev at the end of inflation. Therefore topological defects associated with the vev of this field form at the end of inflation. The grand unified monopoles must then be formed at a previous phase transition, if thermal inflation has to solve the monopole problem.

To summarize, many GUT models can be called self-consistent if inflation emerges naturally from the theory itself; they nevertheless have to face some fine-tuning, as do all models of inflation, which correspond to a suitable choice of initial conditions. However, if the considered theory emerges from a quantum cosmological period, the choice of chaotic initial conditions may be the one which faces the least problems of naturalness.
B. Unified models with intermediate left-right symmetry

The most simple realistic examples of theories beyond the standard model which are self-consistent are unified theories based on rank greater than five gauge groups $G$ which break down to the Standard Model via an intermediate left-right gauge symmetry [16,21]. The superpotential which reduces the rank of the group must be of the form given by Eq.(8) and chaotic initial conditions have to be imposed. In this section, we wish to briefly review the phenomenology of such models and the steps for model building. In the next section, we will use the same spontaneous symmetry breaking pattern and the same set of Higgs fields to get a period of thermal inflation. It will be just a question of changing the form of the superpotential which breaks the $U(1)_{B-L}$ gauge symmetry.

Specifically, the spontaneous symmetry patterns that we consider are the following:

\begin{align*}
G \times \text{SUSY} \xrightarrow{M_{\text{GUT}}} ... & \quad \xrightarrow{M_G} \quad SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times \text{SUSY} \\
& \quad \xrightarrow{M_R} \quad SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \times \text{SUSY} \\
& \quad \xrightarrow{M_{B-L}}(\Phi_{B-L}, \overline{\Phi}_{B-L}) \quad SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2 \times \text{SUSY} \\
& \quad \xrightarrow{M_{B-L}} \quad SU(3)_c \times U(1)_Q \times Z_2)
\end{align*}

where $G$ is a rank greater than five gauge group (not necessarily a semi-simple group), and all cases with or without the intermediate gauge symmetry at $M_R$ or $M_{B-L}$ are relevant. In the latter case, we shall replace $M_R$ by $M_{B-L}$. $G$ can also break directly to the left-right gauge symmetry; this is the case where $M_{\text{GUT}} = M_G$. In Eq.(9), the discrete $Z_2$ symmetry which can be left unbroken at low energy if safe Higgs representations of $G$ (who carry even $B - L$ charge) are used to break $G$ down to the standard model gauge group [22] plays the role of matter parity and can be identified with R-parity. In such models, dimension five operators of the MSSM which lead to rapid proton decay are forbidden and the LSP is stable.

If we study the formation of topological defects in the symmetry breaking pattern (9), we find that topologically stable monopoles form at $M_G$ (or above, where appropriate), more monopoles form at $M_R$ and cosmic strings form at $M_{B-L}$. Therefore a period of inflation is need between $M_G$ ($M_R$) and $M_{B-L}$.

In what follows, we shall often identify $G$ with SO(10) which is the simplest rank five GUT based on a semi-simple gauge group.

In supersymmetric theories, in order to break $B - L$ we need two Higgs fields in nontrivial complex conjugate representation of $G$, which we call $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$. The flatness condition for the D-term leads to the useful relation $\langle |\Phi_{B-L}| \rangle = \langle |\overline{\Phi}_{B-L}| \rangle$. The component of $\Phi_{B-L}$ (and $\overline{\Phi}_{B-L}$) which gets a vev must be an SU(5) singlet. It may transform under $SU(2)_R$ as a doublet or as a triplet. In the case of $SO(10)$, this corresponds to $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ being identified with a 16 and a $\overline{16}$ dimensional Higgs representations or a 126 and a $\overline{126}$ dimensional representations respectively. If the component of $\Phi_{B-L}$ which acquires a vev transforms as a triplet under $SU(2)_R$, the discrete $Z_2$ symmetry is broken, and R-parity is not conserved. Some additional R-parity has to be imposed, or the yukawas couplings of the dimension five operators leading to rapid proton decay must be extremely small. On the other hand, if the component of $\Phi_{B-L}$ transforms as a triplet under $SU(2)_R$, the discrete $Z_2$
symmetry remains unbroken in the $SU(3)_c \times SU(2)_L \times U(1)_Y$ phase, R-parity is conserved and the LSP is stable. Also the right-handed neutrinos can acquire a superheavy Majorana mass via the following coupling:

$$\lambda_R \Phi_{B-L} NN$$

where $N$ is the right-handed neutrino field. In such a case, left-handed neutrinos acquire a mass via the see-saw mechanism [23]. Note that a Majorana mass for the right-handed neutrinos can also emerge in the case of an $SU(2)_R$ doublet, by introducing the following non-renormalisable interaction:

$$\lambda_R \Phi_{B-L} \frac{\Phi_{B-L}}{M_{pl}} NN.$$  \hspace{1cm} (11)

In SO(10), all fermions belonging to a single family, including a right-handed neutrino, are assigned to the 16 dimensional spinorial representation. Majorana masses for the right-handed neutrinos are thus possible via the following couplings:

$$\lambda_{i6}^{ij} 16_i 16_j \mathbf{126}$$

if a pair of $(126 + \mathbf{126})$ dimensional Higgs representations are used to break $U(1)_{B-L}$ and via:

$$\lambda_{i6}^{ij} \mathbf{16}_H 16_H \frac{\mathbf{16}_H}{M_{pl}}$$

if a pair of $(16 + \mathbf{16})$ dimensional Higgs representations are used. Note that it seems impossible to get a 126 dimensional Higgs representation from string theory [24,25]. Therefore, we see that both the $16 + \mathbf{16}$ and the $126 + \mathbf{126}$ dimensional representations have their own advantages and their own disadvantages. For the purpose of thermal inflation both a pair of $16 + \mathbf{16}$ or a pair of $126 + \mathbf{126}$ dimensional representations can be used.

To break $G$ down to the standard model gauge group, we need extra Higgs on top of the $\Phi_{B-L}$ and $\Phi_{B-L}$ fields. In the case of SO(10) for example, we need some Higgs in the adjoint representation (the 45 dimensional representation) and some Higgs in the 54 dimensional representation. The number of extra Higgs depends on the number and/or on the nature of intermediate symmetry that we require in Eq.(9), i.e. whether we want both $3, 2L, 2R, 1_{B-L}$ and $3, 2L, 2L, 1_{B-L}$, only $3, 2L, 2R, 1_{B-L}$ or only $3, 2L, 2L, 1_{B-L}$. We do not consider them here, since they are irrelevant for our analysis. But we suppose that if they couple to the $16 + \mathbf{16}$ ($126 + \mathbf{126}$) sector, the couplings are such that as not to destabilise the required vevs.

The simplest superpotential involving the $\Phi_{B-L}$ and $\Phi_{B-L}$ fields which lowers the rank of the group by one unit, breaking $U(1)_{B-L}$, is given by Eq.(8) if we identify the $\Phi$ and $\Phi$ fields with $\Phi_{B-L}$ and $\Phi_{B-L}$ respectively. The ratio $\frac{\Phi}{\sqrt{\alpha}}$ then sets the $B - L$ breaking scale, $M_{B-L}$. It is well known, as mentioned in the previous section, that this superpotential, with appropriate choice of initial conditions, leads to inflation [17,18,16]. Hence monopoles which are previously formed (see above) are inflated away. At the end of inflation, both $\Phi_{B-L}$ and $\Phi_{B-L}$ acquire a vev, and $B - L$ cosmic strings form. They are called $B - L$ cosmic strings because the Higgs fields which form the strings is the same Higgs fields which are used to
break $U(1)_{B-L}$. These cosmic strings can generate the baryon asymmetry which is observed in our universe [8]. The scenario takes place at end inflation.

The shape of the power spectrum of the cosmic microwave background (CMB) could in principle distinguish between different models of inflation. In such hybrid inflationary scenarios both strings and inflation contribute to the CMB anisotropies, and the string contribution could be as high as 75% [16]. In the hybrid scenario, $M_{B-L}$ is constrained by COBE data to be $M_{B-L} \sim 4.7 \times 10^{15}$ GeV [18,26,16]. Although no full computation of the CMB power spectrum for a mixed scenario with inflation and cosmic strings has been done yet, because this first requires a better understanding of a cosmic string network evolution in an expanding universe [27], one can hope that by the time satellites like MAP or Planck will be launched, a full understanding of the strings evolution will be reached, and the computation of the CMB power spectrum in theories with both strings and inflation will be done. Since no scenario up to date does not seem to fit perfectly the CMB data, a mixed scenario with inflation and cosmic strings may well be the perfect fit! However, in these hybrid models, the strings forming at the end of inflation are very heavy. If we believe the recent string simulation of Vincent et al. [28] which have shown that the main energy loss mechanism for cosmic strings is via particle emission rather than gravitational radiation, the non observation of HECR above $10^{20}$ eV can rule out the existence of such heavy strings [28].

Now, if instead the $B-L$ strings where fat strings, ie arising from a superpotential which is flat in the $\Phi_{B-L}$ and in the $\overline{\Phi}_{B-L}$ directions, with a TeV mass Higgs and a very heavy gauge boson, they could explain the extra galactic diffuse $\gamma$-ray background above $\sim 10$ GeV, together with the highest energy cosmic ray flux above $\sim 10^{11}$ GeV [29]. They would have to form at $T \sim 10^{14}$ GeV. We shall keep in mind the value of this very interesting energy scale of $10^{14}$ GeV for $B-L$ string formation.

Finally, in order to break $U(1)_{B-L}$, instead of introducing a scalar field singlet under $G$ to help the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ fields to get a vev, we could use a superpotential which involves non-renormalisable terms. This is what we shall do in the next section, and see that it may lead to a period of thermal inflation which can make the monopole density in agreement with observation. At the end of this thermal inflationary period, fat $B-L$ cosmic strings form.

IV. INTRODUCING NON-RENORMALISABLE TERMS

Unified theories of the strong, weak and electromagnetic interactions are probably the remnants of a more fundamental theory of nature only valid above some very high energy threshold, possibly the Planck scale. This ultimate theory of nature could manifest itself in the low energy world through Planck suppressed interactions. Note that we do not assume that this ultimate theory is a theory of superstrings, but we do assume that this theory shares similar properties with superstring theory, and in particular that it may lead to interactions which are suppressed by some very high scale which we take to be the Planck scale for simplicity. We therefore now turn to the second possibility for building a superpotential in the $B-L$ sector, see Sec.III B, which involves non-renormalisable terms. With appropriate choice of initial conditions and a set of continuous R-symmetries, we show that a period of thermal inflation emerges which solves the monopole problem.
A. Thermal inflation and the breaking of $U(1)_{B-L}$

Here again we consider unified models based on a rank greater than five gauge group with a spontaneous symmetry breaking pattern of the form given by Eq. (9). The simplest superpotential involving non-renormalisable terms and which leads to the spontaneous symmetry breaking of $U(1)_{B-L}$, involving the $\Phi_{B-L}$ and $\Phi_{B-L}$ fields described in Sec.III B, is given by:

$$W = \frac{\lambda(\Phi_{B-L}\Phi_{B-L})^2}{M_{pl}}$$  \hspace{1cm} (14)

where $M_{pl}$ is the Planck mass and $\lambda$ is a positive coupling constant. Eq.(14) must involve all dimension 4 invariant terms which can be made using the $\Phi_{B-L}$ and $\Phi_{B-L}$ fields. For example, in the case of SO(10), when the $\Phi_{B-L}$ and $\Phi_{B-L}$ fields are a pair of $126 + \bar{126}$ dimensional representations, the superpotential is given by [6]:

$$W = \frac{\lambda(\Phi_{B-L}\Phi_{B-L})^2}{M_{pl}} + \alpha \left(\Phi_{B-L}\gamma^{ab}\Phi_{B-L}\right)^2$$ \hspace{1cm} (15)

where $\gamma^{ab}$ are generalised gamma matrices. As mentioned before, we need more Higgs to complete the spontaneous symmetry breaking of SO(10) down to the standard model gauge group, such as 45's dimensional representations or 54's dimensional ones. We do not consider them here, since they are irrelevant for our analysis.

The effective scalar potential for the flaton fields $\Phi_{B-L}$ and $\Phi_{B-L}$ including the soft supersymmetry breaking terms is given by:

$$V = V_0 + 4\lambda^4 \left[\Phi_{B-L}^4\Phi_{B-L}^4\right] + 4\lambda^4 \left[\Phi_{B-L}^2\Phi_{B-L}^4\right] + \frac{\alpha^2}{2} \left(\Phi_{B-L}^2 - \Phi_{B-L}^2\right)^2$$

$$-m_{\Phi_{B-L}}^2 \Phi_{B-L}^2 - m_{\Phi_{B-L}}^2 \Phi_{B-L}^2 + \lambda A \left(\Phi_{B-L}\Phi_{B-L}\right)^2 + c.c.$$ \hspace{1cm} (16)

where we have used the same notation for the Higgs superfields and their bosonic components, $m_{\Phi_{B-L}}, m_{\Phi_{B-L}}$ and $A$ are soft supersymmetry breaking parameters. The charge $q = 1$ for the SU(5) singlet component of $\Phi_{B-L}$ transforming as an $SU(2)_R$ doublet and $q = 2$ if it transforms as an $SU(2)_R$ triplet (see Sec.III B). $V_0$ will be determined by the requirement that the cosmological constant today vanishes.

We would like to point out that the $\Phi_{B-L}$ and $\Phi_{B-L}$ fields do not necessarily have the same soft mass terms at low energy; $m_{\Phi_{B-L}} \neq m_{\Phi_{B-L}}$ in general. This is because the $\Phi_{B-L}$ and $\Phi_{B-L}$ fields do not couple to the same particles of the theory, and $m_{\Phi_{B-L}}$ and $m_{\Phi_{B-L}}$ depend on all these couplings $^1$. The $\Phi_{B-L}$ field couples to the right handed neutrinos.

$^1$One should in fact compute the renormalisation group equations to confirm both the sign and the magnitude of $m_{\Phi_{B-L}}$ and $m_{\Phi_{B-L}}$; see for example [32], where it is shown that couplings of the Higgs particle to fermions drives its soft mass negative.
whereas the $\Phi_{B-L}$ field does not (see Sec.III B). However, both the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ fields also give a contribution to the mass of the $B-L$ gaugino\(^2\). Furthermore, if we want both the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ fields to acquire a vev, both $m_{\Phi_{B-L}}$ and $m_{\overline{\Phi}_{B-L}}$ must be negative with $m_{\Phi_{B-L}} = m_{\overline{\Phi}_{B-L}}$ if spontaneous symmetry breakdown in the D-flat direction is required. Thus the simplifying assumption $m_{\Phi_{B-L}} = m_{\overline{\Phi}_{B-L}}$ can be justified.

We now turn to the finite temperature effective potential. Both the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ fields couple to the $\Phi_{B-L}$, $A_{B-L}$, $\bar{A}_{B-L}$ fields and $\overline{\Phi}_{B-L}$ also the $N$ and $\bar{N}$ fields. We have used a tilde to denote supersymmetric particles. Therefore the finite temperature effective potential can be calculated \cite{33}. We have:

$$V = V_0 + 4\lambda^4 \frac{|\Phi_{B-L}|^2 |\overline{\Phi}_{B-L}|^2}{M_{pl}} + 4\lambda^4 \frac{|\Phi_{B-L}^2 |^2 |\overline{\Phi}_{B-L}|^2}{M_{pl}} + \frac{g^2 q^2}{2} (|\Phi_{B-L}|^2 - |\overline{\Phi}_{B-L}|^2)^2$$

$$+ (\alpha_{\Phi} T^2 - m_{\Phi_{B-L}}^2) \Phi_{B-L}^2 (\alpha_{\overline{\Phi}} T^2 - m_{\overline{\Phi}_{B-L}}^2) \overline{\Phi}_{B-L}^2 + \lambda A \left( \frac{(|\Phi_{B-L}||\overline{\Phi}_{B-L}|)}{M_{pl}} \right)^2 + c.c.$$  \hspace{1cm} (17)

where $\alpha_{\Phi} \sim \alpha_{\overline{\Phi}} \sim O(1)$.

From the D-flat condition, we must have $\langle |\Phi_{B-L}| \rangle = \langle |\overline{\Phi}_{B-L}| \rangle$ which we call $\eta$. Now minimizing the finite temperature effective potential given by Eq.(17) along the D-flat direction, we find that there are two possible minima:

1. For $T^2 \gg m_{\Phi_{B-L}} (= m_{\overline{\Phi}_{B-L}})$, the minima is at $\eta = 0$ and the symmetry is restored.

2. For $T^2 < m_{\Phi_{B-L}}$,

$$\eta = \left( \sqrt{\frac{13}{12}} + 1 \right) \frac{M_{pl} m}{12 \lambda} \ \ (18)$$

where $m \simeq 10^2 - 10^3$ GeV is the soft supersymmetry breaking scale in gravity mediated supersymmetry breaking scenarios and we have $m \sim m_{\Phi} \sim A$. In that case, the $U(1)_{B-L}$ gauge symmetry is spontaneously broken.

Thus at very high temperatures, the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ fields are kept at the origin by finite temperature corrections to the effective potential. The energy density of the universe is dominated by the vacuum energy density $V_0$, see Eq.(16), and a period of inflation takes place. When the temperature $T$ falls below $\sim m_{\Phi_{B-L}}$, the inflationary period stops. $V_0$ can be determined by requiring that the cosmological constant today vanishes. We find:

$$V_0^{-\frac{1}{2}} \sim 2 \frac{4}{3} m \frac{1}{2} \eta \frac{1}{2}.$$  \hspace{1cm} (19)

\(^2\)Our notation might be confusing, but we hope that it was obvious to the reader that when we say $B-L$ gaugino we really mean a gaugino which is a linear combination of the $U(1)_R$ and the $U(1)_{B-L}$ gauginos, in the same way that when we say $\Phi_{B-L}$ breaks $B-L$ we mean that it breaks a linear combination of $U(1)_R$ and $U(1)_{B-L}$, since the remaining gauge symmetry much be $U(1)_Y$, and that in theories such as SO(10) the hypercharge is given by $Y = I_R + \frac{B-L}{2}$.
We have thus shown that in these models a period of thermal inflation can take place [2]. This is driven by two inflaton fields, the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ fields. The cosmological scenario is as follows. At very high temperatures, the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ fields are kept at the origin by the finite temperature corrections to the effective potential. The spontaneous symmetry breaking of $G$ down to $SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ (or $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, see Sec.III B) takes place and topologically stable monopoles form. The intermediate gauge symmetry remains unbroken as long as the finite temperature corrections are strong enough to keep the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ fields at the origin. When the temperature falls below $\sim m_{\Phi_{B-L}}$, the SU(5) singlet components of $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ acquire a vev, which is given by Eq.(18); the intermediate gauge symmetry spontaneously breaks down to the Standard Model, and $B - L$ cosmic strings form. They are fat strings [33]. These $B - L$ cosmic strings which form at the end of inflation can generate the baryon asymmetry which is observed in our universe [8]. Alternatively, Affleck-Dine baryogenesis may take place [34].

B. The decay of the inflaton fields

We now need to find some decay channel for the inflaton field $\Phi_{B-L}$. Recall that the $\overline{\Phi}_{B-L}$ field gives a Majorana mass to the right-handed neutrinos and hence can decay into right-handed neutrinos, see Sec.III B. But the decay rate of $\overline{\Phi}_{B-L}$ into right-handed neutrinos is very small [34], and it also leads to a reheat temperature far below the required temperature which is needed not to overproduce LSP’s [34]. We thus follow the idea of Ref. [34] and use the vevs of the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ fields which are SU(5) singlets to generate the $\mu$ term of the MSSM.

We thus introduce the following superpotential:

$$W_{\mu} = \beta \left( \overline{\Phi}_{B-L} \Phi_{B-L} \right)^n H_u H_d$$

(20)

where $H_u$ and $H_d$ are the two Higgs doublets of the MSSM, which respectively give mass to up-type quarks and down-type quarks. $\beta$ is a coupling constant and $n$ is an integer. Recall that the superpotential which describes the MSSM is given by [31]:

$$W_{MSSM} = h_u Q U H_u + h_d Q U H_d + h_{16} L E H_d + \mu H_u H_d.$$  

(21)

We thus have:

$$\mu = \beta \left( \langle \overline{\Phi}_{B-L} \rangle \langle \Phi_{B-L} \rangle \right)^n \frac{M_{pl}^{2n-1}}{M_{pl}^{2n-1}}.$$  

(22)

The value of $n$ will be determined once the $B - L$ breaking scale $M_{B-L} \equiv \eta = \langle |\overline{\Phi}_{B-L}| \rangle$ for successful inflation has been calculated.

In the case of SO(10), the $\mu$-term can arise from the following coupling:

$$W_{\mu} = \beta \left( \overline{16}_H 16_H \right)^n \frac{M_{pl}^{2n-1}}{M_{pl}^{2n-1}} \cdot 10_H 10'_{H}$$

(23)

provided that the doublet-triplet splitting has been solved via the mechanism of Ref. [7]. This requires the introduction of two Higgs multiplets in the 10-dimensional representation.
of SO(10) and a Higgs in the adjoint representation with a vev in the $B - L$ direction, and mixing with the spinorial sector. The superpotential which leads to doublet-triplet splitting is given by [7]:

$$W = 10_H 45_H 10'_H + \lambda_1 16_H 16_H 10_H + \lambda_2 \overline{10}_H \overline{10}_H 10'_H.$$  (24)

In that case, the two Higgs doublets of the MSSM belong to two different Higgs multiplets 10 and 10'.

If the doublet-triplet splitting has been solved using the Dimopoulos-Wilczek mechanism [30], the $\mu$-term can then arise via the coupling:

$$W_\mu = \beta \left( \frac{126_H 126_H}{M_{pl}^{2n-1}} \right) H^2_{10}$$  (25)

The Dimopoulos-Wilczek mechanism in SO(10) requires the introduction of a Higgs in the 45 dimensional representation with a vev in the $B - L$ direction and two Higgs in the 10 dimensional representation [30,6]. The superpotential which then leads to doublet-triplet splitting is given by [6]:

$$W = 10_H 45_H 10'_H + M^2 10'_H$$  (26)

where $M$ is a superheavy mass scale. In that case, the two Higgs doublets of the standard model belong to the same Higgs multiplet 10.$\mu$.

We are now ready to calculate the decay rates of the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ fields into standard model particles. By expanding the quantum field operators as:

$$\hat{\Phi}_{B-L} = \Phi_{B-L} + \delta \Phi_{B-L}$$  (27)

$$\hat{\overline{\Phi}}_{B-L} = \overline{\Phi}_{B-L} + \delta \overline{\Phi}_{B-L}$$  (28)

where $\Phi_{B-L} = \langle \Phi_{B-L} \rangle$ and $\overline{\Phi}_{B-L} = \langle \overline{\Phi}_{B-L} \rangle$, we can then deduce from Eqs.(20) and (21) the Lagrangian for the quantum fields. It is given by:

$$-L_{\text{decay}} \sim 2\mu^2 (H^2_u + H^2_d) \left( \frac{\delta \Phi_{B-L}}{\eta} + \frac{\delta \overline{\Phi}_{B-L}}{\eta} \right) + 2\mu h_u Q U H_u \left( \frac{\delta \Phi_{B-L}}{\eta} + \frac{\delta \overline{\Phi}_{B-L}}{\eta} \right) + 2\mu \lambda_d Q U H_u \left( \frac{\delta \Phi_{B-L}}{\eta} + \frac{\delta \overline{\Phi}_{B-L}}{\eta} \right) + 2\mu \lambda_r E H_d \left( \frac{\delta \Phi_{B-L}}{\eta} + \frac{\delta \overline{\Phi}_{B-L}}{\eta} \right) + 2 A_H H_d \left( \frac{\delta \Phi_{B-L}}{\eta} + \frac{\delta \overline{\Phi}_{B-L}}{\eta} \right) - 4 \mu \tilde{H}_u \tilde{H}_d \left( \frac{\delta \Phi_{B-L}}{\eta} + \frac{\delta \overline{\Phi}_{B-L}}{\eta} \right)$$  (29)

where we have written only leading order terms in $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$. From Eq.(29) we can deduce the decay rates for both the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ fields. They are estimated to be [35]:

$$\Gamma_{\Phi_{B-L}} \sim \Gamma_{\overline{\Phi}_{B-L}} \sim \frac{m^3}{\pi \eta^2}$$  (30)

where we have assumed that $\mu \sim A \sim m_{\Phi_{B-L}} \sim m_{\overline{\Phi}_{B-L}} \sim m$. 


C. The dilution factor

At the end of inflation, the two inflaton fields $\Phi_{B-L}$ and $\mathcal{F}_{B-L}$ oscillate and rapidly decay into standard model particles, see Eq. (29), at the same decay rate which is given by Eq. (30). This process reheats the Universe very quickly [14,35]. By assuming that there is no parametric resonance, the reheating temperature at the end of inflation $T_R$ is related to the total decay width $\Gamma_{tot}$ by [35]:

$$\frac{\pi g_N(T_R)T_R^4}{30} \Gamma_{tot} \sim \frac{\Gamma_{tot}^2 M_{pl}^2}{24}$$

(31)

where

$$\Gamma_{tot} = \Gamma_{\Phi_{B-L}} + \Gamma_{\mathcal{F}_{B-L}} \sim 2 \Gamma_{\Phi_{B-L}}$$

(32)

and $g_N$ counts the number of massless degrees of freedom at $T_R$.

Now the dilution factor provided by the period of thermal inflation is given by [2]:

$$\Delta \sim \frac{90V_0}{3\pi^2 g_N(T_c)T_c^3 T_R}$$

(33)

where $T_c \sim m$ is the critical temperature at which the phase transition associated by the spontaneous symmetry breaking of $U(1)_{B-L}$ induced by the vevs of $\Phi_{B-L}$ and $\mathcal{F}_{B-L}$ takes place.

Combining Eqs. (33), (31), (32), (30) and (19), we can determine the $B-L$ breaking scale $M_{B-L} \equiv \eta$. We find:

$$M_{B-L} \sim \left( \frac{\pi \frac{3}{2} g_N(T_c)}{60 g_N(T_R)} \left( \frac{5}{\pi g_N(T_R)} \right)^\frac{1}{4} \Delta m^\frac{1}{2} M_{pl}^\frac{1}{2} \right)^\frac{1}{3}$$

(34)

In Table 1, we give the numerical values obtained for for the scale $M_{B-L}$, the vacuum energy $V_0^\frac{1}{2}$ during thermal inflation and the couplings $\lambda$ and $\beta$, for monopole densities satisfying the neutron stars limit given in Eq. (4) and the stronger limit given in Eq. (6). $V_0^\frac{1}{2}$ is given in Eq. (19), $\lambda$ appears in the superpotential given in Eq. (14) and $\beta$ appears in the $\mu$-term, see Eq. (22). We give results for two values of the soft supersymmetry breaking scale $m = 10^2$ GeV and $m = 10^3$ GeV. We use the values $g_N(T_c) \sim 10^2$, $g_N(T_R) \sim 10$, $M_{pl} = 2.4 \times 10^{18}$ GeV.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta_{NS} = 10^{18}$</th>
<th>$\Delta_{MSNS} = 10^{24}$</th>
</tr>
</thead>
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<tr>
<td>$m$</td>
<td>$10^3$ GeV</td>
<td>$10^2$ GeV</td>
</tr>
<tr>
<td>$M_{B-L}$</td>
<td>$6.6 \times 10^{11}$ GeV</td>
<td>$9.7 \times 10^{10}$ GeV</td>
</tr>
<tr>
<td>$V_0^\frac{1}{2}$</td>
<td>$3.1 \times 10^7$ GeV</td>
<td>$3.7 \times 10^6$ GeV</td>
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<tr>
<td>$\beta$</td>
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</tr>
<tr>
<td>$\lambda$</td>
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</tr>
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<td></td>
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</table>
Table 1: $\Delta_{NS}$ and $\Delta_{MSNS}$ are the dilution factors which are required from the neutron star bound and the strong neutron star bound respectively. $m$ is the soft supersymmetry breaking scale, $M_{B-L}$ is the $U(1)_{B-L}$ breaking scale, $V_0$ is the vacuum energy which dominates the energy density of the universe during thermal inflation, $\lambda$ is a Yukawa coupling which appears in Eq.(14), $\beta$ is a Yukawa coupling which is given by the $\mu$-term and $m_{\nu L}^\tau$ is the tau neutrino mass.

All values for $\beta$ given in Table 1 correspond to $n = 1$ in Eq.(22). In Table 1, we have also given the $\tau$ neutrino mass, assuming that the right-handed neutrino acquires its mass via the renormalisable coupling given by Eq.(10). We thus have $m_{\nu}^\tau \approx \frac{100^2 \text{GeV}^2}{M_{B-L}}$. If neutrino masses arise from the non-renormalisable coupling given in Eq.(11), we get tau neutrino masses far above the limits imposed by SuperKamiokande data [4]. By assuming that SuperKamiokande observation represent $\nu_{\mu} \rightarrow \nu_{\tau}$-oscillations, the observed mass difference $\delta m^2 \approx 10^{-2} - 10^{-3}$ eV$^2$, implies a tau neutrino mass $m_{\nu L}^\tau \approx \frac{1}{10} - \frac{1}{30}$ eV [4]. This corresponds to $M_{B-L} \approx 1 \times 10^{14} - 3 \times 10^{14}$ which then gives a dilution factor $\Delta \approx (0.3 - 9.4) \times 10^{30}$ which is bigger than the strong observational limit given by neutron stars. Thus in such a scenario, the monopole problem is solved. Note that these values correspond to $\beta \approx (2.4 - 0.2) \times 10^{-7}$ and $\lambda \approx (9.2 - 1.0) \times 10^{-8}$ (for $m = 10^3$ GeV). It is particularly interesting that this energy scale $\sim 10^{14}$ GeV required by neutrino masses observations, is the same energy scale which is needed for the fat $B-L$ cosmic strings to explain the extra galactic diffuse $\gamma$-ray background above $\sim 10$ GeV, together with the highest energy cosmic ray flux above $\sim 10^{11}$ GeV [29], as discussed in sec.III.B.

V. CONCLUSIONS

In this paper, we have investigated the possibility of a solution to the GUT monopole problem via the so-called thermal inflation [2]. We first pointed out that the monopole problem is a problem of all unified theories of the strong weak and electromagnetic interaction, even of those based on non semi-simple gauge groups, as long as the $U(1)_Y$ gauge symmetry of the standard model gauge group is embedded in a non abelian gauge symmetry. Also, if there is a theory a quantum gravity which describes the universe above the Planck scale, this does not solve the monopole problem in the sense that monopoles are still formed if a unified theory exists at the grand unified scale. But this ultimate theory could manifest itselfs at low energies by introducing non renormalisable couplings, couplings which are suppressed by some high energy threshold.

We have shown that thermal inflation [2] arises naturally in GUTs based on rank greater than five gauge groups $G$ when non-renormalisable couplings are used to force the GUT Higgs fields which are used to lower the rank of the group by one unit, to acquire a vev. We have also shown that this period of thermal inflation can provide enough e-foldings to solve the monopole problem. The monopoles must be formed before the onset of thermal inflation. Therefore the simplest form of the spontaneous symmetry breaking patterns which solves the monopole problem are given by Eq.(9). The period of thermal inflation is driven by two inflaton fields, $\Phi_{B-L}$ and $\Phi_{\overline{B-L}}$, which also break $U(1)_{B-L}$ when acquiring vevs at the end of inflation. The monopole problem is solved provided that the scale $M_{B-L} \geq 10^{12}$ GeV. At the
end of inflation fat $B - L$ cosmic strings are formed. These strings could explain the baryon asymmetry which is observed in our universe [8]. They could also explain the diffuse gamma ray background observed above the TeV scale, and the HECR observed above $10^{10}$ GeV [29], if $M_{B-L} \sim 10^{14}$ GeV. If SuperKamiokande data correspond to $\nu_\mu \to \nu_\tau$-oscillations, if left-handed neutrinos acquire their masses via the see-saw mechanism [23] and if right-handed neutrino acquire a superheavy Majorana mass via the renormalisable coupling given in eq.(10), $M_{B-L} \sim 10^{14}$ GeV is also the energy scale needed to fit the data.

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