The Solar Neutrino Problem - An Update

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Abstract

The $^8$B solar neutrino flux that has recently been measured by Super-Kamiokande is consistent with the $^{37}$Ar production rate in $^{37}$Cl at Homestake. The gallium solar neutrino experiments, GALLEX and SAGE, continue to observe $^{71}$Ge production rates in $^{71}$Ga that are consistent with the minimal signal expected from the solar luminosity. The observed $^8$B solar neutrino flux is in good agreement with that predicted by the standard solar model of Dar and Shaviv with nuclear reaction rates that are supported by recent measurements of nuclear fusion cross sections at low energies. But, the signals measured by Super-Kamiokande, SAGE and GALLEX leave no room for the contribution from the expected $^7$Be solar neutrino flux. This apparent suppression of the $^7$Be solar neutrino flux can be explained by neutrino oscillations and the Mikheyev-Smirnov-Wolfenstein effect, although neither a flavor change, nor a terrestrial variation, nor a spectral distortion of the $^8$B solar neutrino flux has been observed. Detailed helioseismology data from SOHO and GONG confirm the standard solar model description of the solar core, but helioseismology is insensitive to the fate of $^7$Be in the sun and the production rates of $^7$Be, CNO and $^8$B neutrinos. Thus, helioseismology and the solar neutrino problem do not provide conclusive evidence for neutrino properties beyond the standard electroweak model. The solar neutrino problem may still be an astrophysical problem. The deviations of the experimental results from those predicted by the standard solar models may reflect the approximate nature of our knowledge of nuclear reaction rates and radiation transport in dense stellar plasmas and the approximate nature of solar models. Only future observations of spectral distortions, or terrestrial modulation or flavor change of solar neutrinos in solar neutrino experiments, such as Super-Kamiokande, SNO, Borexino and HELLAZ will be able to establish that neutrino properties beyond the minimal standard electroweak model are responsible for the solar neutrino problem.

Keywords: Sun; Standard Solar Model; Neutrinos; Helioseismology;
1 Introduction

The sun is a typical main sequence star that generates its energy via fusion of hydrogen into helium through the pp and CNO nuclear reaction chains (Fig. 1). Due to the conservation laws of baryon number, electric charge, lepton flavor and energy, the total solar neutrino flux is fixed, practically, by the solar luminosity (e.g., Dar and Nussinov 1991). The neutrino spectrum is essentially a sum of standard beta decay spectra from the $\beta$-decays $2p \rightarrow ^{2}D\ell^{+}\nu_{e}$, $^{8}B \rightarrow 2\alpha e^{+}\nu_{e}$, $^{13}N \rightarrow ^{13}C e^{+}\nu_{e}$, and $^{15}O \rightarrow ^{15}N e^{+}\nu_{e}$ and "lines" from the electron captures $e^{7}Be \rightarrow \nu_{e}$ $\ell^{7}Li$ and pep $\rightarrow D\nu_{e}$. To a good approximation, they are independent of the conditions in the sun. However, the relative contributions of the various solar neutrino sources depend on the chemical composition, temperature and density distributions near the center of the sun. These are usually estimated from standard solar models (SSM). They can be tested also by helioseismology, which, however, is not sensitive to the exact abundance or fate of trace elements (e.g., $^{7}Be$) in the sun.

Solar neutrinos, have been detected on Earth in roughly their expected numbers, in five underground solar neutrino experiments, the Chlorine solar neutrino experiment at Homestake, South Dakota, USA, the Water Cherenkov Experiment, Kamiokande, at Kamioka Japan, the Soviet-American Gallium Experiment, SAGE, at the Baksan, Russia, the European Gallium Experiment, GALLEX, at Gran Sasso, Italy and the large water Cherenkov Experiment, Super-Kamiokande, at Kamioka Japan. These experiments have confirmed that the sun is powered by fusion of hydrogen into helium. This milestone achievement in physics, however, has been overshadowed by the fact that the combined results from the solar neutrino experiments seem to suggest that the solar neutrino flux differs significantly from that expected from the standard solar models. This discrepancy has become known as the solar neutrino problem. Many authors have argued that the solar neutrino problem provides conclusive evidence for neutrino properties beyond the minimal standard electroweak model. However, conclusive evidence for new electroweak physics from solar neutrino observations can be provided only by detecting at least one of the following signals:

1. Solar neutrinos with flavors other than $\nu_{e}$.
2. Spectral distortion of the fundamental $\beta$-decay spectra.
3. Terrestrial modulations of the solar neutrino flux.
4. A clear violation of the luminosity sum rule.

So far, no such conclusive evidence has been provided by the solar neutrino experiments. Therefore, the solar neutrino problem does not provide solid evidence for neutrino properties beyond the standard electroweak model and standard physics solutions to the solar neutrino problem are not ruled out. Moreover, a closer look at the sun through helioseismology, X-ray and UV observations shows that the sun is a bewildering turmoil of complex phenomena. It shows unexpected features and behavior at any scale. It has a strange complex in-
ternal rotation, unexplained magnetic activity with unexplained 11 year cycle, unexpected anomalies in its surface elemental abundances, unexplained explosions in its atmosphere and unexplained mechanism that heats its million degree corona and accelerates the solar wind. Perhaps the surface of the sun is complex because we can see it and the center of the sun is not only because we cannot? Perhaps the SSM which has been improved continuously over the past three decades but which still uses simple plasma physics and assumes an exact spherical symmetry, no mass loss or mass accretion, no angular momentum loss or gain, no differential rotation, zero magnetic field through the entire solar evolution, is a too simplistic picture and does not provide a sufficiently accurate description of the core of the sun.

In this paper we summarize the experimental results from the various solar neutrino experiments, we discuss shortly the solar neutrino problem in a “model independent way” and we compare the experimental results with updated standard solar model calculations. We conclude that (a) there is no conclusive evidence for a \( ^8 \)B solar neutrino problem, (b) the \( ^8 \)B solar neutrino flux as measured by Super-Kamiokande is in good agreement with that predicted by the standard solar model (of Dar and Shaviv) with nuclear reaction rates that are supported by recent measurements of nuclear reaction rates at low energies, (c) the suppression of \( ^7 \)Be solar neutrinos which is suggested by both the chlorine and gallium experiments can be due to neutrino oscillations and the Mikheyev-Smirnov-Wolfenstein effect, although neither a day-night effect, nor a spectral distortion of the \( ^8 \)B solar neutrino flux, nor terrestrial modulation of the flux, has been observed, (d) a deficit of \( ^7 \)Be solar neutrinos, if there is one, may still be explained by standard physics and/or astrophysics (e) only future observations of spectral distortions or flavor change of solar neutrinos in solar neutrino experiments, such as Super-Kamiokande, SNO, Borexino and HELLAZ may establish that neutrino properties beyond the minimal standard electroweak model are responsible for the solar neutrino problem.

2 Solar Neutrino Observations

2.1 The Chlorine Detector

The radiochemical chlorine detector (Davis 1966, Cleveland et al. 1998) which contains 615 tons of tetrachloroethylene, \( C_2Cl_4 \), measures the production rate of \( ^{37}Ar \) by solar neutrinos through the reaction \( \nu_e + ^{37}Cl \rightarrow ^{37}Ar + e^- \) which has a threshold energy of 814 keV. The detector is located at the Homestake gold mine at a depth of 1480 meters underground (\( \sim 4200 \) meters water equivalent) in Lead, South Dakota, USA. After an exposure time of 1 to 3 months the \( ^{37}Ar \) atoms are extracted from the target liquid and counted by observing their electron capture decay back to \( ^{37}Cl \) (half life \( T_{1/2} = 35 \) d) in a proportional counter. The \( ^{37}Ar \) production rate measured in 108 individual runs (runs 18 to
133) from 1970 to 1995 is plotted in Fig. 2. The mean capture rate of solar neutrinos was

\[ <\sigma \phi_{\nu_e} >_{\text{Cl}} = 2.56 \pm 0.16 \text{(stat)} \pm 0.14 \text{(syst)} \text{ SNU} \]  

where 1 SNU = 1 neutrino capture per second in \(10^{36}\) target atoms.

### 2.2 The Light Water Cherenkov Detectors

**Kamiokande II**, the first imaging light water solar neutrino detector, (later upgraded to Kamiokande III) was located in the Kamioka mine in Japan (Fukuda et al. 1996). It measured the Cherenkov light emitted by electron recoils produced by elastic scattering of solar neutrinos from electrons in the inner 680 tons of a large tank filled with a total of 2180 tons of light water. For radioactive background reduction the threshold had to be set to a rather high electron recoil energy (7 MeV). Therefore, the detector was sensitive only to the upper end of the \(^8\)B solar neutrino spectrum. During cumulative live time of 2079 days the detector yielded an average flux of

\[ \phi_{\nu_e}(^8\text{B}) = 2.80 \pm 0.19 \text{(stat)} \pm 0.33 \text{(syst)} \times 10^6 \text{ cm}^{-2}\text{s}^{-1}, \]  

as shown in Fig. 3. Within its experimental sensitivities KII+KIII has not detected temporal variation or spectral distortion of the the \(^8\)B spectrum.

**Super-Kamiokande** (SK) is a 500,00 tons imaging water Cherenkov detector whose inner 22,500 tons are used for the solar neutrino measurements (Fukuda et al 1998). The current energy threshold is 6.5 MeV. It started its operation on April 1 1996. Fig. 4 shows the distribution of events as function of the cosine of the angle of electrons recoiling from neutrino-electron scattering relative to the direction from the sun during live time of 504 days. The solid line is the best fitted histogram due to the \(^8\)B solar neutrino flux of (Suzuki et al. 1998)

\[ \phi_{\nu_e}(^8\text{B}) = 2.44^{+0.06}_{-0.05} \text{(stat)}^{+0.09}_{-0.07} \text{(syst)} \times 10^6 \text{ cm}^{-2}\text{s}^{-1} \]  

and a constant background. The observed energy spectrum of the recoiling electrons is consistent with that expected from elastic scattering of \(^8\)B solar neutrinos. In spite of the large number of events the statistics are not large enough yet to show the expected \(\approx 6.5\%\) periodical variation in the flux due to the periodical variation in the distance of Earth from the sun, as can be seen from Fig. 5. The SK data also does not show any dependence on the path length of solar neutrino in Earth, as can be seen from Fig. 6. The SK limit on a Day-Night asymmetry is

\[ A = \frac{D - N}{D + N} = -0.023 \pm 0.020 \text{(stat)} \pm 0.014 \text{(syst)}. \]
2.3 The Gallium Detectors

The radiochemical gallium detectors measure the production rate of $^{71}\text{Ge}$ by solar neutrinos through the reaction $\nu_e + ^{71}\text{Ga} \rightarrow ^{71}\text{Ge} + e^-$. The energy threshold, 232.2 keV, is well below the maximum energy of the pp neutrinos, 420 keV. After exposure time of a couple of weeks the $^{71}\text{Ge}$ atoms are extracted from the target liquid and counted by detecting the X-ray emission from their electron capture decay back to $^{71}\text{Ga}$ (half life $T_{1/2} = 11.43$ days) in a proportional counter.

The Gallex detector (Hampel et al. 1996; Kirsten 1998) is located in the Gran Sasso underground laboratory in Italy. It contains 30.3 tons of gallium in GaCl$_3$ – HCl solution. The neutrino produced $^{71}\text{Ge}$ atoms form the volatile compound GeCl$_4$ which at the end of the exposure is swept out of the solution by a gas stream and converted into GeH$_4$. The produced $^{71}\text{Ge}$ are counted by detecting their radioactive decay inside a proportional counter. The combined results of 65 individual GALLEX runs corresponding to data taking periods GALLEX I,II,III and IV as shown in Fig. 8 gave a production rate of

$$<\sigma \phi_{\nu_e}>_{\text{Ga}} = 76.4 \pm 6.3(\text{stat})^{+4.5}_{-4.9}(\text{syst}) \text{ SNU}.$$  \hspace{1cm} (5)

GALLEX has also conducted two $^{51}\text{Cr}$ neutrino source experiments to test the overall performance of the detector. The ratio between the measured $^{71}\text{Ge}$ production rate due to the $^{51}\text{Cr}$ source and the expected rate from the known source strength was $1.00 \pm 0.10$ and $0.83 \pm 0.10$, in the two experiments, respectively.

The Soviet American Gallium Experiment (SAGE) is located in the Baksan Neutrino Observatory in the northern Caucasus of Russia at a shielding depth of 4715 meter water equivalent. The detector uses 55 tons of metallic gallium. After the metal is converted to solution the produced $^{71}\text{Ge}$ atoms are removed from the Gallium and counted in a low background proportional counters by a procedure similar to that used by GALLEX. The combined results of 65 individual runs of SAGE corresponding to data taking periods SAGE I,II,III,IV as shown in Fig. 8 gave a production rate of (Abdurashitov et al 1996, Gavrin 1998)

$$<\sigma \phi_{\nu_e}>_{\text{Ga}} = 70 \pm 6.3(\text{stat})^{+4.5}_{-4.9}(\text{syst}) \text{ SNU}.$$  \hspace{1cm} (6)

SAGE has also conducted a $^{51}\text{Cr}$ neutrino source experiment. The ratio between the measured $^{71}\text{Ge}$ production rate due to the $^{51}\text{Cr}$ source and the rate expected from the source strength was 0.95 \pm 0.11.

3 The Luminosity Sum Rule

Due to conservation of baryon number, electric charge, lepton flavor and energy, the net reaction in the sun can be written as

$$2e^- + 4p \rightarrow ^4\text{He} + 2\nu_e + 26.732\text{ MeV}.$$  \hspace{1cm} (7)
Thus, the generation of $Q = 26.732$ MeV in the sun is accompanied by the production of two $\nu_e$'s. If the sun is approximately in a steady state with a nuclear energy production rate that equals its luminosity, then the total solar neutrino flux at Earth is (Dar and Nussinov 1991),

$$\phi_{\nu_e} \approx \frac{2S_\odot}{Q - 2\bar{E}_{\nu_e}} \approx 6.54 \times 10^6 \text{ cm}^{-2} \text{ s}^{-1},$$

(8)

where $S_\odot = L_\odot / 4\pi D_\odot \approx 1.367 \text{ kW m}^{-2}$ is the measured “solar constant” which yields a solar luminosity $L_\odot \approx 4\pi D^2 \approx 3.846 \times 10^{33} \text{ erg s}^{-1}$ for an average distance $D_\odot \approx 1.496 \times 10^{13} \text{ cm}$ of Earth from the sun, and $\bar{E}_{\nu_e} = \Sigma E_{\nu_e}(i)\phi_{\nu_e}(i)/\phi_{\nu_e}$ is the mean energy of solar neutrinos which has been approximated by $\bar{E}_{\nu_e}(pp) \approx 0.214 \text{ MeV}$, the mean energy of the pp solar neutrinos that dominate the solar neutrino flux. Eq. (2) can also be rewritten as a sum rule,

$$\Sigma_i(Q/2 - \bar{E}_{\nu_e}(i))\phi_{\nu_e}(i) \approx S_\odot.$$  

(9)

The summation extends over all the neutrino producing reactions with $\bar{E}_{\nu_e} = 0.265, 1.442, 0.814, 6.710, 0.707, \text{ and } 0.997 \text{ MeV}$ for the pp, pep, $^7\text{Be}$, $^8\text{B}$, $^{13}\text{N}$ and $^{15}\text{O}$ neutrinos, respectively. If the small pep flux, which is proportional to the pp flux, is included in the pp flux, and if the very small $^8\text{B}$ neutrino flux is neglected then the solar luminosity sum rule can be rewritten as

$$0.980\phi_{\nu_e}(pp) + 0.939\phi_{\nu_e}(\text{Be}) + 0.936\phi_{\nu_e}(\text{NO}) \approx 6.377,$$  

(10)

where $\phi_{\nu_e}(i)$ are in units of $10^{10} \text{ cm}^{-2}\text{s}^{-1}$.

4 Helioseismological Constraints

Accurate ground based (e.g., Hill et al. 1996 and references therein) and space based measurements aboard SOHO (e.g., Turck-Chieze et al. 1997 and references therein) of solar photospheric oscillation frequencies provided detailed information about the structure of the solar interior (e.g., Christensen-Dalsgaard 1996). In particular the base of the convection zone has been determined to be at $R_{\text{c}} \approx (0.713 \pm 0.003)R_\odot$ (Basu and Antia 1997) and the photospheric helium abundance has been inferred to be $Y_s = 0.249 \pm 0.003$ (Basu and Antia 1995). Helioseismology is generally in good agreement with the standard solar models (see e.g. Christensen-Dalsgaard 1996). However, there are systematic deviations between the helioseismology determination of the sound speed in the sun and that predicted by the SSM (see, e.g., Fig. 10.) which are similar to all SSMs and whose origin is not clear. Moreover, helioseismology is sensitive only to the average local properties in the sun (temperature, density, average molecular weight) but not to the rates of the rare nuclear reactions in the sun which produce the pep, $^7\text{Be}$, $^8\text{B}$ and NO solar neutrinos. Therefore, helioseismology should not be used to argue that the SSM predict correctly these solar neutrino fluxes.
5 Model Independent considerations

Counting rates in solar neutrino experiments are formally given by

\[ R = N_a \sum_i \phi_{\nu_i}(i) \int_{E_0} \frac{dn_{\nu_i}}{dE} \sigma_{\nu a}(E) dE \]  

(11)

where \( N_a \) is the number of “active” atoms in the detector, \( \sigma_{\nu a}(E) \) is their cross section for neutrinos with energy \( E \), \( E_0 \) is the threshold energy of the detector, \( dn_{\nu_i}/dE \) is the energy spectrum and \( \phi_{\nu_i}(i) \) is the total flux of neutrinos from reaction \( i \) in the sun. Both, \( dn_{\nu_i}/dE \) and \( \sigma_{\nu a} \) follow from the standard electroweak theory and are essentially independent of the sun: \( dn_{\nu_i}/dE \) is practically the standard \( \beta \)-decay spectrum for the \( \beta \)-decays: \( {}^8 \text{B} \rightarrow 2 \alpha_e \nu_e \), \( {}^{13} \text{N} \rightarrow {}^{13} \text{C} \nu_e \) and \( {}^{15} \text{O} \rightarrow {}^{15} \text{Ne} \nu_e \) and is a \( \delta \)-function for the electron captures \( e^{-7} \text{Be} \rightarrow \nu_e \text{Li} \) and \( \text{pep} \rightarrow D
\nu_e \). Thus conclusive evidence for new electroweak physics can be provided only by detecting at least one of the following signals: (1) Spectral distortions of the \( \beta \)-decay spectra of solar neutrinos. (2) Solar neutrino flavors other than \( \nu_e \). (3) Terrestrial modulations of the solar neutrino flux. (4) A clear violation of the luminosity sum rule.

So far, no such conclusive evidence has been provided by the solar neutrino experiments.

1. Spectral Distortion. At present only Super-Kamiokande can measure the spectrum of solar neutrinos (above 6.5 MeV). Within their statistics and systematic uncertainties the energy distribution of the detected electrons which are scattered by solar neutrinos is consistent with that expected from an undistorted spectrum of \( {}^{8} \text{B} \) neutrinos. This can be seen from Fig. 7. A “hint” for a spectral distortion may exist in the SK data, but it depends strongly on events beyond the kinematical limit which are attributed to the detector energy resolution. Also the neutrino spectrum near the “end point” is not known very well because the \( {}^{8} \text{B} \) decays into a virtual short lived \( 2\alpha \) state which has a large energy spread.

Super-Kamiokande, which has been running since April 1, 1996, will finally have a much larger statistics and perhaps a lower threshold energy. These, perhaps, will be able to provide more conclusive evidence.

2. Neutrino Flavor Change. The radiochemical experiments are blind to neutrino flavors other than that of \( \nu_e \)’s. SK is sensitive also to \( \nu_\mu \)’s and \( \nu_\tau \)’s but it cannot distinguish between the neutrino flavors at solar energies. Only future experiments, such as SNO, will be able to obtain information on the flavor content of the solar neutrino flux.

3. Terrestrial Modulations. In spite of the large number of events collected by SK the statistics are not large enough yet to show the expected \( \approx 6.5\% \) periodical variation in the flux due to eccentricity of the Earth’s orbit around the sun, as can be seen from Fig. 5. The SK data also show no dependence on the path length of solar neutrino in Earth, as can be seen from Fig. 6, no
day-night effect and no winter-summer difference. In particular, its limit on the Day-Night asymmetry is $A = (D - N)/(D + N) = -0.023 \pm 0.025$.

4. Violation of the Luminosity Sum Rule. A clear violation of the solar luminosity sum rule could prove that lepton flavor is not conserved. The “minimal” expected signal in gallium which follows from the luminosity sum rule is obtained by assuming that all the solar neutrinos are pp neutrinos. If the mean cross section for the capture of the pp neutrinos in gallium is $\sigma \approx (1.17 \pm 0.03) \times 10^{35} \text{ cm}^{-2}$, it yields a minimal signal of $76 \pm 2$ SNU in $^{71}$Ga. In fact, the $^8$B solar neutrino flux, which is observed by SK contributes additional $<\sigma \phi_{^8\text{B}} >_{\text{Ga}} = 5.85 \pm 1.75$ SNU to the $^{71}$Ge production rate and the minimal expected signal in $^{71}$Ga is $82 \pm 3$ SNU. It is consistent, within the experimental and theoretical uncertainties, with the $76.4 \pm 8$ SNU production rate of $^{71}$Ge by solar neutrinos in gallium which was measured by GALLEX. The gallium experiments, however, appear to leave no extra room for the contribution from $^7$Be solar neutrinos.

A further indication that the $^7$Be and perhaps also the CNO neutrinos are missing is provided by the Cl experiment. The expected production rate of $^{37}$Ar in $^{37}$Cl by the $^8$B solar neutrino flux that was measured by Super-Kamiokande is $<\sigma \phi_{^8\text{B}} >_{\text{Cl}} = 2.68 \pm 0.15$ SNU. It is consistent with $<\sigma \phi_{v_e} >_{\text{Cl}} = 2.56 \pm 0.21$ SNU, the total $^{37}$Ar production rate in $^{37}$Cl, as measured at Homestake but leaves no room for the contribution from the $^7$Be solar neutrino flux.

The SK measurement (3) and the solar luminosity sum rule (10) two observational constraints on the solar neutrino fluxes. Two additional constraints are provided by the gallium and chlorine experiments. Assuming that the neutrino capture cross sections are well represented by their theoretical estimates (e.g., Bahcall 1989; 1998 and references therein) one can write them approximately as

\[
\begin{align*}
\text{Ga} & : \quad (11.7 \pm 0.3) \phi_{v_e} (\text{pp}) + (71.7 \pm 5.0) \phi_{v_e} (^7\text{Be}) + (2.40 \pm 0.78) \times 10^4 \phi_{v_e} (^8\text{B}) \\
& \quad + (87 \pm 12) \phi_{v_e} (\text{NO}) = 77.5 \pm 7.8 \\
\text{Cl} & : \quad (1.11 \pm 0.04) \times 10^4 \phi_{v_e} (^8\text{B}) + (2.4 \pm 0.4) \phi_{v_e} (^7\text{Be}) \\
& \quad + (16 \pm 1) \phi_{v_e} (\text{pep}) + (4.2 \pm 1) \phi_{v_e} (\text{NO}) = 2.56 \pm 0.25
\end{align*}
\] (12) (13)

If the theoretical estimates of the cross sections for nuclear capture of solar neutrinos represent well their true values then the only physical solution of the four constraints is

\[
\begin{align*}
\phi_{v_e} (\text{pp + pep}) & \approx 6.5 \times 10^{10} \text{ cm}^{-2} \text{s}^{-1}, \quad \phi_{v_e} (^8\text{B}) \approx 2.44 \pm 0.11 \times 10^6 \text{ cm}^{-2} \text{s}^{-1}, \\
\phi_{v_e} (^7\text{Be}) & \ll \phi_{v_e}^{\text{SM}} (^7\text{Be}), \quad \phi_{v_e} (\text{NO}) \ll \phi_{v_e}^{\text{SM}} (\text{NO}).
\end{align*}
\] (14)

The confidence level of this solution cannot be quantified in a reliable way because of the unknown origin and nature of all the systematic errors in both the
theoretical cross sections and the experimental results. The only reliable conclusion is that $\phi_{\nu_e}(^{7}\text{Be})$ and $\phi_{\nu_e}(\text{NO})$ appear to be strongly suppressed compared with their standard solar model estimates.

6 Is There a $^{8}\text{B}$ Solar Neutrino Problem?

Table I presents a comparison between the solar neutrino observations and the SSM predictions of Bahcall and Pinsonneault 1995 (BP95), Bahcall, Basu and Pinsonneault 1998 (BSP98), Brun, Turck-Chieze and Morel 1998 (BTM98) and Dar and Shaviv 1996 (DS96). Although BP95 and BSP98 predict a $^{8}\text{B}$ solar neutrino flux that is approximately 2.2 and 2.7 times, respectively, larger than that observed by SK, DS96 predict a flux consistent with that observed by SK. The differences between BP95, BSP98 and DS96 are summarized in Table II (for details see Dar and Shaviv 1996). The differences between the predicted $^{8}\text{B}$ flux are mainly due to the use of different nuclear reaction rates in DS96, differences in the calculated effects of diffusion and differences in the initial solar chemical composition assumed in the two calculations. They reduce the predicted $^{8}\text{B}$ flux relative to those in BP95 (BSP98) by approximate factors of 0.55 (0.70), 0.81, and 0.95, respectively. The remaining differences are mainly due to inclusion of partial ionization effects, premain sequence evolution and deviations from complete nuclear equilibrium in DS96 which were neglected in BP95 and BSP98, and due to different numerical methods, fine zoning and time steps used in the two calculations:

6.1 Initial Chemical Composition

The initial chemical composition of the sun influences significantly the solar evolution and consequently the present density, chemical composition and temperature in the solar core that determine the solar neutrino fluxes. In particular, the radiative opacities, which determine the temperature gradient in the solar interior, are very sensitive to the abundance of heavy elements which are not completely ionized in the sun. Since the initial chemical composition of the sun is unknown, one must infer it indirectly, e.g., from the chemical composition of the solar photosphere, the primitive early solar meteorites and the local interstellar medium (ISM).

The solar photospheric abundances have changed only slightly during the solar evolution by gravitational settling, diffusion and turbulent mixing in the convective layer and by cosmic ray interactions with the surface material. Therefore, in principle, one can adjust the initial chemical composition of the sun in the SSM to yield its measured photospheric composition. Unfortunately, the photospheric abundances of most elements are still not known to sufficient accuracy and there is no direct spectroscopic information on the photospheric abundance of $^{4}\text{He}$. Consequently, the initial mass fraction $Y_i$ of $^{4}\text{He}$ in the sun
has been treated in the SSM as an adjustable parameter. Recently, however, the photospheric mass fraction of \(^4\)He has been inferred from helioseismological measurements of the sound speed in the convective solar layer. The best estimated value is now \(Y_s = 0.249 \pm 0.003\) (Basu and Antia 1997). Since the photospheric mass ratio of metals \((A > 4)\) to hydrogen is \((\text{Grevesse, Noels and Sauval 1996})\) \(Z/X = 0.0244\), one obtains that \(X_s = 0.733\) and \(Z_s = 0.0178\). It is, however, important to note that the metallicity in the present local ISM, essentially measured from analyses of the Orion nebula and of nearby B stars (Gies and Lambert 1992; Cunha and Lambert 1992; Wilson and Rood 1994; Mathis 1996) is lower than the value obtained from the photospheric abundances. This is in contradiction with galactic chemical evolution models which predict an increase with time of metallicity in the ISM. Moreover, gravitational settling and diffusion decrease the solar surface metallicity with time.

The meteoritic elemental abundances are known with much better accuracy. Besides the noble gases, and H, C, N and O which were able to form highly volatile molecules or compounds and escape condensation, all the other elements are believed to have condensed completely in primitive early solar system meteorites. Therefore, their initial relative abundances in the sun are expected to be well represented by their values in type I carbonaceous chondrites. If diffusion and gravitational settling have not changed their ratios significantly, then the relative abundances of these elements in meteorites must be similar to their photospheric values. Over the past decades there have been many initial disagreements between the meteoritic and photospheric abundances. In nearly all cases, when the atomic data were steadily improved and the more precise measurements were made, the photospheric values approached the meteoritic values. The photospheric abundances are now as a rule in very good agreement with the meteoritic values if the conversion factor from the solar abundance scale \(N_H = 10^{12}\) to the meteoritic scale, \(N_S = 10^6\) is \(R = \log(\text{sol})/\text{met}) = 1.56\), as can be seen from Table III borrowed from Grevesse, Noels and Sauval 1996.

In our SSM, \(Y_i\) was left as an adjustable parameter. The initial solar heavy metal abundances were assumed to be equal to the meteoritic (CI chondrites) values of Grevesse, Noels and Sauval (1996). The overwhole initial metallicity ratio \(Z_i/X_i\) and the initial H, C, N, O and Ne abundances were adjusted to yield their present photospheric values \((Z_s/X_s = 0.0244\) and the values quoted in Table III, respectively). Our SSM yields \(Y_s = 0.238 \pm 0.05\) in agreement with the helioseismological estimates.

The photospheric abundances of \(^7\)Li, however is smaller by a factor of nearly 140 than its meteoritic abundance. The origin of such large difference is still not clear. It cannot be explained by nuclear burning during the Hayashi phase although significant lithium burning does takes place during this phase. It may be explained by rotational mixing (e.g., Richard et al 1996). Although the initial solar (meteoritic) abundances of lithium is very small and do not play any significant role in solar evolution, its depletion perhaps can provide a clue to the real history of the convection zone and the sun.

10
6.2 Nuclear Reaction Rates

The nuclear reaction rates for most stellar reactions are inferred by extrapolating measurements at higher energies to stellar reaction energies. The cross sections at center of mass energies well below the Coulomb barrier are usually parametrized as

$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi \eta(E)}$$

(15)

where \(\eta = Z_1 Z_2 e^2/\bar{\hbar} v\) is the Sommerfeld parameter, \(v\) is the relative velocity of the colliding nuclei in the initial state, \(Z_1\) and \(Z_2\) are their charge numbers and \(E\) is their center of mass energy. The exponent is an approximate (WKB) form for the penetration probability of the Coulomb barrier in the initial state. The “astrophysical S factor” is expected to vary only slowly with energy. It is usually extracted either from a polynomial fit to experimental data at low energies or from theoretical calculations normalized to the experimental data.

The uncertainties in the nuclear reaction rates at solar conditions are still large due to (1) uncertainties in the measured cross sections at laboratory energies, (2) uncertainties in their extrapolations to lower solar energies, (3) uncertainties in dense plasma effects (screening, correlations, fluctuations and deviations from pure equilibrium distributions) on reaction rates. Rather than averaging measured cross sections that differ by many standard deviations, DS96 used for their extrapolations only the most recent and consistent measurements of the relevant nuclear cross sections. For sub-Coulomb reactions that take place when the colliding nuclei are far apart, the Optical Model and the Distorted Wave Born Approximation give a reliable description of their energy dependence and can be used for extrapolating the measured sub-Coulomb cross sections to solar energies.

The “astrophysical S factors” which were used in BP95, BSP98 and DS96 are compared in Table II. The origin of the differences are as follows: \(S_{11}(0)\). The value advocated by Adelberger et al. (1998) and used in BSP98 is based essentially on the updated calculation of \(S_{11}(0)\) by Kamionkowski and Bahcall (1994). The authors considered many small corrections. However, they ignored the screening effect of the solar plasma electrons in the outgoing channel. Screening corrections in the nuclear reaction rates were included in BP95 and BSP98, and in all the published SSM calculations, only in the incoming channel. That is justified for radiative captures like \(^3\)He + \(^4\)He \rightarrow \(^7\)Be + \gamma\) and \(p + ^7\)Be \rightarrow \(^8\)B + \gamma\) because the photon is chargeless. The emitted positron in the “beta decay” \(pp \rightarrow D + e^+ + \nu_e\) of the two fusing protons sees essentially a Deuterium nucleus screened by the plasma electrons in the Debye sphere left by the two protons (the mean velocity of the emitted positron is much larger than the mean velocity of the electrons in the Debye sphere). If the wave function of the ejected positron in the Coulomb field of the Deuteron is calculated from the Dirac equation with the Debye screening potential around two fusing protons, one obtains that their fusion rate in the sun is enhanced by approximately 1.75%.
Dar and N. Shaviv, unpublished). Consequently, the “bare” value of $S_{11}(0)$ in SSM calculations that do not include electron screening in the outgoing channel must be increased by 1.75% as in DS96.

Recent low energy measurement of the cross section for the reaction $p + ^7\text{Be} \rightarrow ^8\text{B} + \gamma$ by Hammache et al. (1998) are consistent with the measurements of Vaughn et al. (1970) and of Filippone et al. (1983a,b) which reached energies as low as 134 keV. These measurements disagree with the older measurements of Kavanagh (1960), Parker (1966, 1968) and Kavanagh et al. (1969).

Because of the low binding energy, the radiative capture of $p$ by $^7\text{Be}$ takes place well outside the range of the nuclear forces. Therefore, at low energies the dependence of the cross section on energy is well described by the optical model. When applied to the experimental data of Vaughn (1970); Filippone (1983a,b) and Hammache et al. (1998) it yields $S_{11}(0) = 17.8 \pm 1.0$ eV $\cdot$ b (see also Barker 1995; Nunes et al 1997). This value is consistent with the indirect measurements, $S_{11}(0) = 16.7 \pm 3.2$ eV $\cdot$ b through Coulomb dissociation of $^8\text{B}$ (Motobayashi et al 1994) and $S_{11}(0) = 17.6 \pm 1$ eV $\cdot$ b through proton transfer reactions (Xu et al. 1994). The mean value $S_{17}(0) = 17.5 \pm 1.0$ eV $\cdot$ b is consistent with the value $S_{17}(0) = 17$ eV $\cdot$ b used in DS96, but it is smaller than both $S_{17}(0) = 22.4$ eV $\cdot$ b used in BP95 and $S_{17}(0) = 19$ eV $\cdot$ b that is advocated by Adelberger et al. 1998 and used in BSP98.

The cross section for the reaction $^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2p$ has recently been measured at the Laboratory for Underground Nuclear Astrophysics (LUNA) at energies covering the Gamow peak around $E_G = 21.9$ keV (Junker et al. 1998). At such small lab energies screening by atomic electrons enhances considerably the bare nuclear cross section. In the Born-Oppenheimer adiabatic approximation, $U_e$, the gain in kinetic energy by the colliding nuclei, is bounded by the change in the total binding energy of the atomic electrons when they occupy the atomic ground state of the $^3\text{He} + ^3\text{He}$ “nuclear molecule”. For $^3\text{He}^{++}$ ions incident on $^3\text{He}$ gas target, $U_e \approx 240$ eV. From eq. (15) one obtains that the screening enhancement of the cross section for $U_e \ll E$ is given approximately by

$$\sigma = \sigma_{\text{exp}} e^{\pi U_e/E}.$$  \hspace{1cm} (16)

We stress that one must use a consistent treatment of electron screening enhancement of nuclear cross sections in the lab and in the solar plasma. In particular, $U_e$ cannot be taken as an adjustable parameter when fitting $S(E)$ to the lab measurements but must be fixed to its adiabatic value $U_e = 240$ eV in order to be consistent with the “weak screening” prescription that is used in the SSM. For $U_e = 240$ eV, a best fitted $S(E)$ between 20.7 keV and 1080 keV to the cross section measurements that are consistent at overlapping energies (Dwarkanath and Winkler 1971, Krauss et al. 1987, Greife et al. 1994 and Junker et al. 1998) yields the values $S_{33}(0) = 5.60$ MeV $\cdot$ b, $S_{33}(0) = -4.1b$ and $S_{33}''(0) = 4.60$ MeV$^{-1} \cdot b$. The fit is shown in Fig. 9. Note that if one extracts $S(E)$ directly from the LUNA data at the Gamow peak (Junker et al. 1998) one

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obtains $S_{33}(E_G) = 5.75$ MeV · b. The value $S_{33}(0) = 4.99$ MeV · b was used in BP95, the value $S_{33}(E_G) = 5.30$ MeV · b that was recommended by Adelberger et al. 1998 was used in BSP98, while the value $S_{33}(0) = 5.6$ MeV · b was used in DS96.

$S_{34}(0)$. There are six published measurements of the cross section for the reaction $^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma$ that are based on the detection of the prompt $\gamma$-rays (Parker and Kavanagh 1963; Nagatani et al. 1969; Kräwinkel et al. 1982; Osborne et al. 1982,1984; Alexander et al. 1984; Hilgemeier et al. 1988) and three that are based on the late decay of $^7\text{Be}$ (Osborne et al. 1982,1984; Robertson et al. 1983; Volk et al. 1983). There is a systematic discrepancy of more than $3\sigma$ between these two data sets whose origin is not clear. When theoretical models are used to extrapolate the direct measurement to low energies they yield a weighted mean $S_{34}(0) = 0.507 \pm 0.016$ MeV · b, while the activation measurements yield $S_{34}(0) = 0.579 \pm 0.024$ MeV · b. The weighted mean of all experiments, $S_{34}(0) = 0.53 \pm 0.05$ (Adelberger et al. 1998) was used in BSP98. However, the radiative capture $^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma$ does not take place outside the range of the nuclear forces ($Z_1 Z_2 e^2 / E_B \approx 3.6 f m < R(^3\text{He}) + R(^4\text{He})$). Thus, the ability of the theoretical models to predict correctly the low energy dependence of $S_{34}(E)$ is questionable. Moreover, the energy dependence of the cross section which they predict disagrees with that observed in the measurements of Kräwinkel et al. 1982 who used gas targets, have low statistical errors and extend to the lowest energies. If one uses a polynomial fit of $S_{34}(E)$ to these data, and than uses it to extrapolate the other measurements to low energies, one obtains from the direct measurements a weighted mean $S_{34}(0) = 0.45$ MeV · b which was used in DS96.

6.3 Diffusion

Diffusion, caused by density, temperature, pressure, chemical composition and gravitational potential gradients plays an important role in the sun since it modifies the local chemical composition in the sun. The agreement between helioseismology and the SSM is improved when diffusion is included (see e.g., Fig. 10), but inclusion increases the discrepancy between the SSM and the solar neutrino observations. The relative changes in SSM predictions due to diffusion of all elements are summarized in Table IV. While BP95 and BSP98 found a rather large increase in the predicted $^7\text{Be}$, $^8\text{B}$, $^{13}\text{N}$, $^{15}\text{O}$ and $^{17}\text{F}$ solar neutrino fluxes; 14%, 36%, 52%, 58%, and 61% which result in 36%, 33%, 9% increases in their predicted rates in Super-Kamiokande, Homestake, and in GALLEX and SAGE, respectively, DS96 found only a moderate increase due to diffusion, 4%, 10%, 23%, 24% and 25%, respectively, in the above fluxes, which result in 10%, 10% and 2% increase in the predicted rates in Kamiokande, Homestake, and in GALLEX and SAGE, respectively. Although the two diffusion calculations assumed a different initial solar chemical composition (see below) and BP95 approximated the diffusion of all elements heavier than $^4\text{He}$ by that of fully ion-
ized iron (the DS calculations followed the diffusion of each element separately and uses diffusion coefficients calculated for the local ionization state of each element in the sun as obtained from solving the local Saha equations), these cannot fully explain the above large differences. Independent diffusion calculations by Richard et al. (1996) obtained similar results to those obtained in DS96 as can be seen from Table IV (we interpolated the results from the two models of Richard et al. (1996) to the initial chemical composition assumed in DS96). Note that internal magnetic fields can suppress diffusion significantly.

7 New Neutrino Physics?

Standard solar models, like DS96, perhaps can explain the results reported by Kamiokande and Super-Kamiokande. However, if the neutrino absorption cross sections assumed by the radiochemical experiments are correct, then present standard solar models cannot explain the absence of the expected contributions of the $^7$Be and CNO solar neutrinos to the $^{37}$Ar production rate in $^{37}$Cl and to the $^{71}$Ge production in $^{71}$Ga. Consequently, many authors have claimed that the the solar neutrino observations imply neutrino properties beyond the minimal standard electroweak model (e.g., Bahcall and Bethe 1991).

Neutrino Magnetic Moments. Some authors have suggested that neutrinos may have anomalous magnetic moments large enough so that the solar internal magnetic field in the sun can flip the neutrino helicity and convert part of the left handed weakly interacting solar neutrinos into noninteracting right handed neutrinos (e.g., Okun et al. 1986, Lim and Marciano 1988, Akhmedov 1988). However, the high statistics solar neutrino measurements of SK do not show the time variation of the solar neutrino flux which is predicted by a magnetic helicity flip interpretation of the solar neutrino anomaly.

Neutrino Oscillations: Mikheyev and Smirnov (1985) have discovered that neutrino oscillations in matter (Wolfenstein 1978, 1979) can lead to resonant conversion of neutrino flavor (the MSW effect) in the sun and explain the solar neutrino observations quite neatly. It requires only a natural extension of the minimal standard electroweak theory and it is based on a simple quantum mechanical effect. Many authors have carried out extensive calculations to determine the neutrino mixing parameters which can bridge between the predictions of the standard solar models and the solar neutrino observations. The neutrino mixing parameters can also be deduced analytically directly from the solar neutrino observations (e.g., Dar and Nussinov 1991; Dar 1993): If the $\nu_e$ is mixed with the $\nu_\mu$ (or the $\nu_\tau$) with a vacuum mixing angle $\theta \ll 1$ and a mass difference $\Delta m^2 = m^2_{\nu_\mu} - m^2_{\nu_e}$, then solar $\nu_e$’s which are produced in the sun can flip their flavor on their way out of the sun if they encounter electron density $n_e = \frac{\Delta m^2 c^4 \cos 2\theta}{2 \sqrt{2} G_F E_{\nu_e}} \approx \frac{\Delta m^2 c^4}{2 \sqrt{2} G_F E_{\nu_e}}$. (17)
The probability for the resonant flavor flip (the MSW effect) is given approximately by (e.g., Haxton 1986; Parke 1986; Dar et al. 1987)\[ P(\nu_e \rightarrow \nu_\mu) \approx 1 - e^{-\epsilon/E_{\nu_e}} \]

where \[ \epsilon \approx \pi H \Delta m^2 c^4 \sin^2 \theta \approx \pi H \Delta m^2 c^4 \theta / \bar{h}c , \]

with \( H = -n_e/(dn_e/dr) \) being the density scale-height at the resonance.

Strong suppression of the contribution from the pep, \(^7\)Be and CNO solar neutrinos to the Cl experiment requires a complete flavor change of these neutrinos in the sun, i.e., that they encounter a resonant density with \( \epsilon \gg E_{\nu_e} \sim 1 \) MeV. But the results from GALLEX and SAGE suggest that the lower energy pp solar neutrinos evade such a flavor flip. This is possible if the resonance condition (17) is satisfied for the pep, \(^7\)Be and CNO solar neutrinos (\( E_{\nu_e} \geq 0.862 \) MeV) but not for the pp neutrinos (\( E_{\nu_e} \leq 0.420 \) MeV). Since the maximal (central) electron density in the sun is \( n_e \approx 10^2 N_A \) the last condition reads (e.g., Dar 1993)

\[ 0.5 \times 10^{-5} eV^2 \leq \Delta m^2 c^4 \leq 1 \times 10^{-5} eV^2 \]

For a given \( \Delta m^2 \), eq. (18) can be used to adjust the mixing angle to produce the required suppression of the SSM \(^8\)B solar \( \nu_e \) flux:

\[ \Delta m^2 \sin^2 2\theta \approx 3 \times 10^{-8} eV^2 c^{-4}, \]

\[ \Delta m^2 \sin^2 2\theta \approx 1 \times 10^{-8} eV^2 c^{-4}, \]

for respectively, the BSP98 and the DS96 standard solar models.

8 Are The \(^7\)Be Solar Neutrinos Missing?

Electron capture by \(^7\)Be into the ground state of \(^7\)Li produces 862 keV neutrinos. The threshold energy for neutrino absorption by \(^37\)Cl is 814 keV. Thus, absorption of \(^7\)Be neutrinos by \(^37\)Cl produces 48 keV electrons. The maximum energy of the pp solar neutrinos is 420 keV. The threshold energy for neutrino absorption in \(^71\)Ga (\( 3/2^- \)) is 233 keV into the ground state (\( 1/2^- \)) and 408 into its first excited state (\( 5/2^- \)). The produced electrons have therefore energies below 187 and 12 keV, respectively. If the theoretical cross sections for neutrino absorption near threshold overestimate their true values significantly then the predicted rates will significantly overestimate the expected signals in the Chlorine and Gallium experiments. An indication that final state interactions effects are not completely understood is provided by Tritium \( \beta \)-decay. Although final state interactions in Tritium \( \beta \)-decay have been studied extensively, they do not explain well the end-point \( \beta \)-decay spectrum (\( E_e \sim 18.6 \) keV). In all recent measurements, the measured spectrum yields a negative value for the fitted squared mass of the electron neutrino. Final state interactions effects (screening of the nuclear charge by atomic electrons, exchange effects, radiative corrections, nuclear recoil against the electronic cloud, etc) in neutrino captures
near threshold in $^{37}$Cl and $^{71}$Ga may be much larger because their Z values are much larger and because the de Broglie wave lengths of the produced electrons are comparable to the Bohr radii of the atomic K shells in Cl and Ga. If final state interactions reduce considerably the near threshold absorption cross sections of pp neutrinos in $^{71}$Ga (making room for the expected contribution of $^7$Be solar neutrinos in Gallium) and of $^7$Be neutrinos in $^{37}$Cl, perhaps they can make the solar neutrino observations of Kamiokande and the Homestake experiment compatible. Such an explanation of the solar neutrino problem implies that experiments such as BOREXINO and HELLAZ will observe the full $^7$Be solar neutrino flux.

9 Astrophysical Solutions To The SNP

Even if the $^7$Be solar neutrino flux is strongly suppressed, it does not eliminate standard physics solutions to the solar neutrino problem: The ratio between the fluxes of $^7$Be and $^8$B solar neutrinos is given by

$$R = \frac{\phi_{\nu_\odot}(^7\text{Be})}{\phi_{\nu_\odot}(^8\text{B})} = \frac{\int n_n n_7 <\sigma v >_{e7} d^3r}{\int n_p n_7 <\sigma v >_{p7} d^3r}. \quad (21)$$

Because of the decreasing temperature and Be7 abundance as function of distance from the center of the sun on the one hand, and the $\sim r^2$ increase in radial mass on the other, the production of $^7$Be and $^8$B solar neutrinos in the SSM peaks around an effective radius, $r_{\text{eff}} \approx 0.064 R_\odot$ (where $r_{\text{eff}}$ is approximately the radius within which 50% of the flux is produced). The SSM also predicts a ratio of electron to proton densities near the center of the sun, $n_e/n_p \sim 2$, consistent with helioseismology observations. Consequently, the SSMs predict

$$R \approx \frac{2 <\sigma v >_{e7}}{<\sigma v >_{p7}} \approx 1.27 \times 10^{-14} S_{17}^{-1} F_{17}^{-1/6} T_7^{3/2} \epsilon^{47.825/T_7^{1/3}}, \quad (22)$$

where $F_{17}$ is the screening correction to the p-capture rate by $^7$Be, $T_7$ is the temperature in $10^7 K$ at the effective radius and $S_{17}$ is in $ev$ barn units. The SSMs yield $T_7(r_{\text{eff}}) \approx 1.45$. Using $S_{17}(0) = 17 ev$ barn and $\phi_{\nu_\odot}(^8\text{B}) = 2.44 \times 10^6 cm^{-2} s^{-1}$ as observed by Super-Kamiokande, one can reproduce the SSM prediction (e.g., Dar and Shaviv 1996)

$$\phi_{\nu_\odot}(^7\text{Be}) = R \phi_{\nu_\odot}(^8\text{B}) \approx 3.7 \times 10^9 cm^{-2} s^{-1}. \quad (23)$$

Astrophysical solutions of the solar neutrino problem aim towards suppressing the value of $R$. Three alternatives are currently investigated:

**Plasma Physics Effects**: The effects of the surrounding plasma on nuclear reaction rates in dense stellar plasmas, and in particular on proton and electron capture by $^7$Be in the sun are known only approximately. In order to explain
the deficit of $^7$Be solar neutrinos, without much affecting the SSM, plasma screening effects must reduce considerably the electron/proton capture ratio by $^7$Be, relative to the predictions of the weak screening theory (Salpeter and Van Horne 1969). The screening enhancement of bare nuclear cross sections is not well understood even in laboratory measurements with gas targets. Also the applicability of the weak screening theory to the dense plasma in the solar core is questionable. Moreover, correlations and fluctuations, which are neglected in the weak screening theory can affect strongly the screening enhancement of nuclear reaction rates in the solar core. This possibility is currently studied, e.g., by Shaviv and Shaviv (1998) using numerical methods. Because of accidental cancellations the weak screening corrections to the rates of all nuclear reactions do not change the predicted $^8$B solar neutrino flux, but perhaps a more exact treatment of screening may change R considerably.

Because the sub-Coulomb nuclear reactions in the core of the sun take place mainly between nuclei with kinetic energies much larger than their mean kinetic energies, their rates are very sensitive to the high energy tail of their velocity distribution in the sun. Diffusion, radiative flows, energetic nuclear products, internal fluctuating (equipartition ?) electric and magnetic fields and other collective effects may change the assumed Maxwell-Boltzmann tails of the energy distribution of the energetic particles in the core of the sun. This may shift the position of the Gamow peaks for the nuclear reaction rates and change considerably the ratios between nuclear reaction rates in the sun which have very different temperature dependence (see, e.g., Kaniadakis et al. 1998).

In principle, collective plasma physics effects, such as very strong magnetic or electric fields near the center of the sun, may also polarize the plasma electrons, and affect the branching ratios of electron capture by $^7$Be (spin 3/2$^-$) into the ground state (spin 3/2$^-$, $E_{\nu_e} = 0.863$ MeV, BR=90%) and the excited state (spin 1/2$^-$, $E_{\nu_e} = 0.381$ MeV, BR=10%) of $^7$Li. Since solar neutrinos with $E_{\nu_e} = 0.381$ MeV are below the threshold (0.81 MeV) for capture in $^{37}$Cl and have a capture cross section in $^{71}$Ga that is smaller by about a factor of 6 relative to solar neutrinos with $E_{\nu_e} = 0.863$ MeV, therefore a large suppression in the branching ratio to the ground state can produce large suppressions of the $^7$Be solar neutrino signals in $^{37}$Cl and in $^{71}$Ga. However, such an explanation requires anomalously large fields near the center of the sun.

**Temporal and Spatial Variations in T:** Davis (1996) has been claiming persistently that the solar neutrino flux measured by him and his collaborators in the $^{37}$Cl radioc hemical experiment is varying with time. Because of the possibility that neutrinos may have anomalous magnetic moments, much larger than those predicted by minimal extensions of the standard electroweak model, which can solve the solar neutrino problem, attention has been focused on anticorrelation between the solar magnetic activity (the 11 year cycle) and the $\nu_\odot$ flux (see, e.g., Davis 1996). Also a day-night effect (e.g., Cribier et al 1986; Dar and Mann 1987) due to resonant conversion of the lepton flavor of solar neutrinos which cross Earth at night before reaching the solar neutrino detector
was not found by Kamiokande. However, the basic general question whether the solar neutrino flux varies on a short time scale, has not been fully answered, mainly because of the limited statistics of the first generation of solar neutrino experiments. The SSM predict no significant variation of the solar neutrino flux on time scales shorter than millions of years. However, the sun has a differential rotation. It rotates once in $\sim 25$ days near the equator, and in $\sim 33$ days near the poles. Moreover, the observed surface rotation rates of young solar-type stars are up to 50 times that of the sun. It suggest that the sun has been losing angular momentum over its lifetime. The overall spin-down of a sun-like star by mass loss and electromagnetic radiation is difficult to estimate from stellar evolution theory, because it depends on delicate balance between circulations and instabilities that tend to mix the interior and magnetic fields that retard or modify such processes. It is quite possible that the differential rotation extends deep into the core of the sun and causes there spatial and temporal variations in the solar properties due to circulation, turbulences and mixing. Since $R$ is very sensitive to the temperature, even small variations in temperature can affect $R$ significantly without affecting significantly the pp solar neutrino flux (the $^7$Be and $^8$B solar neutrinos will come mainly from temperature peaks, while the pp neutrinos will reflect more the average temperature). If the solar neutrino flux is time dependent, then cross correlation analysis of the various data sets from the Homestake, Kamiokande, GALLEX, SAGE and Superkamiokande may reveal such unexpected correlations: If arbitrary time lags are added to the different solar neutrino experiments, the cross correlation is maximal when these time lags vanish. Moreover, a power spectrum analysis of the signals may show peaks, if the time variation is periodic. In particular, Super-Kamiokande with its high statistics should examine whether data from different fiducial volumes are cross correlated in time. Relevant information on time variability in the solar core may come soon also from SOHO and GONG.

**Mixing of $^3$He:** The SSM $^3$He equilibrium abundance increases sharply with radius. Cummings and Haxton (1996) have recently suggested that the $^7$Be solar neutrino problem could be circumvented in models where $^3$He is transported into the core in a mixing pattern involving rapid filamentary flow downward. We note that if this mixing produces hot spots (due to enhanced energy release) they can increase the effective temperature for p capture by $^7$Be in a cool environment, reducing $R$ while keeping the $^8$B solar neutrino flux at the observed level. Perhaps, helioseismology will be able to test that. Cummings and Haxton (1996) also noted that such mixing will have other astrophysical consequences. For example, galactic evolution models predict $^3$He abundances in the presolar nebula and in the present interstellar medium (ISM) that are substantially (i.e., a factor of five or more) in excess of the observationally inferred values. This enrichment of the ISM is driven by low-mass stars in the red giant phase, when the convective envelope reaches a sufficient depth to mix the $^3$He peak, established during the main sequence, over the outer portions of the star. The $^3$He is then carried into the ISM by the red giant wind. The
core mixing lowers the main sequence $^3$He abundance at large $r$.

10 Conclusions

The solar neutrino problem does not provide conclusive evidence for neutrino properties beyond the standard electroweak model. The solar neutrino problem may be an astrophysical problem. The deviations of the experimental results from those predicted by the standard solar models may reflect the approximate nature of our knowledge of nuclear reaction rates and radiation transport in dense stellar plasmas and the approximate nature of the solar models which neglect angular momentum effects, differential rotation, magnetic field, angular momentum loss and mass loss during evolution and do not explain yet, e.g., solar activity and the surface depletion of lithium, relative to its meteoritic value (which may or may not be relevant to the solar neutrino problem). Improvements of the standard solar model should continue. In particular, dense plasma effects (screening, correlations, fluctuations and deviations from Maxwell-Boltzmann distributions) on nuclear reaction rates and radiative opacities, which are not well understood, may affect the SSM predictions and should be further studied, both theoretically and experimentally. Relevant information may be obtained from studies of thermonuclear plasmas in inertial confinement experiments. Useful information may also be obtained from improved data on screening effects in low energy nuclear cross sections of ions, atomic beams and molecular beams incident on a variety of gas, solid and plasma targets.

Better knowledge of low energy nuclear cross sections is still needed. Improved measurement of the low energy nuclear cross sections for the radiative captures $p + ^7$Be $\rightarrow ^8$Be + $\gamma$ and $^3$He + $^4$He $\rightarrow ^7$Be + $\gamma$ by photodissociation of $^8$B and $^7$Be in the Coulomb field of heavy nuclei can help determine whether there is a $^8$B solar neutrino problem.

Neutrino oscillations, and in particular the MSW effect, may be the correct solution to the solar neutrino problem. But, only future experiments, such as SNO, BOREXINO and HELLAZ, will be able to supply a definite proof that Nature has made use of this beautiful effect.

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Table Ia: Comparison between the solar neutrino fluxes predicted by the SSM of BP95, BSP98, BTM98 and DS96, and measured by the solar neutrino experiments.

<table>
<thead>
<tr>
<th>Flux</th>
<th>BP95</th>
<th>BSP98</th>
<th>BSP98</th>
<th>DS96</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_\nu(pp)$ [$10^{10},\text{cm}^{-2}\text{s}^{-1}$]</td>
<td>5.91</td>
<td>5.94</td>
<td>6.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_\nu(\text{pep})$ [$10^{9},\text{cm}^{-2}\text{s}^{-1}$]</td>
<td>1.39</td>
<td>1.39</td>
<td>1.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_\nu(^7\text{Be})$ [$10^{9},\text{cm}^{-2}\text{s}^{-1}$]</td>
<td>5.18</td>
<td>4.80</td>
<td>3.71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_\nu(^8\text{B})$ [$10^{6},\text{cm}^{-2}\text{s}^{-1}$]</td>
<td>6.48</td>
<td>5.15</td>
<td>4.82</td>
<td>2.49</td>
<td>2.44 ± 0.11</td>
</tr>
<tr>
<td>$\phi_\nu(^{13}\text{N})$ [$10^{6},\text{cm}^{-2}\text{s}^{-1}$]</td>
<td>6.4</td>
<td>6.05</td>
<td>3.82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_\nu(^{15}\text{O})$ [$10^{8},\text{cm}^{-2}\text{s}^{-1}$]</td>
<td>5.15</td>
<td>5.32</td>
<td>3.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_\nu(^{17}\text{F})$ [$10^{8},\text{cm}^{-2}\text{s}^{-1}$]</td>
<td>6.48</td>
<td>6.33</td>
<td>4.53</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table Ib: Characteristics of the BP95, BTM98, and DS96 Solar Models in Table Ia (c=center; s=surface; bc=base of convective zone; $\bar{N} = \log([N]/[H]) + 12$).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>BP95</th>
<th>BTM98</th>
<th>DS96</th>
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</thead>
<tbody>
<tr>
<td>$T_c$ [$10^7\text{K}$]</td>
<td>1.584</td>
<td>1.567</td>
<td>1.561</td>
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<tr>
<td>$\rho_c$ [g cm$^{-3}$]</td>
<td>156.2</td>
<td>151.9</td>
<td>155.4</td>
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<td>$X_c$</td>
<td>0.3333</td>
<td>0.3442</td>
<td>0.3424</td>
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<tr>
<td>$Y_c$</td>
<td>0.6456</td>
<td>0.635</td>
<td>0.6380</td>
</tr>
<tr>
<td>$Z_c$</td>
<td>0.0211</td>
<td>0.02084</td>
<td>0.01940</td>
</tr>
<tr>
<td>$R_{\text{conv}}$ [$R/R_\odot$]</td>
<td>0.712</td>
<td>0.715</td>
<td>0.7130</td>
</tr>
<tr>
<td>$T_{bc}$ [$10^6\text{K}$]</td>
<td>2.20</td>
<td>2.172</td>
<td>2.105</td>
</tr>
<tr>
<td>$X_s$</td>
<td>0.7351</td>
<td>0.739</td>
<td>0.7512</td>
</tr>
<tr>
<td>$Y_s$</td>
<td>0.2470</td>
<td>0.243</td>
<td>0.2308</td>
</tr>
<tr>
<td>$Z_s$</td>
<td>0.01798</td>
<td>0.0181</td>
<td>0.0170</td>
</tr>
<tr>
<td>$\bar{N}_s(^{12}\text{C})$</td>
<td>8.55</td>
<td>8.55</td>
<td>8.55</td>
</tr>
<tr>
<td>$\bar{N}_s(^{14}\text{N})$</td>
<td>7.97</td>
<td>7.97</td>
<td>7.97</td>
</tr>
<tr>
<td>$\bar{N}_s(^{16}\text{O})$</td>
<td>8.87</td>
<td>8.87</td>
<td>8.87</td>
</tr>
<tr>
<td>$\bar{N}_s(^{20}\text{Ne})$</td>
<td>8.08</td>
<td>8.08</td>
<td>8.08</td>
</tr>
<tr>
<td>$T_{\text{eff}}$ [K]</td>
<td>5800</td>
<td>5800</td>
<td>5803</td>
</tr>
</tbody>
</table>
**Table II:** Comparison between the SSM of Bahcall and Pinsonneult (1995) and of Dar and Shaviv (1996).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>BSP98</th>
<th>DS96</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\odot$</td>
<td>$1.9899 \times 10^{33} , g$</td>
<td>$1.9899 \times 10^{33} , g$</td>
</tr>
<tr>
<td>$L_\odot$</td>
<td>$3.844 \times 10^{33} , erg , s^{-1}$</td>
<td>$3.844 \times 10^{33} , erg , s^{-1}$</td>
</tr>
<tr>
<td>$R_\odot$</td>
<td>$6.9599 \times 10^{10} , cm$</td>
<td>$6.9599 \times 10^{10} , cm$</td>
</tr>
<tr>
<td>$\tau_\odot$</td>
<td>$4.566 \times 10^9 , y$</td>
<td>$4.57 \times 10^9 , y$</td>
</tr>
<tr>
<td>Rotation</td>
<td>Not Included</td>
<td>Not Included</td>
</tr>
<tr>
<td>Magnetic Field</td>
<td>Not Included</td>
<td>Not Included</td>
</tr>
<tr>
<td>Mass Loss</td>
<td>Not Included</td>
<td>Not Included</td>
</tr>
<tr>
<td>Angular Momentum Loss</td>
<td>Not Included</td>
<td>Not Included</td>
</tr>
<tr>
<td>Premain Sequence Evolution</td>
<td>Not Included</td>
<td>Included</td>
</tr>
</tbody>
</table>

**Initial Abundances:**
- $^4$He: Adjusted  Adjusted
- C, N, O, Ne: Adjusted Adjusted
- All Other Elements: Adjusted Meteoritic

**Photospheric Abundances:**
- $^4$He: Predicted Predicted
- C, N, O, Ne: Photospheric Photospheric
- All Other Elements: Meteoritic Predicted

**Radiative Opacities:**
- OPAL 1996
- Straniero 1996?
- DS 1996

**Equation of State:**
- Not Included
- Included

**Partial Ionization Effects:**
- Not Included
- Included

**Diffusion of Elements:**
- Included
- Included

**Heavier Elements:**
- Approximated by Fe
- All Included

**Partial Ionization Effects:**
- Not Included
- Included

**Nuclear Reaction Rates:**
- $S_{11}(0) \, eV \cdot b$: $4.00 \times 10^{-19}$  $4.07 \times 10^{-19}$
- $S_{13}(0) \, MeV \cdot b$: $5.3 \times$  $5.6$
- $S_{24}(0) \, keV \cdot b$: $0.53$  $0.45$
- $S_{17}(0) \, eV \cdot b$: $19$  $17$

**Screening Effects:**
- Included
- Included

**Nuclear Equilibrium:**
- Imposed
- Not Assumed
Table IV: Fractional change in the predicted $\nu_\odot$ fluxes and counting rates in the $\nu_\odot$ experiments due to the inclusion of element diffusion in the SSM calculations of Bahcall and Pinsonneault (BP95), Brun, Turck-Chieze and Morel (BTM98), Dar and Shaviv (DS96) and Richard, Vauclair, Charbonnel and Dziembowski (RVCD96). The results of models 1 and 2 of RVCD96 were extrapolated to the initial solar composition which was used in DS96.

<table>
<thead>
<tr>
<th>$\phi_{\nu_\odot}$</th>
<th>BP95</th>
<th>BTM98</th>
<th>DS96</th>
<th>RVCD96</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pp$</td>
<td>$-1.7%$</td>
<td>$-0.3%$</td>
<td>$-0.8%$</td>
<td></td>
</tr>
<tr>
<td>$pep$</td>
<td>$-2.8%$</td>
<td>$-0.3%$</td>
<td>$-0.4%$</td>
<td></td>
</tr>
<tr>
<td>$^7\text{Be}$</td>
<td>$+13.7%$</td>
<td>$+4.2%$</td>
<td>$+6.5%$</td>
<td></td>
</tr>
<tr>
<td>$^8\text{B}$</td>
<td>$+36.5%$</td>
<td>$+11.2%$</td>
<td>$+10.7%$</td>
<td></td>
</tr>
<tr>
<td>$^{13}\text{N}$</td>
<td>$+51.8%$</td>
<td>$+22.7%$</td>
<td>$+19.8%$</td>
<td></td>
</tr>
<tr>
<td>$^{15}\text{O}$</td>
<td>$+58.0%$</td>
<td>$+24.0%$</td>
<td>$+20.8%$</td>
<td></td>
</tr>
<tr>
<td>$^{17}\text{F}$</td>
<td>$+61.2%$</td>
<td>$+24.9%$</td>
<td>$+21.8%$</td>
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</tr>
<tr>
<td>Rates</td>
<td>$+36.5%$</td>
<td>$+11.2%$</td>
<td>$+10.7%$</td>
<td></td>
</tr>
<tr>
<td>H$_2$O</td>
<td>$+32.9%$</td>
<td>$+9.5%$</td>
<td>$+12.3%$</td>
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<tr>
<td>Cl</td>
<td>$+8.7%$</td>
<td>$+2.6%$</td>
<td>$+3.7%$</td>
<td></td>
</tr>
</tbody>
</table>
Figure Captions

Fig. 1. The principal branches of the pp cycle and the CNO bi-cycle.

Fig. 2. The solar neutrino capture rate in $^{37}\text{Cl}$ as measured in the Homestake experiment runs nos. 18 to 133. The dashed line shows the average value.

Fig. 3. The $^{8}\text{B}$ solar neutrino flux as function of time as measured by Kamiokande between 1986-1996.

Fig. 4. The cosine of the angle between the electron direction and the radius vector from the sun in Super-Kamiokande. The solid line shows the best fit for a $^{8}\text{B}$ solar neutrino flux.

Fig. 5. The time variation of the $^{8}\text{B}$ solar neutrino flux as measured by Super-Kamiokande from June 96 to June 97. The solid line shows the expected variation of the flux due to the eccentricity of the Earth’s orbit around the sun.

Fig. 6. The ratio between the $^{8}\text{B}$ solar neutrino flux observed by Super-Kamiokande and the flux predicted by the SSM of BP95 as function of zenith angle of the sun.

Fig. 7. The ratio between the observed number of electrons scattered by solar neutrinos in Super-Kamiokande and their expected number in the SSM of BP95 as function of electron recoil energy.

Fig. 10. The $^{71}\text{Ge}$ production rate in $^{71}\text{Ga}$ by solar neutrinos as measured by GALLEX and SAGE between 1990 and 1997.

Fig. 9. The astrophysical S(E) factor for the reaction $^{3}\text{He} + ^{3}\text{He} \rightarrow ^{4}\text{He} + 2\text{p}$ as measured by various low energy experiments. The dotted line is a best polynomial fit (solid line) to the data with maximal screening enhancement ($U_e = 240 \text{ eV}$).

Fig. 10. The relative difference between the speed of sound squared as inferred from the helioseismological measurements of GOLF and LOWL on board SOHO and that calculated in BTM98 from the SSM with diffusion (solid line) and without diffusion (dotted line).