The Cremmer-Scherk mechanism is generalised in a non-Abelian context. In the presence of the Higgs scalars of the standard model it is argued that fields arising from the low energy effective string action may contribute to the mass generation of the observed vector bosons that mediate the electroweak interactions and that future analyses of experimental data should consider the possibility of string induced radiative corrections to the Weinberg angle coming from physics beyond the standard model.

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The standard model of fundamental particle interactions is based on families of charged leptons, quarks and scalar fields interacting via gauge fields associated with Lie groups [1],[2]. This structure is general enough to accommodate gravitation and it is a common belief that an underlying geometrical theory associated with some extended structure in higher dimensions will offer a unified framework from which the standard model will emerge as an effective low energy approximation. Although such a unique unified geometrical theory is currently elusive, there are strong hints that it may manifest itself in a number of different guises. These alternative representations are said to be related by certain duality transformations. They promise to relate different spectra associated with dual coupling regimes and offer an explanation of the observed masses of the fundamental particles. At the level of low energy effective actions, however, current phenomenology relies on a number of alternative mechanisms for mass generation. These may be briefly classified as (Kaluza-Klein) harmonic expansions in dimensional reduction, dynamical symmetry breaking and the Higgs mechanism based on the existence of fundamental self-coupled scalar fields. Each mechanism makes particular predictions and experimental guidance is required for preferring one without prejudice.

In this note we explore the role of the Abelian gauge vector potential \( A \), the dilaton \( \phi \) and the antisymmetric 3-form gauge field \( H \) that arise in certain low energy effective actions on the Higgs mechanism of the standard model. In lowest order \( H \) is usually interpreted as describing a scalar (axion) field that, together with the dilaton scalar \( \phi \), modifies Einsteinian gravitation. \(^1\) However, we propose here that \( H \) couples to vector fields via an interaction with Higgs fields in such a way that it contributes to mass generation via a spontaneous breakdown of gauge symmetry. The coupling is inspired by an earlier incarnation of string theory as a dual model for the strong interactions and the recognition that sectors associated with “Reggeons” and “Pomerons” are “dual”. The gauge potential \( A \) is analogous to a Reggeon state in the hadron model and the Abelian gauge symmetry in the action is inherited from a representation of the Virasoro symmetry. The antisymmetric tensor field described here in terms of the 2-form \( B \), where to lowest order \( H = dB \), corresponds to a state in the Pomeron sector of the dual model. In 1974, Cremmer and Scherk suggested on the basis of a 1-loop summation that both these fields could gain mass by radiative corrections [3],[4]. Furthermore they gave a gauge invariant effective action that maintained a duality between the Reggeon and Pomeron descriptions.

In the context of low energy effective string theory, the mechanism arises from the effective action density 4-form

\[
\Lambda[g, \phi, A, B] = \kappa R - \left(2\alpha - 3\right) d\phi \wedge * d\phi + \frac{1}{2} e^{-2\phi} dB \wedge * dB + \frac{1}{2} e^{-2\phi} dA \wedge * dA + \lambda A \wedge dB
\]

where \( A \) is a 1-form, \( B \) a 2-form, \( \phi \) the dilaton 0-form on spacetime \( M \) with a metric \( g \), curvature scalar \( R \) and

\(^1\)The interpretation of a massless axion \( A \) is based on the field equation \( d(e^{-2\phi} \ast dB) = 0 \) so that locally \( dB = e^{2\phi} \ast dA \) and hence \( d(e^{2\phi} \ast dA) = 0 \).
associated Hodge map \( \ast \). The action \( \int_M \Lambda \) is invariant under the Abelian gauge symmetries \( A \rightarrow A + df_0, B \rightarrow B + df_1 \) where \( f_0 \) and \( f_1 \) are arbitrary 0 and 1 forms, respectively. In terms of the Weyl scaled metric \( \hat{g} = e^{-\phi}g \), with the corresponding Hodge map \( \ast \) and curvature scalar \( \hat{R} \), the effective action may be written

\[
\int_M \{ e^{-\phi}(\kappa\hat{R} \ast 1 - \frac{\alpha}{2} d\phi \land \ast d\phi + \frac{1}{2} dB \land \ast dB) + \frac{1}{2} e^{-2\phi} dA \land \ast dA + \lambda A \land dB \}. \tag{2}
\]

In this form one can make contact with low energy effective axi-dilaton gravity when \( \lambda = 0 \) [5],[6]. Having established this link, we concentrate next on the dynamics of the field system \( \{ \phi, B, A \} \) in a flat Minkowski background (the implications for the gravitational sector will be discussed elsewhere):

\[
d(e^{-2\phi} \ast dA) + \lambda dB = 0, \tag{3}
\]

\[
d(e^{-2\phi} \ast dB) - \lambda dA = 0, \tag{4}
\]

\[
d \ast d\phi = \frac{2}{(2\alpha - 3)} e^{-2\phi} (dB \land \ast dB + dA \land \ast dA). \tag{5}
\]

Since \( M \) is topologically trivial (3) and (4) imply

\[
d\tilde{A} = \lambda e^{2\phi} \ast \tilde{B}, \tag{6}
\]

\[
d\tilde{B} = \lambda e^{2\phi} \ast \tilde{A}, \tag{7}
\]

in terms of the variables \( \tilde{A} = A - \frac{1}{\lambda} df_0, \tilde{B} = B - \frac{1}{\lambda} df_1 \) in the gauge equivalence classes \([A]\) and \([B]\), respectively. One may fix gauges by taking solutions with particular \( f_0 \) and \( f_1 \). Remarkably the entire theory can be described in terms of either the fields \( \{ \phi, \tilde{A} \} \) or \( \{ \phi, \tilde{B} \} \). Each description refers to a different dual sector of the same theory. Moreover, in terms of the \( \{ \phi, \tilde{A} \} \) description the theory admits a vector field satisfying the dilaton-Proca system:

\[
d(e^{-2\phi} \ast d\tilde{A}) + \lambda^2 e^{2\phi} \ast \tilde{A} = 0, \tag{8}
\]

\[
d \ast d\phi = -\frac{2\lambda^2}{2\alpha - 3} e^{2\phi} \tilde{A} \land \ast \tilde{A} + \frac{2}{(2\alpha - 3)} e^{-2\phi} d\tilde{A} \land \ast d\tilde{A}. \tag{9}
\]

These equations admit a “vacuum” solution \( \tilde{A} = 0, \phi = \phi_0 \) for some constant \( \phi_0 \). Linearising about this solution with \( \tilde{A} = \epsilon \tilde{A}^{(1)}, \phi = \phi_0 + \epsilon \phi^{(1)} \) yields

\[
d \ast dA^{(1)} + \lambda^2 e^{4\phi_0} \ast \tilde{A}^{(1)} = 0, \tag{10}
\]

\[
d \ast d\phi^{(1)} = 0, \tag{11}
\]

showing that the Abelian gauge field acquires a mass \( \mu_0 = \lambda e^{2\phi_0} \). In a similar way the theory has a dual description in terms of \( \{ \phi, \tilde{B} \} \):

\[
d(e^{-2\phi} \ast d\tilde{B}) - \lambda^2 e^{2\phi} \ast \tilde{B} = 0, \tag{12}
\]

\[
d \ast d\phi = -\frac{2\lambda^2}{2\alpha - 3} e^{2\phi} \tilde{B} \land \ast \tilde{B} + \frac{2}{(2\alpha - 3)} e^{-2\phi} d\tilde{B} \land \ast d\tilde{B}, \tag{13}
\]

showing that the Kalb-Ramond 2-form field \( \tilde{B} \) also acquires a mass \( \mu_0 \).

The role of the fields \( \{ \phi, \tilde{A}, \tilde{B} \} \) is elusive in the standard model. However, guided by the \( \tilde{A} \leftrightarrow \tilde{B} \) duality in the presence of the Cremmer-Scherk mechanism we propose that the Abelian gauge potential \( A \) be identified with the weak hypercharge gauge potential in the electroweak \( SU(2) \times U(1) \) gauge group and that the Cremmer-Scherk mass generation mechanism be generalised with the aid of the standard Higgs multiplet. The low energy effective action
where $F = dA + gA \wedge A$ with $A$ an $SU(2)$ Lie algebra (with basis $T_j$) valued 1-form and $D \Phi = d \Phi + gA \Phi + igF \Phi$ with $A = A_j T_j$ in terms of the Pauli matrices $T_j$. The dynamics of the fields $\{A, A\}$ arise from the field equations

$$D(e^{-2\phi} * F) + g\lambda (\Phi^i T_j) T_j \rho dB + \frac{g}{2} (\langle D \Phi \rangle T_j) T_j + \frac{1}{2} (D \Phi) T_j = 0,$$

$$d(e^{-2\phi} * dA) - \frac{g'}{4} \lambda \Phi^i T_j dB - i \frac{g'}{4} (\Phi^i D \Phi + (D \Phi)^i) = 0$$

where $D = d + gA$ with $A$ in the adjoint representation of $SU(2)$. In the Higgs vacuum $\Phi = (v, 0)^T$, the new interaction $i\lambda \Phi^i D \Phi$ becomes $\frac{\lambda v^2}{2} (gA_3 - g'A) \wedge dB$ (cf. the interaction in (1)) and one expects that the mass generation for the vector fields will be modified. This is indeed the case. Expanding about the vacuum, $\Phi = (v, 0)^T$, $\phi = \phi_0$, with $dB = i \lambda e^{2\phi} * (\Phi^i D \Phi)$, the mass eigenstates are given by the equations

$$d(e^{-2\phi} * dA_1) + \frac{g^2}{4} |v|^2 * A_1 = 0,$$

$$d(e^{-2\phi} * dA_2) + \frac{g^2}{4} |v|^2 * A_2 = 0,$$

$$d(e^{-2\phi} * dA_3) + \frac{\lambda^2}{4} e^{2\phi_0} g |v|^2 * (gA_3 - g'A) + \frac{g^2}{4} |v|^2 * A_3 - \frac{g'g}{4} |v|^2 A = 0,$$

$$d(e^{-2\phi} * dA) - \frac{g'}{4} |v|^2 (\lambda^2 e^{2\phi_0} |v|^2 + 1) * (gA_3 - g'A) = 0.$$
References