Constraints on the Formation and Evolution of Circumstellar Disks in Rotating Magnetized Cloud Cores

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ABSTRACT

We use magnetic collapse models to place some constraints on the formation and angular momentum evolution of circumstellar disks which are embedded in magnetized cloud cores. Previous models have shown that the early evolution of a magnetized cloud core is governed by ambipolar diffusion and magnetic braking, and that the core takes the form of a nonequilibrium flattened envelope which ultimately collapses dynamically to form a protostar. In this paper, we focus on the inner centrifugally-supported disk, which is formed only after a central protostar exists, and grows by dynamical accretion from the flattened envelope. We estimate a centrifugal radius for the collapse of mass shells within a rotating, magnetized cloud core. The centrifugal radius of the inner disk is related to its mass through the two important parameters characterizing the background medium: the background rotation rate $\Omega_b$ and the background magnetic field strength $B_{\text{ref}}$. We also revisit the issue of how rapidly mass is deposited onto the disk (the mass accretion rate) and use several recent models to comment upon the likely outcome in magnetized cores. Our model predicts that a significant centrifugal disk (much larger than a stellar radius) will be present in the very early (Class 0) stage of protostellar evolution. Additionally, we derive an upper limit for the disk radius as it evolves due to internal torques, under the assumption that the star-disk system conserves its mass and angular momentum even while most of the mass is transferred to a central star.

Subject headings: accretion, accretion disks - circumstellar matter - ISM: clouds - ISM: magnetic fields - MHD - stars: formation - stars: pre-main-sequence
1. INTRODUCTION

Ever since the discovery that the planets in our solar system have nearly circular and coplanar orbits, the presence of a gaseous circumstellar disk has been thought to be intimately associated with star and planet formation. At the present time, circumstellar disks can be detected around a large number of young stellar objects (YSO’s) by a variety of techniques, including infrared excess (e.g., Cohen, Emerson, & Beichman 1989; Strom et al. 1989), measurement of their millimeter and submillimeter flux (e.g., Beckwith et al. 1990; Adams, Emerson, & Fuller 1990; Ohashi et al. 1991; Osterloh & Beckwith 1995), and even direct optical imaging of disk silhouettes in the Orion Nebula using HST (McCaugrean & O’Dell 1996). The millimeter and submillimeter measurements, which are sensitive to the outer parts of the disk, usually find disk radii $\lesssim 100$ AU. The radii of the Orion silhouettes range from 25 AU to 500 AU. Altogether, a disk radius of $\sim 100$ AU is often taken to be a typical value. However, circumstellar disks of size $\sim 1000$ AU are also found around some older stars such as $\beta$ Pictoris.

Theoretically, disks are believed to be a natural consequence of the collapse of a rotating molecular cloud core. Therefore, in principle, one should be able to derive some physical properties of a circumstellar disk directly from the properties of a cloud core in which it is formed, or even from the properties of the ambient molecular cloud in which the core itself was formed. A crucial quantity that should be derived from a collapse model is the centrifugal radius for infalling mass shells; this quantity depends certainly on the existing density and angular velocity profile near a protostar at the time that it is formed. Previous estimates of the centrifugal radius have tended to use hydrodynamic models. For example, Terebey, Shu, & Cassen (1984) have derived a centrifugal radius $r_c$ under the assumption that the core is uniformly rotating and has the static density profile of a singular isothermal sphere at the moment that a central protostar is formed.

In this paper, we estimate a centrifugal radius for the collapse of a rotating, magnetized core. We use results from numerical magnetohydrodynamic (MHD) models of the formation of cores by ambipolar diffusion and magnetic braking (Basu & Mouschovias 1994, 1995a, 1995b, hereafter BM94, BM95a,b) and their subsequent collapse once they have a supercritical mass-to-flux ratio. The nonrotating aspects of these MHD models are discussed in detail by Fiedler & Mouschovias (1992, 1993, hereafter FM93) and Ciolek & Mouschovias (1993, 1994, 1995, hereafter CM93, CM94, CM95). In a previous paper, (Basu 1997), we used these numerical models to build a semianalytic model for the supercritical collapse phase. Our analytic expressions for the inner solution could be extrapolated to the instant that a central protostar is formed. At this moment, labeled $t = 0$ in the terminology of isothermal similarity solutions for cloud collapse (Larson 1969; Shu 1977;
Hunter 1977), the cloud core takes the form of a flattened envelope around the protostar, with a nonequilibrium power-law radial density profile \( \rho \propto r^{-2} \) and supersonic infall velocity. The core is also differentially rotating, with angular velocity \( \Omega \propto r^{-1} \) in the innermost region. Although the semianalytic solution applies to the time period \( t < 0 \), we can extrapolate to \( t > 0 \) for the purpose of estimating the centrifugal radius, since angular momentum should continue to be conserved in this more dynamical phase of accretion onto the central protostar. A centrifugal radius is expected to exist during the accretion phase \( (t > 0) \), even though it does not exist during the earlier runaway collapse phase \( (t < 0) \).

We relate the centrifugal radius \( r_c \) to the disk mass \( m \) though two fundamental parameters characterizing the ambient cloud: the background rotation rate \( \Omega_b \), and the background magnetic field strength \( B_{\text{ref}} \). The mass accretion rate \( \dot{m} \) is dependent on two other fundamental parameters: the isothermal sound speed \( c_s \) and the universal gravitational constant \( G \). However, \( \dot{m} \) is not expected to be constant for \( t > 0 \), for reasons given by Basu (1997). This has been demonstrated by recent numerical models of magnetized clouds (Tomisaka 1996; Ciolek & Königl 1998). We discuss the implications of several recent papers (Basu 1997; Saigo & Hanawa 1998; Ciolek & Königl 1998; Tomisaka 1996) on the mass accretion rate, and use them to place constraints on its likely value.

Once formed, or perhaps during the formation period itself, the disk is expected to undergo significant angular momentum evolution in order to give birth to star-disk systems of the type that are observed. Given the current theoretical uncertainties in understanding these processes, we present a constraint on the disk evolution if driven by internal torques (e.g., viscous or gravitational). Regardless of the details of the process, we find an upper limit to the disk radius if its mass and angular momentum are conserved.

We also discuss how estimates of disk sizes in the early protostellar phases (before the protostar has assembled most of its mass) may help to distinguish between various collapse scenarios which yield different physical properties in the near-protostellar environment. In essence, we find that a differentially rotating nonequilibrium pre-stellar core is necessary in order to obtain a significant centrifugal disk in the early (Class 0) protostellar phase.

2. Formation of an Inner Centrifugal Disk

2.1. The Centrifugal Radius

In the presence of dynamically significant magnetic fields, star formation is expected to occur within a flattened cloud core aligned perpendicular to the mean magnetic field direction. FM93 found that an initially magnetically subcritical (i.e., mass-to-flux ratio
below the critical value for collapse) cloud will first relax to equilibrium along field lines as it begins a quasistatic, ambipolar diffusion driven radial contraction toward a local density peak. This radial contraction becomes dynamic within a central region after it has achieved a supercritical mass-to-flux ratio; however, near-equilibrium is maintained along field lines. The supercritical regions, which are identified with molecular cloud cores, provide a flattened envelope within which stars form and grow by accretion. These disklike structures are in fact much larger (size up to \( \sim 10,000 \) AU) than a possible inner centrifugally-supported disk, and are not in equilibrium (they are supercritical). They are also not to be confused with the “pseudodisks” of Galli & Shu (1993a,b), which form after the formation of a central protostar in a cloud that has the density profile of a singular isothermal sphere when the protostar is formed, and is threaded by a dynamically weak magnetic field. When magnetic fields are dynamically significant, as implied by some Zeeman measurements (Goodman et al. 1989; Crutcher et al. 1993), the formation of the outer magnetic disk precedes star formation.

The collapse of magnetic cores has been further investigated numerically by CM94, 95, and BM94, 95a, 95b, using the thin-disk approximation. Basu (1997) found analytic expressions for the inner profiles of physical variables in these collapsing cores. A natural limiting form for isothermal collapse is when a power-law density profile (established due to self-similarity in the innermost region) extends inward to radius \( r = 0 \), creating a central singularity. Since a finite mass now exists at \( r = 0 \), this moment is usually associated physically with the formation of a central protostar.\(^1\) Basu (1997) shows that the magnetic collapse models of BM94, 95a, 95b typically lead to the limiting column density (integrated along the mean magnetic field direction) profile

\[
\sigma(r) \simeq \frac{c_s^2}{Gr}
\]

where \( c_s \) is the isothermal sound speed and \( G \) is the universal gravitational constant. The above equation describes a nonequilibrium profile (see §2.2). The angular velocity, which becomes aligned with the mean magnetic field direction during the core formation epoch (Mouschovias & Paleologou 1979, 1980), achieves the limiting profile

\[
\Omega(r) \simeq \frac{2\pi \Omega_b c_s^2}{B_{\text{ref}} G^{1/2} r}
\]

(Basu 1997), where \( \Omega_b \) is the ambient rotation rate of the cloud and \( B_{\text{ref}} \) is a uniform background (or “reference”) magnetic field. This differential rotation profile is a result

\(^1\)Isothermality is not really expected to be valid all the way to \( r = 0 \), i.e., for all densities, but it is a reasonable first step to pursue, since the non-isothermal gas occupies an inner region of size \( \lesssim 10 \) AU (Larson 1969).
of magnetic braking enforcing uniform rotation $\Omega \simeq \Omega_b$ until a critical column density is reached, and subsequent near angular momentum conservation during the rapid collapse phase, during which the column density profile of equation (1) is built up (BM94). Based on the BM94 results, Basu (1997) showed that angular momentum conservation effectively begins at a column density $\sigma_{\text{crit}} \simeq 2B_{\text{ref}}/\mu_{\text{crit}}$, where $\mu_{\text{crit}} = (2\pi G^{1/2})^{-1}$ is the critical mass-to-flux ratio for a magnetized thin disk (Nakano & Nakamura 1978). The result is a rotation rate many orders of magnitude greater than $\Omega_b$ near the cloud center. Equations (1) and (2) yield the following relation between specific angular momentum $j = \Omega r^2$ and the enclosed mass $m$:

$$j \simeq \frac{\Omega_b G^{1/2}}{B_{\text{ref}}} m. \quad (3)$$

A typical (mean) rotation rate for dense molecular cloud cores or starless Bok globules is about $10^{-14}$ rad s$^{-1}$ (Goodman et al. 1993; Kane & Clemens 1997). Using this as a typical value for the pre-collapse rotation rate $\Omega_b$, and using a canonical magnetic field value of 30$\mu$G (see Goodman et al. 1989; Crutcher et al. 1993), equation (3) yields $j \simeq 10^{20} - 10^{21}$ cm$^2$ s$^{-1}$ for $m \simeq 1 - 10$ M$_\odot$. The upper range is in agreement with estimates for $j$ in dense cores or Bok globules (Goodman et al. 1993; Kane & Clemens 1997), and the lower range is in good agreement with estimates of $j$ for smaller scale protostellar envelopes (Ohashi et al. 1997b), for which the lower mass range may be more relevant.

The $j - m$ relation of equation (3) is preserved during the dynamic collapse phase before a central protostar is formed ($t < 0$), since the collapse timescale becomes much shorter than the magnetic braking timescale (BM94). An inner centrifugally-supported disk may be expected to form near the cloud center, where the rotation rate is the highest. However, during the time period $t < 0$, the ratio of centrifugal acceleration to gravitational acceleration (the centrifugal support) remains constant in the central region, i.e., both accelerations increase in proportion to $r^{-3}$, where $r$ is a Lagrangian radius. This is a property of the self-similar collapse, and occurs in a rotating, magnetic disk (BM95a; Basu 1997), where efficient magnetic braking prior to collapse keeps the centrifugal support at very low levels anyway. This effect is also present in non-magnetic rotating disks (Norman et al. 1980; Narita, Hayashi, & Miyama 1984; Hayashi 1987), where the centrifugal support is much greater but also cannot grow and thereby halt the collapse. After a central protostar is formed ($t > 0$), the collapse becomes even more dynamic in an inner region where the infall resembles free-fall onto a central point mass, e.g., the similarity solutions of Shu (1977) and Hunter (1977). Hence, angular momentum is more likely to be conserved during this phase. More importantly, the nature of self-gravity in the flattened envelope is different at this stage, particularly near the cloud center. At $t = 0$, an $r^{-1}$ column density profile yields a gravitational field $g_r = -Gm/r^2$, where $m$ is the enclosed mass, i.e., the same as in spherical geometry. For example, equation (1) yields $m(r) = 2\pi c_s^2 r/G$, where $c_s$ is the sound speed in the envelope.
so that the integral equation for a thin-disk gravitational field (see equations [12] and [13] of BM94) yields \( g_r = -2\pi c_s^2/r = -Gm/r^2 \). When \( t > 0 \), the inner near free-fall region has infall velocity \( v_r \propto r^{-1/2} \), and column density \( \sigma \propto r^{-1/2} \) if the mass accretion rate \( \dot{M} = -2\pi \sigma v_r \) is uniform in this region\(^2\). The \( \sigma \propto r^{-1/2} \) profile does not yield the exact relation \( g_r = -Gm/r^2 \), but numerical integration shows that \( g_r = -g_0 Gm/r^2 \), where \( g_0 \approx 0.7 \) (Ciolek & K"onigl 1998). Hence, during \( t \geq 0 \), when the column density profile is a combination of \( r^{-1/2} \) and \( r^{-1} \), we expect \( g_r = -g_0 Gm/r^2 \), with \( g_0 \) falling in the range \( 0.7 - 1 \). For the remainder of this paper, we simply use \( g_r \approx -Gm/r^2 \), and not consider the multiplicative constant in front that is close to unity.

Since the gravitational field increases less rapidly than the centrifugal acceleration during \( t > 0 \), each mass shell (of fixed \( j \)) hits a centrifugal barrier at a radius \( r_c \) determined (to within a factor of order unity) by the relation

\[
\frac{j^2}{r_c^3} \simeq \frac{Gm}{r_c^2}.
\]

Incorporating equation (3), we find the centrifugal radius

\[
r_c \simeq \left( \frac{\Omega_b}{B_{\text{ref}}} \right)^2 m.
\]

Hence, the centrifugal radius can be related to the disk mass via two fundamental parameters characterizing the ambient molecular cloud. Using canonical values for molecular cloud cores, we find that

\[
r_c \simeq 15 \left( \frac{\Omega_b}{10^{-14} \text{ rad s}^{-1}} \right)^2 \left( \frac{30 \mu \text{G}}{B_{\text{ref}}} \right)^2 \left( \frac{m}{1 M_\odot} \right) \text{AU}.
\]

The squared dependence on \( \Omega_b \) and \( B_{\text{ref}} \) means that \( r_c \) can be quite sensitive to the actual ambient cloud conditions. The inverse dependence on \( B_{\text{ref}} \) occurs since a stronger ambient magnetic field implies a longer subcritical phase during which magnetic braking can enforce \( \Omega \simeq \Omega_b \). The relation (5) should be used only when \( B_{\text{ref}} \) is sufficiently strong that the core begins dynamic collapse from a magnetically critical state rotating at a background rate \( \Omega_b \). One may also choose to write equation (5) in terms of a critical column density \( \sigma_{\text{crit}} \approx 2B_{\text{ref}}/\mu_{\text{crit}} \); see § 2.1) at which dynamical contraction begins.

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\(^2\)A uniform mass accretion rate is a property of self-similar collapse and results in the inner density profile \( \rho \propto r^{-3/2} \) in spherical similarity solutions (e.g., Shu 1977; Hunter 1977). It also results in an inner column density profile \( \sigma \propto r^{-1/2} \) in thin-disk similarity solutions, as first noted by Nakamura, Hanawa, & Nakano (1995).
2.2. Self-Similarity and Dynamical Accretion from the Envelope

2.2.1. Analysis of Self-Similar Profiles

The mass $m$ of the star-disk system grows by dynamical accretion from the nonequilibrium flattened envelope. The accretion rate ($\dot{m}$) during the time $t > 0$ determines how rapidly the star-disk system is built up. Since the self-similar column density profile given by equation (1) is valid in at least an innermost region, it is worthwhile to see what predictions are made by self-similar solutions for the collapse from this profile. We make comparisons with self-similar models and discuss below the likely evolution of the mass accretion rate for $t > 0$ based on existing semianalytic and numerical models in the literature.

An important property of equation (1) is that it represents a nonequilibrium column density profile, i.e., the column density exceeds the equilibrium value. For comparison, the equilibrium solution for a non-magnetic thin disk, the singular isothermal disk (SID), is

$$\sigma_{\text{SID}} = \frac{c_s^2}{2\pi Gr}$$

(see Basu 1997). This profile may be regarded as the thin-disk counterpart to the density profile of the equilibrium singular isothermal sphere (SIS), $\rho_{\text{SIS}} = c_s^2/(2\pi Gr^2)$. The numerical results of BM94 yield a limiting profile which is overdense relative to the SID by the factor $k_{\text{SID}} \approx 2\pi$. The factor $k_{\text{SID}}$ overdensity may be partially accounted for by the presence of a magnetic field, as shown below.

The magnetized solution can be recovered from the non-magnetic one in thin-disk geometry by a simple scaling if the mass-to-flux ratio is spatially uniform (Shu & Li 1997; Nakamura & Hanawa 1997; Basu 1997); the magnetic pressure force is then proportional to the thermal pressure force and the magnetic tension is proportional (with opposite sign) to gravity. Basu (1997) also shows that this scaling is only valid in an inner region where the local magnetic field strength is much greater than the background field strength $B_{\text{ref}}$ of the ambient molecular cloud. Under these circumstances, the hydrodynamic equations can be transformed to the MHD equations by simply replacing $c_s^2$ with

$$c_{s,\text{eff}}^2 = (1 + a_{\text{M,P}}/a_T)c_s^2,$$

and $G$ with

$$G_{\text{eff}} = (1 + a_{\text{M,T}}/g_r)G,$$

where $a_{\text{M,P}}/a_T$ is the ratio of the accelerations due to magnetic pressure and thermal pressure, and $a_{\text{M,T}}/g_r$ is the ratio of the accelerations due to magnetic tension and
gravity. The three papers by Shu & Li (1997), Nakamura & Hanawa (1997) and Basu (1997) all show that \( \alpha_{M,T}/g_r = -\mu^{-2} \), where \( \mu \) is the mass-to-flux ratio in units of \( \mu_{\text{crit}} = (2\pi G^{1/2})^{-1} \). However, they find slightly different values for \( \alpha_{M,P}/a_T \), although they are all essentially proportional to \( \mu^{-2} \). The proportionality constant can vary due to different assumptions about the vertical structure of the cloud or inclusion in the magnetic pressure of finite-thickness corrections to the magnetic tension force, as pointed out by Shu & Li (1997). For example, Nakamura & Hanawa (1997) find that \( \alpha_{M,P}/a_T = \mu^{-2} \), but they do not include any finite thickness corrections to the magnetic tension force. Basu (1997) finds that \( \alpha_{M,P}/a_T = 2\mu^{-2} \), since his magnetic pressure term actually incorporates a finite thickness correction term from the magnetic tension. Since the magnetic pressure force is itself an effect of the finite-thickness of the disk, we consider the inclusion of the finite thickness magnetic tension term a necessity. In the following discussion, we scale hydrodynamic thin-disk solutions using \( \alpha_{M,P}/a_T = 2\mu^{-2} \), both for this reason and also because we are comparing results with the simulations of BM94, for which the analysis of Basu (1997) is most applicable.

Given that detailed numerical simulations of core formation and evolution with ambipolar diffusion (e.g., FM93; CM94; BM94) show that \( \mu \approx 2 \) has approximate spatial uniformity during the late stages before protostar formation, we obtain the scaled magnetic equilibrium solution, the singular isothermal magnetic disk (SIMD), from the SID profile. Using the exact value \( \mu = 2.1 \) from the late stages of BM94’s standard model, we find that

\[
\sigma_{\text{SIMD}} = \frac{(1 + 2\mu^{-2})}{(1 - \mu^{-2})} \frac{c_s^2}{2\pi G r} \approx 1.88 \frac{c_s^2}{2\pi G r},
\]

so that the limiting profile of BM94 (eq. [1]) exceeds the magnetic equilibrium value by the factor \( k_{\text{SIMD}} \approx 2 \pi (1/1.88) \approx 3.3^4 \). This feature of nonequilibrium column density profiles, accompanied by supersonic infall velocities, is common to several independent models of magnetized cloud collapse (FM93; CM94; BM94; Nakamura et al. 1995, 1998; Tomisaka 1996; Basu 1997), and is reminiscent of the spherical similarity solution found by Larson.

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3The leading term in the finite thickness correction to the magnetic tension is proportional to \( B_z^2 \nabla Z \), where \( B_z \) is the vertical component of the magnetic field, \( Z \) is the half-thickness, and \( \nabla \) is the gradient in the plane of the disk. This term exactly cancels out one of the terms in the total magnetic pressure, proportional to \( -\nabla (Z B_z^2) \). The canceled term is exactly \(-1/2 \) times the total magnetic pressure in the late supercritical phase, so that the value of \( \alpha_{M,P}/a_T \) calculated by Basu (1997) is exactly twice that estimated by Nakamura & Hanawa (1997). A detailed derivation of the thin-disk equations that were used by Basu (1997) can be found in CM93.

4Centrifugal support is very negligible at this stage (BM94; Basu 1997), and can be ignored when considering sources of support.
(1969) and Penston (1969) for the runaway collapse phase \((t < 0)\), in which the density profile exceeds the SIS by the factor 4.4. This solution has been extended to the accretion phase \((t > 0)\), when a central point mass exists, by Hunter (1977); we subsequently refer to the combined solution as the Larson-Penston-Hunter (LPH) solution. Given that there are an infinite number of spherical hydrodynamic similarity solutions, with the highly dynamic LPH solution and the static (at \(t = 0\)) Shu solution at two extremes (Hunter 1977; Whitworth & Summers 1985), it is remarkable that various numerical calculations show a convergence to the LPH solution near the cloud center (Larson 1969; Hunter 1977; Foster & Chevalier 1993). The convergence to the LPH solution has been justified recently by Hanawa & Nakayama (1997), who performed a stability analysis of various solutions. Given this history in the hydrodynamic case, it is (at least in retrospect) not surprising that the MHD models also converge to a dynamic similarity solution. However, the similarity solution in this case is quantitatively different from LPH due to the distinctive character of self-gravity in thin-disk geometry.

Basu (1997) used an approximate similarity solution (in which the self-similar profiles were predetermined by comparison with numerical simulations) to analyze the approach to self-similarity during \(t < 0\) in a thin-disk magnetic cloud. There has also been recent progress in finding exact solutions to the self-similar equations for a thin disk. Here, we utilize these solutions to gain further insight into the collapse properties both before and after protostar formation. Li & Shu (1997) found a similarity solution for \(t > 0\) which starts from a static SIMD profile (as given by eq. [10]) at \(t = 0\), and is similar in spirit to the spherical similarity solution of Shu (1977) for the collapse of a cloud that has a static SIS profile at \(t = 0\). In contrast, Saigo & Hanawa (1998, hereafter SH) found hydrodynamic similarity solutions which exhibit dynamic collapse for \(t < 0\); they also extended these solutions to \(t > 0\). The magnetic solution can be recovered from the hydrodynamic one using the scaling laws given above. Their model also includes rotation, which leads to a centrifugally-supported inner region when \(t > 0\), as we expect based on the arguments given in § 2.1. SH’s solutions all converge to a spatially uniform infall velocity and \(r^{-1}\) column density profile at \(t = 0\), and for \(t > 0\) have an inner region with \(r^{-1/2}\) profiles in infall velocity and column density, both characteristic of free-fall. The rotating models also have an innermost quasi-equilibrium region with zero infall velocity and \(r^{-1}\) column density profile. The latter profile for an equilibrium centrifugal disk can also be inferred from the mass-radius relation of equation (5). SH’s solutions are parametrized by varying values of the rotation parameter \(\omega = c_s j / G m\). Using equations (1) and (3), we find that the appropriate value for a rotating, magnetized core is

\[
\omega \approx \frac{c_s \Omega_b}{B_{\text{ref}} G^{1/2}}
\]
\[ \Omega_b = 0.26 \left( \frac{c_s}{10^{-14} \text{ rad s}^{-1}} \right) \left( \frac{30 \mu G}{B_{\text{ref}}} \right). \]  

Equation (11) shows that the typical rotating, magnetized core will be an extremely slow rotator. It is best fit by SH’s \( \omega = 0 \) model, since even their \( \omega = 0.1 \) solution agrees with the \( \omega = 0 \) solution to within a few percent in the column density, infall velocity and mass accretion rate. Although the nonrotating \( \omega = 0 \) solution cannot give the inner centrifugal disk size for \( t > 0 \), this quantity can be obtained from equation (5). The \( \omega = 0 \) solution yields a column density \( 3.61 c^2_{\text{eff}}/(2\pi G_{\text{eff}} r) \) and infall velocity \(-1.73 c_{\text{eff}} \) at \( t = 0 \), so that the scaled magnetic solution is

\[ \sigma = 3.61 \frac{(1 + 2 \mu^{-2})}{(1 - \mu^{-2})} \frac{c^2_s}{2\pi Gr} \approx 6.78 \frac{c^2_s}{2\pi Gr}, \]

\[ v_r = -1.73 (1 + 2 \mu^{-2})^{1/2} c_s \approx -2.09 c_s. \]

The latter term in each case represents the scaled magnetic solution when \( \mu = 2.1 \). The limiting profile of BM94 (eq. [1]) is within 10% of the SH value, but over 3 times the equilibrium value. Hence, the inner regions of magnetized thin disks (where a similarity solution is expected to be valid) appear to converge to the dynamic similarity solution of SH rather than the static (at \( t = 0 \)) solution of Li & Shu (1997) during the approach to protostar formation. The infall velocity of equation (13) agrees less well with the simulations in regions where the column density is in good agreement, but Basu (1997) points out that the infall velocity begins to converge to the self-similar value in a much smaller region (see his Figures 3a and 4b and associated discussion in § 5.1.2). The maximum value of the infall velocity \( (\approx -c_s) \) does not appear to have yet reached a terminal value in the BM94 model. This late convergence of the infall velocity, relative to the column density, has also been noted by Nakamura et al. (1995, 1998) In any case, a limiting infall velocity of \( v_r \approx -2c_s \) is in agreement with the limiting \( (t = 0^-) \) velocity estimated by Basu (1997) if the central magnetic force becomes significantly less than thermal-pressure force, and is also in agreement with the limiting infall velocity obtained in the thin-disk magnetic collapse calculations of Nakamura et al. (1995) and the two-dimensional magnetic simulation of Tomisaka (1996).

### 2.2.2. Expected Mass Accretion Rates

The late convergence of the infall velocity means that only the collapse of the innermost mass shells can be described by the similarity solution when extended to \( t > 0 \); later mass shells will fall in at a lower rate since they start out with lower infall velocities at \( t = 0 \).
(see further discussion in § 5.1.3 of Basu 1997). A time-dependent accretion rate has been confirmed by Ciolek & Königl (1998) for an initially subcritical magnetic cloud in which ambipolar diffusion is operative. Additionally, Tomisaka (1996) shows that an initially supercritical magnetic cloud with flux-freezing also yields a time-dependent accretion rate. This feature was also seen in the hydrodynamic collapse calculations of Hunter (1977) and Foster & Chevalier (1993). The likely outcome of a time-dependent accretion rate has also been discussed recently by Henriksen, André, & Bontemps (1997), who present a simplified hydrodynamic model for its development.

Given the reasonable fit of the SH solution, we estimate its predicted mass accretion rate for a magnetic cloud. SH find that \( \dot{m}(t < 0) = 6.27 \frac{c_{s,\text{eff}}^3}{G} \) and \( \dot{m}(t > 0) = 10.96 \frac{c_{\text{eff}}^3}{G} \) in their \( \omega = 0 \) solution. Therefore, scaling to the magnetic solution using \( \mu = 2.1 \) yields

\[
\dot{m}(t = 0^-) = 6.27 \frac{(1 + 2\mu^{-2})^{3/2}}{(1 - \mu^{-2})} \frac{c_s^3}{G} \approx 14 \frac{c_s^3}{G}
\]

just before protostar formation, and

\[
\dot{m}(t = 0^+) = 10.96 \frac{(1 + 2\mu^{-2})^{3/2}}{(1 - \mu^{-2})} \frac{c_s^3}{G} \approx 25 \frac{c_s^3}{G}
\]

just afterwards. The former value is in good agreement with the limiting value \( \dot{m}(t = 0^-) \approx 13 \frac{c_s^3}{G} \) estimated by Basu (1997), but the latter value could not be obtained from his model which only applied to \( t < 0 \). These values can be compared with the values \( \dot{m}(t = 0^-) = 29 \frac{c_s^3}{G} \) and \( \dot{m}(t = 0^+) = 47 \frac{c_s^3}{G} \) of the spherical hydrodynamic LPH solution.

Equations (14) and (15) represent the limiting forms for the inner (where \( B_z \gg B_\text{ref} \)) solution of BM94 if the contraction continues with flux-freezing at a terminal value \( \mu = 2.1 \), and also if full convergence to the scaled SH similarity solution is achieved. For comparison, if the solution converged to the SIMD profile implied by \( \mu = 2.1 \) at \( t = 0 \), the mass accretion rates would be \( \dot{m}(t = 0^-) = 0 \) and (using the model of Li & Shu 1997)

\[
\dot{m}(t = 0^+) = 1.05(1 + H_0)\frac{c_s^3}{G} \approx 2 \frac{c_s^3}{G},
\]

where \( 1 + H_0 = (1 + 2\mu^{-2})/(1 - \mu^{-2}) \approx 1.88 \) is the overdensity factor supported by magnetic fields (see eq.[10]). The \( t = 0^- \) limit of the BM94 simulation is better fit by the scaled SH solution; the nonequilibrium column density is in good agreement and the magnitude of the infall velocity is far above zero, although still a factor \( \sim 2 \) below the limiting SH value when the simulation ends. Despite the reasonable agreement of the BM94 simulation with the scaled SH solution for \( t = 0^- \), there is much important physics that is left out of a flux-frozen thin-disk similarity solution, as discussed below.

In reality, ambipolar diffusion plays an important role in the supercritical collapse phase. Basu (1997) showed that the mass-to-flux ratio increases as a weak power of the
column density during $t < 0$:

$$\mu \propto \sigma^\epsilon,$$

(16)

where $\epsilon \simeq 0.05$ when using the simplified ionization model of BM94. He also showed that this significantly reduces the level of magnetic support $a_M/|g_r| \propto \sigma^{-2\epsilon}$, so that the contraction of the initially near-equilibrium core becomes increasingly dynamic. However, the estimate of equation (14), which assumes constant $\mu$, is expected to be reasonably good since much of the innermost $1 M_\odot$ in the supercritical core is in regions with $\mu \simeq 2$. When $t > 0$, Ciolek & Königl (1998) have shown that ambipolar diffusion has an even more dramatic effect; an increased magnetic tension force in the vicinity of the newly formed protostar halts the inward advection of magnetic field lines and ions, even as the neutral particles tend to continue their infall. This results in a hydromagnetic shock at a larger radius where the ions and neutrals become better coupled (see also Contopoulos, Ciolek, & Königl [1998] for a semianalytic treatment). Ciolek & Königl (1998) find that $\dot{m}(t = 0^+) \simeq 6 c_s^3/G$. The discrepancy with the estimate on the far right hand side of equation (15) is accounted for by three factors: 1) the mean mass-to-flux ratio $\mu$ in their supercritical core is somewhat higher than in the model of BM94, 2) the infall velocity magnitude is $\sim c_s$ rather than $\sim 2 c_s$ when the central zone is converted to a sink cell, and 3) the retarding effect of the ambipolar diffusion induced hydromagnetic shock. In regard to point 2) above, Ciolek & Königl (1998) do find that the maximum value of the infall velocity is still increasing when a sink cell is introduced at a central density $n_{n,c} \approx 10^{11}$ cm$^{-3}$, when non-isothermal effects are expected to become significant. Hence, it has not yet reached a terminal value, as expected if full convergence to an isothermal similarity solution has occurred. This raises the interesting point that nature may not provide enough dynamic range between cloud core densities and the formation of a dense opaque core for the infall velocity to fully converge to that of the SH isothermal similarity solution.

The previous discussion highlights the important fact that increasingly better understanding of the innermost regions of star-forming cores will depend crucially on detailed understanding of the microphysics of the attachment of neutral particles to the magnetic field and also of the conditions under which opacity effects become important and a hydrostatic stellar core is formed. This will lead to increasingly more accurate estimates of $\dot{m}$ at and near $t = 0$.\footnote{A detailed understanding of the extent of magnetic coupling in the innermost region will also shed light on the possible dynamical role of magnetic fields in the centrifugal disk phase.} Based on current thin-disk MHD numerical solutions, we conclude that the solutions are converging to a dynamic thin-disk similarity solution (approximately a scaled SH solution), but that the departure from flux-freezing has two effects: 1) it accelerates the collapse somewhat during $t < 0$ by increasing the mass-to-flux...
ratio $\mu$ (Basu 1997), and 2) it significantly decelerates infall during $t > 0$ for mass shells that start collapse outside the hydromagnetic shock caused by rapid ambipolar diffusion in the innermost region (Ciolek & Königl 1998).

All in all, the detailed physics is too complicated to be described by a single similarity solution. The numerical simulations must guide us in the right direction. It is also worth remembering that the vertical equilibrium assumption of the thin-disk approximation is expected to break down in the vicinity of the protostar, where dynamical infall takes place. Here, the mass accretion will occur in the vertical direction as well, so that the thin-disk accretion rates, including the simulated value of Ciolek & Königl (1998), must be regarded as lower limits to the actual value. Indeed, Tomisaka (1996) found a mass accretion rate $\dot{m}(t = 0^+) \approx 40 \frac{c_s^3}{G}$ in his two-dimensional axisymmetric MHD calculation, very close to the LPH value. This simulation does include vertical infall, and the initial cloud is magnetically supercritical, unlike the model of Ciolek & Königl (1998). Furthermore, it does not include ambipolar diffusion, which acts to significantly retard the accretion rate at this stage. Altogether, we believe that the true accretion rate must lie somewhere between the Ciolek & Königl (1998) and Tomisaka (1996) values, with

$$\dot{m}(t = 0^+) \gtrsim 10 \frac{c_s^3}{G}$$

(17)

a safe assumption. In other words, one cannot pin down an exact value for the constant $m_0(0)$ due to the multiple complex phenomena involved, but all models imply that it is of order 10. Based on these models, we would further surmise that it falls within the range $10 - 20$.

All the hydrodynamic and MHD models do show that $\dot{m}$ decreases with time during $t > 0$. The lower rates are due to infall of mass shells which were moving slowly at $t = 0$, as discussed by Basu (1997). In the hydrodynamic case, Foster & Chevalier (1993) show that the collapse of Bonnor-Ebert spheres which have a long outer tail of mass in a distribution matching the SIS yield a mass accretion rate which approaches the value $\approx \frac{c_s^3}{G}$ implied by the Shu (1977) solution. This is due to the outer mass shells having very low infall velocities when the central point mass is formed. In the MHD case, a static (at $t = 0$) self-similar model (i.e., that of Li & Shu 1997) is less likely to predict with any certainty an accretion rate for the outer core material. This is due to the non-constancy of the mass-to-flux ratio in the outer core and the important effect of the background magnetic field $B_{\text{ref}}$ in that region (e.g., CM94; BM94; Basu 1997). Both effects are not included in the self-similar Li & Shu (1997) model. These effects also yield a density profile that is somewhat shallower than $r^{-2}$ in the outer core. Nevertheless, one may be able to treat an appropriate value from the Li & Shu (1997) model (applied to the outer core) as a lower limit to (but not the actual value of) the mass accretion rate from the outer core (see Tomisaka 1996; Ciolek &
The mass accretion rate can actually decrease even further if and when the core mass is exhausted. The mechanism for ultimately terminating the accretion, whether due to a limited mass supply or the effect of outflows from the YSO, remains an unsolved problem.

3. Constraints on Disk Evolution

A centrifugal disk that is formed in the manner described in § 2, with radius $r_c$ given by equation (5), will contain most of its matter in the outer disk. For typical parameters, less than 1% of the mass will fall directly onto the star. Hence, star formation requires that the bulk of the protostellar material accrete through an initially self-gravitating centrifugal disk. In order to accrete onto the central star, most mass shells must shed the bulk of their angular momentum.

Despite significant uncertainties in determining disk masses, or even the central star mass, various determinations yield YSO disk masses which are about an order of magnitude below the stellar mass (see reviews by Beckwith & Sargent 1993; Mundy 1997). This implies that the phase of a massive disk around a YSO is relatively short lived, and that an efficient process is available for transferring most of the system mass onto the central star.

The accretion may be driven by angular momentum loss from the outer disk via a disk wind (e.g., Königl 1989; Pelletier & Pudritz 1992). Alternatively, if outflows are generated within the innermost part of the disk, nearby or at the star-disk interface, then the outer mass shells must find some other means to shed their angular momentum. This can still be accomplished by internal redistribution of angular momentum within the disk. In the remainder of this paper, we assume that the disk evolution is driven by internal torques; either viscous or gravitational. Gravitational torques may dominate in the early stages, when the disk mass is greater than or comparable to the stellar mass. These can be driven by nonaxisymmetric spiral instabilities (e.g., Cassen et al. 1981; Heemskerk, Papaloizou, & Savonije 1992; Laughlin & Bodenheimer 1994; Laughlin & Rozyczka 1996), which may, if the instability is severe enough, also result in the formation of a secondary star, as argued by Adams, Ruden & Shu (1989). In contrast, viscous effects may induce a more gentle but steady accretion which will be the primary mover when the disk mass is less than the stellar mass. In the rest of this paper, we focus on the case of single-star formation. However, if disk instabilities lead to the formation of a close binary, the general feature of disk expansion discussed below should also be present in a circumbinary disk.

In either the gravitational or viscous case, the inward transport of mass also requires
that some mass move outward in order to carry the angular momentum of the system; therefore, the disk radius continually increases. The end result is that the disk radius increases dramatically, even though it contains progressively less mass in relation to the central star. This evolutionary scenario is consistent with the general tendency of any disk of a given angular momentum to achieve its lowest energy state when all of the mass is brought to \( r = 0 \) and an infinitesimal mass carries all of the angular momentum at an infinite radius (Lynden-Bell & Pringle 1974). Clearly, the observed YSO-disk systems have evolved significantly in this direction if they started out with the massive disks envisioned by collapse calculations. The recent time-dependent calculations of Hartmann et al. (1998) illustrate the radial expansion of viscously evolving disks.

We next proceed to derive an estimate for the disk radius \( r_d \) after it has undergone significant evolution due to internal torques. We let \( m = m_s + m_d \), where \( m_s \) is the mass of the star and \( m_d \) is the mass of the disk. If \( m_s \gg m_d \) at this late stage, as suggested by observations, then the Keplerian angular velocity in the disk is

\[
\Omega(r) = \frac{(Gm_s)^{1/2}}{r^{3/2}}.
\]

Consequently, if the disk of mass \( m_d \) contains essentially all the angular momentum \( J \), then its radius is

\[
r_d = \frac{\ell}{Gm_s} \left( \frac{J}{m_d} \right)^2,
\]

where \( \ell \) is a constant of order unity which depends on the assumed column density profile in the disk. For \( \sigma \propto r^{-p} \) \((p < 2)\), we find that \( \ell = \left(\frac{5/2 - p}{2 - p}\right)^2 \). We make contact with our collapse model by integrating equation (3) to yield the expected angular momentum

\[
J \approx \frac{1}{2} \frac{\Omega_b G^{1/2}}{B_{\text{ref}}} m^2.
\]

Combining equations (19) and (20), we find that

\[
r_d \simeq \frac{\ell}{4} \left( \frac{\Omega_b}{B_{\text{ref}}} \right)^2 \left( \frac{m}{m_s} \right) \left( \frac{m}{m_d} \right)^2 m.
\]

The factor \( \ell/4 \) will be of order unity for most plausible column density distributions, and will exactly equal 1 if \( \sigma \propto r^{-3/2} \), which is the inferred profile for the protosolar nebula (Weidenschilling 1977). In the following discussion we simply let \( \ell/4 = 1 \). For the case \( m \approx m_s \gg m_d \), and using equation (5), we find

\[
r_d \simeq \left( \frac{\Omega_b}{B_{\text{ref}}} \right)^2 \left( \frac{m}{m_d} \right)^2 m \simeq r_c \left( \frac{m}{m_d} \right)^2.
\]
Equation (22) illustrates the growth of the disk radius $r_d$ from the initial centrifugal value $r_c$ as mass is transferred from the disk to the star. Normalizing this equation to standard values yields

$$r_d \approx 1500 \left( \frac{\Omega_b}{10^{-14} \text{ rad s}^{-1}} \right)^2 \left( \frac{30 \mu G}{B_{\text{ref}}} \right)^2 \left( \frac{m}{1 M_\odot} \right) \left( \frac{m/m_d}{10} \right)^2 \text{AU.}$$  \hspace{1cm} (23)

Taken together, equations (5) and (22) bracket the possible radii for circumstellar disks. Collapse of a molecular cloud core tends to build up a centrifugal disk of size $r_c$, which then grows in size (even as it loses mass to the central star) towards the size $r_d$. The latter value should be treated as an upper limit; processes such as outflows will at least partially compromise the assumed conservation of mass and angular momentum and yield smaller final disk sizes.

4. Discussion

The general evolutionary picture that we have presented, in which an inner centrifugal disk is formed within a flattened nonequilibrium envelope (the remnant supercritical core), is consistent with several millimeter wave studies of protostellar environments. For example, Hayashi, Ohashi, & Miyama (1993) detected dynamical infall in an outer flattened envelope of radius $\sim 1400$ AU surrounding the T Tauri star HL Tau. The star is also observed to have an inner (presumably centrifugally supported) disk of radius $\lesssim 100$ AU (e.g., Beckwith et al. 1990; Lay et al. 1994; Mundy et al. 1996). Recent studies of the environment around the embedded protostars L1551-IRS 5 (Ohashi et al. 1996) and L1527-IRAS 04368+2557 (Ohashi et al. 1997a) also reveal evidence for dynamical infall in an envelope of size $\sim 1000 - 2000$ AU. L1551-IRS 5 is a well studied object and contains an inner disk of radius $\lesssim 100$ AU (Keene & Masson 1990; Lay et al. 1994; Looney, Mundy, & Welch 1997; the latter argue that the disk contains a binary system). These observations, particularly of HL Tau, imply that infall from the envelope continues even after the inner disk has undergone significant evolution. Hence, the transition from a inner disk of radius $r_c$ to one with radius approaching $r_d$ may not be a simple two-step process; the star can be accreting from the inner disk even as the disk is growing by dynamical infall from the envelope.

The estimated typical centrifugal radius $\sim 10$ AU in our model is a reassuringly low value if the observed low mass $\sim 100$ AU inner disks are believed to be the evolved (i.e., expanded) counterparts of these initial states. Normally, dense cores of density $\sim 10^4$ cm$^{-3}$ and rotation rate $\sim 10^{-14}$ rad s$^{-1}$ would, if they conserve angular momentum, yield centrifugal disks of radius $\sim 100$ AU. Hence, the observational constraints would mean
that these disks could hardly expand during accretion. However, magnetic braking enforces background rotation rates until much higher typical densities \((10^5 - 10^6 \text{ cm}^{-3})\), yielding typical centrifugal radii \(\sim 10 \text{ AU}\). We have shown that the expansion of such initial disks will still lead to final configurations of size \(\sim 1000 \text{ AU}\). This has also been elegantly demonstrated by the time-dependent calculations of Hartmann et al. (1998). Therefore, an explanation for the size of most YSO systems\(^6\) and our own solar system requires a mechanism to limit the disk expansion. Since the expansion is driven by the need to carry the angular momentum of the system, it will not be so dramatic if outflows can indeed carry away a significant amount of angular momentum from the outer disk, as suggested by Königl (1989). Another possibility is the photoevaporation of the outer disks (Shu, Johnstone, & Hollenbach 1993; Hollenbach et al. 1994). The latter process is likely effective in the Orion Nebula (Johnstone, Hollenbach, & Bally 1998), where the influence of the massive star \(\theta^1\) C may be truncating the outer disks of low-mass stars.

As shown in § 2.1, the initial centrifugal radius \(r_c\) is a simple consequence of the existing density and angular velocity profiles in the vicinity of the protostar as it is formed. Hence, it is useful to compare our value of \(r_c\) with that obtained in other environments. Terebey et al. (1984, hereafter TSC) have presented such a value under the assumption that the core is rigidly rotating and has the density profile of a singular isothermal sphere \(\rho_{\text{SIS}} = c_s^2/(2\pi G r^2)\). Magnetic braking is invoked to justify the rigid rotation at a background value \(\Omega_b\) at this late stage. In this case, TSC show that

\[
 r_c = \frac{\Omega_b^2 G^3 m^3}{16 c_s^8}.
\]  

(24)

This expression (even more than eq. [5]) is very sensitive to input parameters, but a typical value is

\[
 r_c = 39 \left(\frac{\Omega_b}{10^{-14} \text{ rad s}^{-1}}\right)^2 \left(\frac{0.2 \text{ km s}^{-1}}{c_s}\right)^8 \left(\frac{m}{1 M_\odot}\right)^3 \text{ AU}.
\]  

(25)

Hence, both equations (5) and (24) predict typical centrifugal radii \(\sim 10 \text{ AU}\) using different profiles of physical variables at \(t = 0\). However, equation (5) incorporates the magnetic field and is based on the result of a detailed calculation of the collapse phase \((t < 0)\). BM94 have shown that magnetic braking becomes relatively ineffective when a core becomes supercritical, so that a period of angular momentum conservation precedes the formation of a central protostar; differential rotation is established in the core during this time. The column density also exceeds the equilibrium value, as discussed in § 2.

\(^6\)A counterexample is the probable main-sequence star \(\beta\) Pic (and others of its class) which do have disks of size \(\sim 1000 \text{ AU}\).
An important difference between equations (5) and (24) is the rate at which the centrifugal disk is built up. The TSC disk is built up with radius proportional to $m^3$, whereas the centrifugal disk within an infalling magnetic envelope is built up with radius proportional to $m$. A relation very similar to equation (24), differing only by a multiplicative constant, would apply to the magnetized disk as well if it were rigidly rotating at $t = 0$. Figure 1 illustrates the growth of $r_c$ versus mass in both cases, using typical molecular cloud parameters. In the magnetic case, the disk grows more rapidly with mass early on, but eventually lags in growth rate due to the $m^3$ growth of the TSC disk. The growth of mass versus time (the mass accretion rate) also differs in the two models, as discussed in § 2. The TSC model assumes self-similarity for all mass shells, hence $\dot{m} = m_0 c_s^3 / G$ with $m_0 = 0.975$, as implied by the self-similar collapse model for a singular isothermal sphere (Shu 1977). The centrifugal disk in a realistic magnetized cloud grows due to a mass accretion rate $\dot{m} = m_0(t) c_s^3 / G$, where $m_0(0)$ is conservatively estimated here to be $\approx 10$ (see § 2.2.2). Since $m_0(t)$ will gradually decrease with time when $t > 0$, we can only use $m_0 \approx 10$ at early times.

This early phase of protostellar evolution, when the two values of $r_c$ differ the most, can be associated with the Class 0 phase (André, Ward-Thompson, & Barsony 1993). The age of these protostars is inferred to be only up to a few $\times 10^4$ yr. Circumstellar disks are not easily detected in this highly embedded phase. However, Pudritz et al. (1996) find indirect evidence for a disk around the prototypical Class 0 source VLA 1623 and place an upper limit to its radius of 175 AU. It is not clear what fraction of the flattened structure is a centrifugal disk. Like most Class 0 sources, VLA 1623 drives a powerful outflow. In fact, Class 0 objects tend to have the most powerful CO outflows (Bontemps et al. 1996; Bachiller 1996). If outflows are in fact powered by the presence of a centrifugal disk, we should expect that significant (though deeply embedded) disks exist in this early phase. However, a model that assumes rigid rotation at $t = 0$ predicts a very slow early growth of a disk. For example, using the TSC model with $t = 3 \times 10^4$ yr, $c_s = 0.2$ km s$^{-1}$, and $\Omega_b = 10^{-14}$ rad s$^{-1}$ yields a radius $r_c = 6.5 \times 10^{-3}$ AU $= 1.38 R_\odot$. This is less than a typical protostellar radius of $\approx 1.5 - 3 R_\odot$ (Stahler 1988), so that infall is still occurring directly onto the protostar; there is no centrifugal disk. The magnetic model presented here, with differential rotation (therefore higher rotation rate near $r = 0$) and a higher initial infall rate, in which we conservatively choose $m_0(0) = 10$, yields $r_c = 8.4$ AU at the same time. Aside from the additional parameter $B_{\text{ref}} = 30 \mu$G, all other parameters are the same as in the previous case. Hence, in this model, a centrifugal disk is predicted to exist in the early protostellar phase given standard molecular cloud input parameters.
5. Summary

Using the results of earlier numerical and semianalytic models, we have found a connection between ambient conditions in a magnetized molecular cloud and the size of disks around YSO’s. Our analytic expression (eq. [5]) relates the centrifugal radius to the mass via two fundamental parameters characterizing the ambient molecular cloud: the background rotation rate $\Omega_b$ and a background magnetic field strength $B_{\text{ref}}$. This represents an estimate for the initial disk radius formed from the collapse of a molecular cloud core. An upper limit to the disk radius after it has undergone significant evolution due to internal torques is given by equation (22); it is simply the initial value scaled by the square of the total system mass to the disk mass $(m/m_d)^2$ ($\approx 100$ according to observations).

We have reviewed a variety of numerical and semianalytic models which support the general view that a centrifugal disk is built up from an infalling envelope which begins dynamic collapse before a central protostar is formed at time $t = 0$. This leads to a mass accretion rate $\dot{m} = m_0(t)c_s^2/G$, where $m_0$ is likely $\gtrsim 10$ at $t = 0$, but subsequently decreases with time. This is due to the fact that convergence to self-similarity (especially in the infall velocity) takes place only in an innermost region near $r = 0$ and $t = 0$. The density profiles converge to self-similar values over a larger range. The inner regions of thin-disk MHD simulations are shown to be in good agreement (at least during $t < 0$) with hydrodynamic thin-disk similarity solutions found by Saigo & Hanawa (1998), when the latter are scaled in an appropriate manner.

We conclude that the detection of centrifugal disks around very young ($\sim 10^4$ yr) Class 0 protostars will support the view that a cloud core begins dynamic collapse and develops differential rotation before a central protostar is formed. The consequent high rotation rates in the deep interior and the rapid infall mean that an inner centrifugal disk is initially built up more rapidly than in equilibrium, rigidly rotating cores. Therefore, a significant centrifugal disk is present in the early Class 0 stage and can help to drive the strong outflows from these sources.

Our model places constraints on the size of circumstellar disks in the early and late stages of evolution, regardless of the internal processes which may drive their evolution. These constraints should aid interpretation of observations and act to sharpen our intuition for future numerical simulations of the accretion phase in rotating, magnetized cloud cores.

I thank Glenn Ciolek for many illuminating discussions. This work was supported by the Natural Sciences and Engineering Research Council of Canada.
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This preprint was prepared with the AAS \LaTeX{} macros v4.0.
Fig. 1.— A comparison of the growth of centrifugal radius $r_c$ versus mass $m$ for two cases. Solid line: a magnetized core with column density given by equation (1) and differential rotation given by equation (2) at $t = 0$. Dashed line: a singular isothermal sphere that is rigidly rotating at $t = 0$. Standard input parameters $c_s = 0.2 \text{ km s}^{-1}$ and $\Omega_b = 10^{-14} \text{ rad s}^{-1}$ are used in both cases, and $B_{\text{ref}} = 30 \mu \text{G}$ is also used for the magnetized case.