A Simple Theory for Signals Induced by a Point Charge
Moving in a Resistive Plate ChamberTh. Heubrandtner^a, B. Schnizer^a and H. Schöpf^a^a*Institut für Theoretische Physik, Technische Universität, Graz, Austria*

A simple model for computing the current through the pick-up strips of a resistive plate chamber (RPC) is developed; a simple approximate but quite accurate formula for this current is found. The model is an infinite plane condenser filled with two layers; one is vacuum and represents the gas gap. The other one represents the anode and is a dielectric with a relative dielectric constant between 2 and 4; its weak conductivity is neglected as it has been shown by Schöpf and Schnizer [3] that for the high values of the specific resistance of melamine-phenolic laminate or glass it has no influence on the signals so that the quasi-static approximation can be applied. It turns out that the influence of the dielectric on the signal is also small and may be neglected; so a formula derived for an empty condenser and giving the strip current as function of the distance between the anode and the cathode, of the width of the strip and of the distance between the strip and the trajectory of the charge is a good approximation to the real case. The deviations due to a gap between the strips are discussed.

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1 Introduction

In this paper a simple model is presented, which permits us to derive a simple formula for the current induced in a pick-up strip by a point charge moving with a given speed in a Resistive Plate Chamber (RPC) [1], [2]. This model is a plane condenser filled with two dielectric layers (see Fig.1). The layer just below the pick-up strips corresponds to the resistive anode; its relative dielectric constant is about 2 to 4 typical for melamine-phenolic laminate or glass. The lower layer represents the gas gap; its electric properties are those

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² Corresponding author. email: schnizer@itp.tu-graz.ac.at.

of the vacuum. Comparing our Fig.1 to Fig.1 of [2] shows that our model is roughly one half of a real RPC, which is obtained by inserting a conducting plane (then identified with the cathode of the model) in the symmetry plane of the RPC. We approximate the electron avalanche generated in the gas gap by a constant point charge moving with a given velocity $v \approx 5 \times 10^4$ m/s along a trajectory perpendicular to the electrodes. We know from an earlier investigation (cf. Fig.3 of [3]) of the corresponding transient problem that the low conductivity (of about 10^{-9} to 10^{-10} S/m) of the anode has negligible influence on the signal and that then the charge distribution on the electrodes can be calculated in a quasi-static approximation (s. discussion in sect.3 of [3]). So we calculate the potential in the condensor for a point charge at rest at position z_0 . From this we get the charge distribution on a continuous plate in place of the pick-up strips. The total charge q_s on a strip is calculated by integrating the charge distribution over a strip of the sheet; it is a function of z_0 . This is rendered time-dependent by assuming $z_0 = vt$. The current is the derivative $dq_s/dt = I$. It is obvious that we neglect the gaps between the pick-up strips. In this way we get the following simple formula for the current flowing through a pick-up strip of width w :

$$\begin{aligned} \frac{ID}{Qv} &:= J_{00}(x_0, w, D) = \\ &= \frac{1}{4} \frac{\sinh(\frac{\pi(x_0 - w/2)}{D})}{\cos(\frac{\pi z_0}{D}) + \cosh(\frac{\pi(x_0 - w/2)}{D})} - \frac{1}{4} \frac{\sinh(\frac{\pi(x_0 + w/2)}{D})}{\cos(\frac{\pi z_0}{D}) + \cosh(\frac{\pi(x_0 + w/2)}{D})}. \end{aligned} \quad (1)$$

This formula does not depend on the relative dielectric constant of the anode. So we find that both electric parameters of the anode have negligible influence on the signal as soon as we assume the value and the motion of the charge as given.

2 The model and its theory

The model is shown in Fig.1 and discussed in the introduction. The potential within the dielectric anode due to a point charge Q located at $(0, 0, z_0)$ in the gas gap is expressed as a Fourier integral:

$$\begin{aligned} \Phi(x, y, z; z_0) &= \\ &= \frac{Q}{\varepsilon_0} \left(\frac{1}{2\pi} \right)^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x dk_y \frac{2}{k} \frac{\sinh[k(D - z)] \sinh(kz_0) e^{ik_x x} e^{ik_y y}}{(\varepsilon + 1) \sinh(kD) + (\varepsilon - 1) \sinh[k(D - d/2)]}; \end{aligned} \quad (2)$$

with $k = \sqrt{k_x^2 + k_y^2}$. The total charge in a "pick-up strip" is obtained by integrating the charge density over the area of this strip, whose median line

Fig. 1. Relative position of anode strip and trajectory of point charge. Anode is shown in gray and has relative dielectric constant ε .

has a distance x_0 perpendicular to the straight trajectory of the charge Q :

$$q_s(x_0, w; z_0) = \varepsilon \int_{-\infty}^{\infty} dy \int_{x_0-w/2}^{x_0+w/2} dx \left. \frac{\partial \Phi(x, y, z; z_0)}{\partial z} \right|_{z=D}. \quad (3)$$

Inserting the slow motion $z_0 = vt$ of the charge Q into the above expression renders the charge of the strip time-dependent; then the current flowing through an anode strip is derived as $I = dq(x_0, w; vt)/dt$ and, after performing three of the integrations and changing the remaining variable of integration, it is expressed by the following integral:

$$\frac{ID}{Qv} = \frac{1}{2\pi} \int_0^{\infty} d\zeta \left[\sin\left(\zeta \frac{x_0 - w/2}{D}\right) - \sin\left(\zeta \frac{x_0 + w/2}{D}\right) \right] \frac{\cosh(\zeta z_0/D)}{\sinh(\zeta)} \times f(\varepsilon, d/D, \zeta). \quad (4)$$

with

$$f(\varepsilon, \frac{d}{D}, \zeta) = \frac{2\varepsilon}{(\varepsilon + 1) + (\varepsilon - 1) \sinh[\zeta(1 - 2d/D)] / \sinh(\zeta)}. \quad (5)$$

The factor $f(\varepsilon, d/D, \zeta)$ accounts for the dielectric layer. It equals unity if the relative dielectric constant $\varepsilon = 1$. Then the integral in (4) can be evaluated analytically to give eq.(1). If the factor f is not constant then the integrals in (4) must be evaluated in several steps. The factor is expanded into powers of $(\varepsilon - 1)/(\varepsilon + 1)$ and the ensuing series is integrated term by term. This gives:

$$\frac{ID}{Qv} = J_{00}(x_0, w, D) + \frac{\varepsilon - 1}{\varepsilon + 1} J_{00}(x_0, w, D) + \quad (6)$$

Fig. 2. a) Current through anode strip versus distance from trajectory of the charge. Gas gap width $D - d = 6mm$, dielectric layer width $d = .8mm$, relative dielectric constant $\varepsilon = 2$ (lines without dots) or 1 (lines with dots), anode strip width $w = 10mm$. $z_0 = 6mm$: — and $\cdots$; $z_0 = 4.5mm$: --- and $-\cdots-$; $z_0 = 0mm$: — — — and $-\cdots-\cdots-$. The black bar shows the extension of the pick-up strip. The gray bar reminds of the resistive plate. b) Same as a) but with $D - d = z_0 = 8mm$: — and $\cdots$; $D - d = z_0 = 4mm$: --- and $-\cdots-$; $D - d = z_0 = 2mm$: — — — and $-\cdots-\cdots-$.

$$+ \sum_{n=1}^{\infty} \left[\frac{\varepsilon - 1}{\varepsilon + 1} \right]^n \left[J_n \left(\frac{x_0 - w/2}{D}, \frac{z_0}{D}, 1 - \frac{2d}{D} \right) - J_n \left(\frac{x_0 + w/2}{D}, \frac{z_0}{D}, 1 - \frac{2d}{D} \right) \right].$$

with the integrals defined as ($0 \leq b < 1$, $0 \leq c < 1$)

$$J_n(a, b, c) := \frac{1}{2\pi} \int_0^{\infty} d\zeta \sin(\zeta a) \cosh(\zeta b) \sinh^n(\zeta c) / \sinh^{n+1}(\zeta). \quad (7)$$

These integrals can be evaluated analytically and expressed by trigonometric and hyperbolic functions and polynomials in a, b, c . The corresponding lengthy evaluations can be done by computer algebra [4] only and are described in [5]. For $n \geq 6$ they may be evaluated numerically. For lower values numeric integrations encountered trouble. As can be seen from Figs. 2 and 3, these accurate evaluations obtained by heavy efforts yield results, which differ only by a few percent (maximum 10 %) from the approximate results found from eq.(1).

3 Results, discussion and conclusions

As can be seen in Figs.2 and 3 the influence of the dielectric on the signal is quite small. Of course, the dielectric will change the static electric field strength prevailing in the gas gap, so influence the development and velocity of the electron avalanche. But these effects are not noticable in our approach as we assume the value of the charge and the velocity as given. When the charge is near to the interface between the gap and the anode, the current through the

pick-up strip has a pronounced minimum just inside the boundary of the strip; and a maximum just outside of this boundary. Note that these do not arise from the effects of a sharp edge as our model of a pick-up strip is a continuous sheet from which a strip is just taken by the limits of the integration. These extremes taper off quite fast when the distance of the charge from the interface increases. The signal strength decreases also quite fast with the distance of the trajectory from the pick-up strip. So the two strips enclosing the strip will be the only ones to receive appreciable signals.

In Figs.3 the current is shown for various positions of the charge along the trajectory for various trajectories. But here we must add a word of warning. In the simple model described above there is a continuous sheet without gap. As we find that the influence of the dielectric on the signal strength is quite small we started to investigate a two-dimensional configuration including a gap by conformal mapping. In Fig.4 we compare the signal strength of a line charge moving towards a continuous electrode to that due to a line charge moving towards an electrode with a gap. These results show that the simple model tends to overestimate the signals. In addition, for trajectories just inside or outside the edge the tendencies may be even opposite. But the positions of the current maxima and minima for trajectories just inside or outside the edge remain the same. These researches will be extended, refined and published in the near future.

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