Quantum field and uniformly accelerated oscillator

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Abstract

We present an exact treatment of the influences on a quantum scalar field in its Minkowski vacuum state induced by coupling of the field to a uniformly accelerated harmonic oscillator. We show that there are no radiation from the oscillator in the point of view of a uniformly accelerating observer. On the other hand, there are radiations in the point of view of an inertial observer. It is shown that Einstein-Podolsky-Rosen (EPR) like correlations of Rindler particles in Minkowski vacuum states are modified by a phase factor in front of the momentum-symmetric Rindler operators. The exact quantization of a time-dependent oscillator coupled to a massless scalar field was given.

Keywords: Particle detector model, uniform acceleration, Rindler space.

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I. INTRODUCTION

It is well known that the Minkowski vacuum state has spacelike pair-wise [Einstein-Podolsky-Rosen (EPR) type] correlations of Rindler particles between the left and the right Rindler wedge, which makes the Minkowski vacuum to be described by a thermal bath of Rindler particles [1]. In this respect, it is understood that a uniformly accelerating detector detects thermal radiation of the field. This was analyzed in more detail by Unruh and Wald [2] who discussed several apparently paradoxical aspects related to causality, energy considerations, and particle creation from the detector. The conclusion for the presence of the radiation by Unruh and Wald was criticized by Grove [3] who claimed that the appearance of particle is not a dynamical process but an artifact of the state reduction. Massar, Parentani, and Brout [4], and Hinterleitner [5] resume the discussions by use of a solvable model presented by Raine, Sciama, and Grove [6]: In the model of a harmonic oscillator coupled to a scalar quantum field they investigated the mean value of the energy-momentum tensor of the field. They concluded that a uniformly accelrated oscillator does not radiate.

This contradiction was discussed by Audretsch and Müller [7] using two level detector. They pointed out the difference between the stress tensor and the particle number and then
stressed the importance of the built-up correlations and the quantum measurement process.

Lau [8] also discussed the radiation from the oscillator and the disappearance of the EPR-correlations. However, in these literatures, the authors assume the coupling of the detector to the quantum field is weak. So, the back reaction to the field was not counted exactly, and the mechanism of particle creation is unclear. This is the reason why the previous literatures fail to provide all of non-perturbative aspects. In this paper, we focus on obtaining the exact vacuum to vacuum relations. We summarize the main results in this paper in Fig. (1), which describes the relations between several different vacua.
1), 4) The Minkowski vacuum state is a coherent state of Bogoliubov-Bardeen-Cooper-Schrieffer type, in which particles are pairwise correlated (a particle in the positive Rindler wedge are correlated with a particle in the left Rindler wedge.) [1,9]

2) The Rindler vacuum is the same with the instantaneous vacuum for a uniformly accelerating observer. There are only some phase shift for momentum symmetric states with non-zero number of particles.

3) The phase shift in 2) makes crucial differences for the vacuum structures between Minkowski and the instantaneous observers. There are net creation of particles from the oscillator.

The relation 1) is well known. In the present paper, we clarify the other three relations
in exact forms which need to calculate the back reaction of the oscillator to the field. In
due course, we define each ground state from the creation and annihilation operators and
find the relations between each sets of operators. During these calculations, we succeed in
quantizing the complete system with a time-dependent coupling constant, which is given as
an appendices.

The outline of this work is as follows. In Sec. II, we introduce our notation and briefly
sketch our calculational procedure. Section III is devoted to compute the particle creation
of from the oscillator and the EPR-correlations of the Minkowski vacuum. In Sec. IV,
we summarize and discuss the result. There are two appendices which are devoted (A) to
develope an exact quantization scheme for the coupled system in the respect of Rindler
observer and (B) to get the formal relations between the operators at initial and final time
in Rindler space.

II. DEFINITIONS, NOTATIONS, AND SKETCH OF PROCEDURE

The (1+1) dimensional Minkowski space can be devided into 4 regions in the respect of
a Rindler observers. Let us call each regions by Future (F), Past (P), Right (R, \( \sigma = + \)),
and Left (L, \( \sigma = - \)) quadrant of Rindler space. We parametrize the region R using \((\tau, \rho)\):

\[
x = \frac{1}{a} e^{\alpha \rho} \cosh a \tau, \quad t = \frac{1}{a} e^{\alpha \rho} \sinh a \tau,
\]

(1)
where \((t, x)\) is the Minkowski coordinate basis. L also can be parametrized by similar coordinates patch, but we do not write it explicitly in this paper.

The free field in (1+1) dimension is given by

\[
\phi^0(t, x) = \int dk [a_k U_k(t, x) + a_k^\dagger U_k^*(t, x)] \text{ in Minkowski space,}
\]

\[
= \sum_\sigma \int dk [b_k^{(\sigma)} u_k^{(\sigma)}(\tau, \rho) + b_k^{(\sigma)*\dagger} u_k^{(\sigma)*}(\tau, \rho)] \text{ in Rindler space,}
\]

where the mode solutions are

\[
u_k^{(\sigma)} = \frac{1}{\sqrt{4\pi|\omega|}} e^{-i\sigma \Omega \tau + i\kappa \rho} \quad \text{and} \quad U_k = \frac{1}{\sqrt{4\pi|\omega|}} e^{-i\omega t + ikx}.
\]

From now on we represent \(\omega = |k|\), and \(\Omega = |\kappa|\). The relation between \(a_k\), \(a_k^\dagger\), \(b_k^{(\sigma)}\), \(b_k^{(\sigma)^*}\) is given by Bogoliubov transformation

\[
b_k^{(\sigma)} = \int dk \left( a_k^{(\sigma)} a_k + b_k^{(\sigma)^\dagger} a_k^\dagger \right),
\]

which is simplified to be

\[
b_k^{(\sigma)} = \left[ 1 + N(\pi \Omega/a) \right]^{1/2} d_k^{(\sigma)} + N^{1/2}(\pi \Omega/a) d_k^{(-\sigma)^*}.
\]

Where \(N(\alpha) = \frac{1}{e^{2\alpha} - 1} \) and \(d_k^{(\sigma)} = \int_{-\infty}^{\infty} dk p_k^{(\sigma)}(k) a_k\). In Eq. (4), the function

\[
p_k^{(\sigma)}(k) = \frac{1}{\sqrt{4\pi a\Omega}} \left( \frac{a}{\omega} \right)^{-i\sigma \kappa / a} \quad \text{for} \quad \kappa k > 0,
\]

\[
= 0 \quad \text{for} \quad \kappa k \leq 0
\]

is complete and orthonormal in the following sense (see Ref. [9] for higher dimensions):
\[
\int_{-\infty}^{\infty} dk p^{(\sigma)\ast}_\kappa(k)p^{(\sigma')}_\kappa(k) = \delta_{\sigma\sigma'}\delta(\kappa - \kappa'),
\]
\[\sum_{\sigma} \int dk p^{(\sigma)\ast}_\kappa(k)p^{(\sigma)}_\kappa(k') = \delta(k - k').\]

Therefore, \(d^{(\sigma)\dagger}_\kappa\) creates and \(d^{(\sigma)}_\kappa\) annihilates a Minkowski particle and satisfies
\[
[d^{(\sigma)}_\kappa, d^{(\sigma')\dagger}_\kappa] = \delta_{\sigma,\sigma'}\delta(\kappa - \kappa'),
\]

Minkowski vacuum is defined by
\[
d^{(\sigma)}_\kappa|0\rangle_M = 0, \quad \sigma = \pm,
\]
and Rindler vacuum also is defined by
\[
b^{(\sigma)}_\kappa|0\rangle_R = 0.
\]

By solving Eq. (9) using Eq. (4) we get the well known non-local pair-wise correlation of the Minkowski vacuum with respect to an accelerated observer, which can be summarized by the following formulae:
\[
|0\rangle_M = \prod_{\kappa} (1 - r^2)^{1/2}\exp(rb^{(\sigma)\dagger}_\kappa b^{(-\sigma)\dagger}_\kappa)|0\rangle_R,
\]
where
\[
r = e^{-\pi\Omega/a}.
\]
Up to this point, we follow the beautiful review of Takagi [9]. In fact, we rederived these results because his calculation for $p_\kappa^{(\sigma)}(k)$ cannot be directly applicable to $1 + 1$ dimension.

Now let us consider a detector of oscillator $q(\tau)$ which is minimally coupled to a massless real scalar field $\phi(t, x)$ in two dimensions. This model is already discussed in Ref. [10], in which authors calculated the radiation from the oscillator with a time-varing coupling constant. The only difference of the model from other Refs. [5,4,11] are the time dependency of the coupling constant. The action of the system is

$$ S = \int dxdt \frac{1}{2} \left\{ \left( \frac{\partial}{\partial \tau} \phi(t, x) \right)^2 - \left( \frac{\partial}{\partial x} \phi(t, x) \right)^2 \right\}$$

$$+ \int d\rho d\tau \left\{ \frac{1}{2} m \left( \frac{d q(\tau)}{d \tau} \right)^2 - \frac{1}{2} m \omega_0^2 q^2(\tau) - e(\tau) q(\tau) \frac{d \phi}{d \tau}(t, x) \right\} \delta(\rho).$$

The oscillator follows the uniformly accelerated trajectory ($\rho = 0$) in R.

We decompose the field using mode solutions even in the presence of the oscillator. The mode solution outside the oscillator satisfies the free field equation. So we continually use $U_k$ and $u_\kappa^{(\sigma)}$ as the mode solutions in each regions. The instantaneous creation and annihilation operators depend on time and the mode expansion of the field is

$$ \phi(t, x) = \int dk [A_k(t) U_k(t, x) + A_k^*(t) U_k^*(t, x)] \text{ in Minkowski space,}$$

$$= \sum_\sigma \int dk [B_k^{(\sigma)}(\tau) u_\kappa^{(\sigma)}(\tau, \rho) + B_k^{(\sigma)*}(\tau) u_\kappa^{(\sigma)*}(\tau, \rho)] \text{ in Rindler space.}$$

The position operator of the oscillator is described by
\[ q(\tau) = \frac{a(\tau) + a^\dagger(\tau)}{\sqrt{2m\omega_0}}, \]  

(15)

where \( a(\tau), \ a^\dagger(\tau) \) are the time-dependent ladder operators of the oscillator.

We are to consider a system which evolves from the initial state that the scalar field and the oscillator are completely decoupled. Therefore, the vacuum \(|0\rangle_M\) for initial state is described by the direct product of the Minkowski vacuum and the ground state of the oscillator at \( \tau_0 \):

\[ a_k |0\rangle_M = 0 = a(\tau_0) |0\rangle_M. \]  

(16)

Where we use the same notation as the vacuum state without oscillator. Similarly, we define the initial Rindler vacuum state \(|0\rangle_R\) as

\[ b_k^{(\sigma)} |0\rangle_R = 0 = a(\tau_0) |0\rangle_R. \]  

(17)

The Fock space at time \( t \) of an inertial observer should be defined by the instantaneous eigenmodes. From (14) we define the instantaneous vacuum state \(|0\rangle_A\) to be

\[ A_k(t) |0\rangle_A = 0 = a(\tau) |0\rangle_A \text{ in Minkowski space}, \]  

(18)

where \( \tau \) is the propertime of the oscillator at the hypersurface given by \( t = \text{const} \). Similarly, we define the instantaneous vacuum of a uniformly accelerating observer at \( \tau \) as

\[ B_k^{(\sigma)}(\tau) |0\rangle_B = 0 = a(\tau) |0\rangle_B \text{ in Rindler space}. \]  

(19)
In this equation $B^{(\sigma)}(\tau)$ and $B^{(\sigma)\dagger}(\tau)$ can be obtained from the formula:

$$B^{(\sigma)}(\tau) = \int d\rho \ u^{(\sigma)*}(\tau, \rho)i\Pi(t, x) - \int d\rho \ \phi(t, x)i\partial_{\tau}u^{(\sigma)*}(\tau, \rho), \quad (20)$$

$$B^{(\sigma)\dagger}(\tau) = -\int d\rho \ u^{(\sigma)}(\tau, \rho)i\Pi(t, x) + \int d\rho \ \phi(t, x)i\partial_{\tau}u^{(\sigma)}(\tau, \rho),$$

where $\Pi$ is the conjugate momentum of $\phi$. With this form, $B^{(\sigma)}(\tau)$, $B^{(\sigma)\dagger}(\tau)$ automatically satisfy

$$[B^{(\sigma)}(\tau), B^{(\sigma)\dagger}(\tau)] = \delta_{\sigma \sigma'}\delta(\kappa - q), \quad (21)$$

$$[B^{(\sigma)}(\tau), a(\tau)] = [B^{(\sigma)}(\tau), a^{\dagger}(\tau)] = 0,$$

which allows a natural definition of Fock space as eigenstates of $B^{(\sigma)\dagger}B^{(\sigma)}$ and $a^{\dagger}a$.

From Eq. (14) we get the following relations between the two sets of the annihilation and the creation operators:

$$A_k = \int dk \left[ (1 + N)^{1/2}p^{(\sigma)}(k)B^{(\sigma)} - N^{1/2}p^{(\sigma )\dagger}(k)B^{(\sigma)\dagger} \right] \quad (22)$$

$$B^{(\sigma)} = (1 + N)^{1/2}D^{(\sigma)} + N^{1/2}D^{(-\sigma)\dagger} \quad (23)$$

where $D^{(\sigma)} = \int dk p^{(\sigma)}(k)A_k$. These relations are exactly the same with Eq. (4), which means the ground state of $A$ is a EPR-like correlated state of Rindler particles created by $B^{(\sigma)}\dagger$.

What we would like to know is the inter-relations between the instantaneous operators.
$B^{(\sigma)}_k$, $a(\tau)$, $A_k$ and the initial operators $a_k, b^{(\sigma)}_k, a(\tau_0)$. Note that $B^{(\sigma)}_k$, $A_k$, and $a(\tau)$ depend on time but $a_k$ and $b^{(\sigma)}_k$ are independent of time. So we set the initial value of $B^{(\sigma)}_k$, $A_k$ as $b^{(\sigma)}_k$, $a_k$. The time evolution of the system is completely determined by the relations between these two sets of operators $[A_k, B^{(\sigma)}_k, a(\tau); a_k, b^{(\sigma)}_k, a(\tau_0)]$ in appendix A. In appendix B, we calculate the asymptotic limit of the system by taking $\tau_0 \to -\infty$ and $e(\tau) = e$. In fact, we only use the final result of appendix B (74), which relates the in and out creation and annihilation operators in Rindler frame. The details which leads to (74) is not relevant for later discussions.

For later convinence, we define the momentum symmetric and asymmetric creation and annihilation operators

$$a_{\omega \pm} = \frac{a_\omega \pm a_{-\omega}}{\sqrt{2}},$$

$$b^{(\sigma)}_{\Omega \pm} = \frac{b^{(\sigma)}_{\Omega} \pm b^{(\sigma)}_{-\Omega}}{\sqrt{2}}.$$  

We use similar notations for other operators.

**III. PARTICLE CREATION BY ACCELERATING OSCILLATOR**

We now consider the system in the asymptotic limit $\tau_0 \to -\infty$. We set the initial state at $\tau = \tau_0$ to be the direct product of Minkowski vacuum and the $n^{th}$ state of the oscillator.
In this respect, we regard the instantaneous ground state (18) and (19) as the out vacuum state for the inertial observers and the uniformly accelerating observers each other.

A. Uniformly accelerating observer point of view

At first, let us consider the system in the point of view of the uniformly accelerating observers. The transformation (74) is most simple when we use the momentum symmetric and asymmetric creation and annihilation operators:

\[
B^{(+)}_{\Omega+}(\tau) = P(\Omega) b^{(+)}_{\Omega+}, \quad B^{(-)}_{\Omega-}(\tau) = b^{(-)}_{\Omega-}, \quad B^{(\pm)}_{\Omega\pm}(\tau) = b^{(\pm)}_{\Omega\pm},
\]

where \( P(\Omega) = \frac{\chi(\Omega)}{\chi^*(\Omega)} \). The interaction modifies only the phase of the symmetric operators. In the case of a creation operator of a momentum eigenstate, there occurs relative phase shift due to the phase factor \( P \) in Eq. (25). The maximal phase shift occurs at \( \Omega = \omega_0 \), which is the resonance frequency of the oscillator. In Eq. (25), there is no mixing of the creation and the annihilation operators. Therefore, the two ground states in Eqs. (65) and (19) are the same for the scalar field.

From Eqs. (23) and (25) we get

\[
P(\Omega) B^{(+)}_{\Omega+} = (1 + N)^{1/2} d^{(+)}_{\Omega+} + N^{1/2} d^{(-)\dagger}_{\Omega+},
\]

\[
B^{(-)}_{\Omega+} = (1 + N)^{1/2} d^{(-)}_{\Omega+} + N^{1/2} d^{(+)\dagger}_{\Omega+}.
\]
By solving Eq. (16) using (26) we get the relation between the initial state and the out
vacuum state:

\[ |0\rangle_M = \prod_{\Omega}(1 - r^2) \exp \left[ r \left( P(\Omega)B^{(+)}_{\Omega+}B^{(-)}_{\Omega+} + B^{(+)}_{\Omega-}B^{(-)}_{\Omega-} \right) \right] |0\rangle_B, \tag{27} \]

where \( r \) is given by Eq. (12). The oscillator does not influence on the field but is determined
completely by the symmetric mode of the field [See Eq. (72)]. The number of Rindler
particles in the initial state (27) is

\[ \langle B^{(+)}_\kappa B^{(+)}_q \rangle_M = N \left( \frac{\Omega}{\alpha} \right) \delta(\kappa - q) = \langle b^{(+)}_\kappa b^{(+)}_q \rangle_M \tag{28} \]

This result vividly shows there are no additional particle creation from the oscillator in
the point of view of a uniformly accelerating observer. We claim stringent constraint for
the particle creation that there are no additional creation of particle from the oscillator for
every initial states of the oscillator and the field. The proof for this assertion is simple
because \( B^{(+)}_{\Omega+} B^{(+)}_{\Omega'} = b^{(+)}_{\Omega+} b^{(+)}_{\Omega'} \) is an operator level identity for the momentum symmetric
or asymmetric particles. On the other hand, the transition amplitude for different number
states of symmetric particles acquires the phase factor.

**B. Inertial observer point of view**

Next, let us consider the same system in the inertial observers point of view.
There are delta function type contribution along the past event horizon of the oscillator 
\((t + x = 0)\) but it depends on the initial condition of the oscillator and commutes with \(B_\kappa^{(\sigma)}\).

Therefore we ignore it in the following calculations. From Eqs. (23) and (26), we get the following relations:

\[
D_{\Omega+}^{(+)} = [P + N(P - 1)]d_{\Omega+}^{(+)} + \sqrt{N(1 + N)}(P - 1)d_{\Omega+}^{(-)\dagger},
\]

\[
D_{\Omega+}^{(-)\dagger} = \sqrt{N(1 + N)}(1 - P)d_{\Omega+}^{(+)} + [1 + N(1 - P)]d_{\Omega+}^{(-)\dagger}.
\]

We solve Eq. (16) by use of (29), then we get the instantaneous states representation of the initial state in Minkowski space:

\[
|0\rangle_M = \prod_{\Omega}(1 - |R|^2)^{1/2}\exp\left(RE^{(+)\dagger}D_{\Omega+}^{(-)\dagger}\right)|0\rangle_A
\]

where

\[
R = \frac{4iem\Omega\chi(\Omega)\sqrt{N(1 + N)}}{1 - 4iem\Omega\chi(\Omega)N}.
\]

Its size is

\[
|R|^2 = \frac{16m^2e^2\Omega^2|\chi(\Omega)|^2N(N + 1)}{1 + 16m^2e^2\Omega^2|\chi(\Omega)|^2N(N + 1)} = \frac{4m^2e^2\Omega^2|\chi(\Omega)|^2}{\sinh^2(\pi\Omega/a) + 4m^2e^2\Omega^2|\chi(\Omega)|^2}.
\]

The presence of oscillator results in the mixing of the Minkowski annihilation and creation operators, so, there are net creations of particles from the oscillator. The number of particles created by the oscillator in unit time and space is
\[ N = M \langle 0 | \int dk A_k^\dagger A_k | 0 \rangle_M = M \langle 0 | \sum_\sigma \int d\Omega D^{(\sigma)}_\Omega \dagger D^{(\sigma)}_\Omega | 0 \rangle_M \]

\[ = 8m^2\epsilon^2 \int_0^{\infty} d\Omega \Omega^2 N(N + 1) |\chi(\Omega)|^2. \]

\[ = 2m^2\epsilon^2 \int_0^{\infty} d\Omega \Omega^2 \frac{|\chi(\Omega)|^2}{\sinh^2 (\pi \Omega/a)} \]

As can be seen in Eq. (33) the number of created particles vanishes for zero acceleration and monotonically increases according to the acceleration. We explicitly integrate the integration in Eq. (33) using the contour in Fig. (2) after changing the integration range \( \int_0^{\infty} \) to \( \frac{1}{2} \int_{-\infty}^{\infty} \).

Fig. (2). contour in complex \( \Omega \) plane. There are two kinds of poles which should be considered. First is poles given by the properties of the oscillator which is located at \( \Omega = \pm \omega_0 \pm i\epsilon \) and second is the reaction due to acceleration which are located at \( \Omega = ina \) along the imaginary axis. Here \( n \) is an integer.

The number of created particles from the uniformly accelerated oscillator is
\[ \mathcal{N} = \frac{\pi \epsilon}{\omega_0} m^2 Re \left[ \frac{\omega_0 + i \epsilon}{\sinh^2 \{ \pi (\omega_0 + i \epsilon)/a \}} \right] + \frac{8 \epsilon^2 a^3}{\pi} \sum_{n=1}^{\infty} n m^4 |\chi(\Omega)|^4 |(na)^4 - \omega_0^4| \right]. \quad (34) \]

Note that \( \mathcal{N} \) is of order \( O(\epsilon^2) \) and is monotonically increasing function of \( a \). Each term in Eq. (34) is positive definite for \( a > 0 \).

IV. SUMMARY AND DISCUSSIONS

We have obtained the vacuum to vacuum relations of the a massless scalar field interacting with a harmonic oscillator both for the inertial and the uniformly accelerating observers. There are four kinds of vacuums each are related with the inertial observers (M) or the uniformly accelerating observers (R) and the in (I) or out (O) Fock space. We set the initial vacuum state to be (MI) the direct product of the Minkowski vacuum and the ground state of the oscillator.

The relation between the MI and the RO vacuums is modified by the phase factor attached at the creation operator of Rindler particle \( (B_{\Omega L}^{(+)}_{\Omega L}) \). The phase factor does not give any noticeable change to physical observables localized in R or L of Rindler wedge. We present two comments in order: First, there is no radiation of particles to all order of \( \epsilon \) when viewed by an uniformly accelerating observer. Second, the final state of the oscillator is completely determined by the field [See Eq. (72)]. If we set the acceleration of the oscillator to be zero, we get the following uncertainty relation of the oscillator:
\[ \langle \Delta p \Delta q \rangle^2 = \frac{\omega_0^2 (\pi - \xi)}{4\pi^2 \omega_f^2} \left[ (\pi - \xi) - \frac{2\epsilon \Omega_f}{\omega_0^2} \left( \gamma + \ln \frac{\omega_0}{\Gamma} \right) \right], \quad (35) \]

where \( \Gamma \) is the UV cutoff which is introduced in Ref. [10] and \( \gamma \) is Euler constant. Note that the uncertainty go to \( \frac{1}{2} \) when \( \epsilon \to 0 \). This is consistent with Anglin, Laflamme, Zurek, and Paz [12], where they showed that the final state of the oscillator is the ground state if the coupling is extremally small.

In the point of view of an inertial observer, there are radiated particles with its number given by Eq. (34). The number of the radiated particles is of order \( e^2 \) for small \( e \) and vanishes for \( a = 0 \). Therefore, these particles are created by the acceleration of the oscillator. In fact, there are another particles created by the oscillator which we neglected in this paper. As pointed out by Unruh [13] there is delta function like radiation along the past lightcone of the oscillator. This term only depends on \( a(\tau_0), a^\dagger(\tau_0) \) which commute with \( B_{\kappa}^{(\sigma)} \), and \( A_k \). So we can justify the ignorance of it in this paper.

Another point to note is that there is symmetries on the Minkowski vacuum (27) and (30) for the pair interchanges:

\[
\left( B^{(\sigma)\dagger}_{\Omega^+}, B^{(\sigma)\dagger}_{\Omega^-} \right) \leftrightarrow \left( B^{(-\sigma)\dagger}_{\Omega^+}, B^{(-\sigma)\dagger}_{\Omega^-} \right), \quad (36)
\]

\[
\left( B^{(+)}_{\Omega^+}, B^{(-)}_{\Omega^-} \right) \leftrightarrow \left( \frac{\chi^*(\Omega)}{\chi(\Omega)} B^{(+)}_{\Omega^-}, B^{(-)}_{\Omega^-} \right), \quad (37)
\]

These are the modified symmetries which represent the EPR like correlations of Minkowski
vacuum viewed by a uniformly accelerated observer. If the oscillator decouple with the field, the phase factor $\chi^*/\chi$ in Eq. (37) becomes 1 and Eqs. (36) and (37) reproduce complete EPR correlation.

A new non-trivial fact, from Eq. (30), is that there remain symmetries

$$D_{n+}^{(\sigma)\dagger} \leftrightarrow D_{n+}^{(-\sigma)\dagger}.$$  \hspace{1cm} (38)

to an inertial observer. This symmetries leave correlations between the created Minkowski particles in pair.
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VI. APPENDIX A: QUANTIZATION AND TIME EVOLUTION OF THE COUPLED SYSTEM

The uniformly accelerated oscillator always stay in R and its motion is simplest in this coordinates patch. Therefore, in this appendix, we only consider the system in R.

Varying the action (13) with respect to $\phi(t,x)$ and $q(\tau)$, we get the Heisenberg equation of motion for the field and the oscillator

$$\square \phi(t,x) = \frac{dc(\tau)q(\tau)}{d\tau} \delta(\rho),$$  \hspace{1cm} (39)

$$m \left( \frac{d}{d\tau} \right)^2 q(\tau) + m\omega_0^2 q(\tau) = -e(\tau) \frac{d\phi}{d\tau}(t(\tau), x(\tau)),$$  \hspace{1cm} (40)

The formal solution to Eq. (39), by using the two-dimensional Green's function, is

$$\phi(t,x) = \phi_0(t,x) + \frac{e(\tau_{ret})}{2} q(\tau_{ret}),$$  \hspace{1cm} (41)

where $\tau_{ret}$ is the value of $\tau$ at the intersection of the past lightcone of $(t,x)$ and the trajectory of the oscillator. After the substitution of the solution (41) into Eq. (40) we get
\[ \ddot{q}(\tau) + \frac{d \ln M(\tau)}{d \tau} \dot{q}(\tau) + \omega^2(\tau)q(\tau) = F(\tau). \]  

In this equation

\[ M(\tau) = m \exp \left( 2 \int_{\tau_0}^{\tau} \epsilon(\tau) d\tau \right), \]  

\[ \omega^2(\tau) = \omega_0^2 + \frac{e^2(\tau)}{4m}, \]  

\[ F(\tau) = -\frac{e(\tau)}{m} \frac{d \phi_0}{d \tau} (t(\tau), x(\tau)), \]

where \( \epsilon(\tau) = e^2(\tau)/(4m) \). Eq. (42) is the equation of motion of a damping harmonic oscillator subject to an external force \( F(\tau) \).

Now let us find \( B_Q(t) \) which satisfies the commutation relation

\[ [B_Q(\tau), B_Q^\dagger(\tau)] = 1, \quad [B_Q(\tau), b_\kappa^{(+)}] = [B_Q(\tau), b_\kappa^{(+)}] = 0, \]  

and satisfies the first order differential equation \(^1\)

\[ \frac{d}{d \tau} B_Q(\tau) = -i \frac{\omega_l}{M(\tau)g_-(\tau)} B_Q(\tau), \]

whose solution is

\[ B_Q(\tau) = e^{-i\Theta(\tau)} B_Q(\tau_0), \]

\(^1\)This equation was found in Ref. [14], where the authors quantized the time-dependent-forced harmonic oscillator.
where $\Theta(\tau) \equiv \int d\tau \frac{\omega_I}{M(\tau) g_-(\tau)}$ and $\omega_I^2 \equiv g_+ g_- - g_0^2$ is invariant under time evolution. The function $g_i$’s are defined in Refs. [14,15] are given by

$$g_-(\tau) \equiv f(\tau) f^*(\tau), \quad g_0(\tau) \equiv -\frac{M(\tau)}{2} \dot{g}_-(\tau), \quad g_+(\tau) \equiv M^2(\tau) |\dot{f}(\tau)|^2,$$

(49)

where $f(\tau)$ is a homogeneous solution of Eq. (42). The explicit form of $B_Q(\tau)$ can be found by setting

$$B_Q(\tau) \equiv b(\tau) + e^{-i\Theta(\tau)} \beta(\tau),$$

(50)

and then by finding a differential equation which is satisfied with $\beta(\tau)$:

$$\frac{d}{d\tau} \beta(\tau) = -i \sqrt{\frac{g_-(\tau)}{2\omega_I}} M(\tau) F(\tau).$$

(51)

Here $b(\tau) = \sqrt{\frac{g_+(\tau)}{2\omega_I}} e^{i\xi(\tau)} q(\tau) + i \sqrt{\frac{g_-(\tau)}{2\omega_I}} \frac{M(\tau)}{m} p(\tau)$ is the annihilation operator of an oscillator [15] and

$$\tan \xi(\tau) \equiv \frac{g_0(\tau)}{\omega_I}.$$

(52)

By explicitly integrating Eq. (51) one get

$$\beta(\tau) = \beta_{Q0} + \sqrt{\frac{2\omega_I}{g_-(\tau)}} \int_0^\infty d\Omega \sqrt{2} \left[ b^{(+)}_{\Omega+} g^*(-\Omega, \tau) - b^{(+)*}_{\Omega+} g^*(\Omega, \tau) \right],$$

(53)

where $\beta_{Q0}$ is a constant operator which should be fixed by the initial condition of $B_Q(\tau)$ and $g(\Omega, \tau)$ is
\begin{align*}
g(\Omega, \tau) &= \frac{g_-(\tau)}{2m \omega_I} \int_{\tau_0}^{\tau} \mathrm{d}\tau' \sqrt{g_-(\tau')} M(\tau') e(\tau') e^{-i\Theta(\tau')} u_{\Omega}(\tau', 0). \\
\text{(54)}
\end{align*}

Let us set the invariant operators to describe the initial state of the oscillator at \( \tau_0 \):

\begin{align*}
B_Q(\tau_0) &= a(\tau_0) = \sqrt{\frac{m \omega_0}{2}} q(\tau_0) + i \sqrt{\frac{1}{2m \omega_0}} p(\tau_0). \\
\text{(55)}
\end{align*}

From the requirement (55) we determine the unknown constant operator in Eq. (53)

\begin{align*}
\beta_{Q_0} &= (1 - c_1) e^{i\Theta(\tau)} B_Q(\tau) - c_2 e^{-i\Theta(\tau)} B_Q^\dagger(\tau), \\
\text{(56)}
\end{align*}

where

\begin{align*}
c_1 &= \frac{1}{2} \sqrt{\frac{g_+(\tau_0)}{m \omega_0 \omega_I}} \left[ e^{i \xi(\tau_0)} - m \omega_0 \left( \frac{g_- (\tau) }{g_+ (\tau_0)} \right) \right], \\
\text{(57)}

c_2 &= \frac{1}{2} \sqrt{\frac{g_+(\tau_0)}{m \omega_0 \omega_I}} \left[ e^{i \xi(\tau_0)} + m \omega_0 \left( \frac{g_- (\tau) }{g_+ (\tau_0)} \right) \right]. \\
\text{(58)}
\end{align*}

Now we finally find

\begin{align*}
\bar{B}_Q(\tau) &= \sqrt{\frac{m}{M(\tau)}} e^{i\Theta} B_Q(\tau) \\
&= \alpha \sqrt{\frac{mg_+(\tau)}{2M(\tau) \omega_I}} q(\tau) + i \beta \sqrt{\frac{M(\tau) g_- (\tau)}{2m \omega_0}} p(\tau) \\
&+ \sqrt{\frac{2m \omega_I}{M(\tau) g_- (\tau)}} \int d\Omega \sqrt{2} \left[ b_{\Omega^+}^{(+)} G_1(\Omega, \tau) - b_{\Omega^-}^{(+)} G_1(-\Omega, \tau) \right] \\
\text{(59)}
\end{align*}

where \( \alpha = c_1^* e^{i(\Theta + \xi(\tau))} + c_2 e^{-i(\Theta + \xi(\tau))} \), \( \beta = c_1^* e^{i\Theta(\tau)} - c_2 e^{-i\Theta(\tau)} \), and

\begin{align*}
G_1(\Omega, \tau) &= c_1^* g^*(-\Omega, \tau) + c_2 g(\Omega, \tau). \\
\text{(60)}
\end{align*}
By inverting Eq. (59) one get the time evolution of the oscillator

\[ q(\tau) = q_O(\tau) + q_F(\tau) \]

\[ = \sqrt{\frac{g(\tau)}{2\omega_I}} \left( c_1 + c_2 e^{2i\Theta(\tau)} \right) B_Q(\tau) + B_F(\tau) + \text{H.C.}, \]

where \( q_F(\tau) = B_F(\tau) + B_F^\dagger(\tau) \) and

\[ B_F(\tau) = \int_0^\infty d\Omega \sqrt{2} G_-(\Omega, \tau) b_{\Omega+}^{(\dagger)}. \]

In this equation,

\[ G_-(\Omega, \tau) = e^{i\Theta(\tau)} g(\Omega, \tau) - e^{-i\Theta(\tau)} g^*(\Omega, \tau) \]

is a classical solution of the forced harmonic oscillator equation (42) with \( F(\tau) = -\frac{ie(\tau)}{m} u_\Omega^{(\dagger)}(\tau, 0) \) and its initial conditions are

\[ G_-(\Omega, \tau_0) = 0, \quad \dot{G}_-(\Omega, \tau_0) = 0. \]

If the coupling does not vary, \( B_Q \) in Eq. (59) asymptotically vanishes because \( M(\tau) \) increase exponentially. So, the dynamics of the oscillator is completely determined by the field asymptotically.

The commutation relations (46) allow the natural definition of the Fock space as eigenstates of \( B_Q^\dagger B_Q \) and \( b_k^{(\sigma)} b_k^{(\sigma)} \). The eigenvalues of these operators are invariant under time evolution of the system, so we usually call them invariants. The ground state \( |0\rangle_R \) is
\[ b^{(\sigma)}_\kappa |0\rangle_R = B_Q(\tau) |0\rangle_R = 0. \]  

In fact, \(|0\rangle_R\) is the Rindler vacuum state at \(\tau_0\) because \(B_Q(\tau)\) are simply proportional to \(a(\tau_0)\) and \(b^{(\sigma)}_\kappa\) are invariant on time.

By inserting Eqs. (41) and (61) into (20) we obtain the relations between the two sets of operators. \(B^{(-)}_\kappa\) is not affected by the presence of the oscillator and \(B^{(+)}_\kappa\) is given by

\[ B^{(+)}_\kappa(\tau) = b^{(+)}_\kappa(\tau) + \Omega \int_{\tau_0}^{\tau} u^{(+)*}_\kappa(\tau', 0) e(\tau') q(\tau') d\tau' \]

\[ = b^{(+)}_\kappa + \Omega \int_{\tau_0}^{\tau} u^{(+)*}_\kappa(\tau', 0) e(\tau') q(\tau') d\tau' \]

\[ + \Omega \int d\Omega' \Omega' \tilde{G}(\Omega, \Omega', t) \sqrt{2} b^{(+)}_{\Omega'} + \Omega \int d\Omega' \Omega' \tilde{G}^*(-\Omega, \Omega', \tau) \sqrt{2} b^{(+)}_{\Omega'}, \]

where

\[ \tilde{G}(\Omega, \Omega', \tau) = \int_{\omega}^{\tau} d\tau' e(\tau') u^{(+)}_{\Omega}(\tau', 0) G_{-}(\Omega', \tau'). \]  

From Eq. (59) we get the instantaneous annihilation operator of the oscillator

\[ a(\tau) = \sqrt{\frac{mg_-(\tau) \omega_0}{4 \omega_I}} \left[ \left\{ (c_1 + c_2 e^{2i\Theta}) B_Q(\tau) + H.C. \right\} \right. \]

\[ + \frac{\omega_I}{M(\tau) g_-(\tau) \omega_0} \left\{ (c_1 - c_2^* e^{2i\Theta}) B_Q(\tau) + H.C. \right\} \]

\[ + \sqrt{\frac{m \omega_0}{2}} \int d\Omega \Omega \left\{ b^{(+)}_{\Omega_1} G_-(\Omega, \tau) + b^{(+)}_{\Omega_1} G^*_-(\Omega, \tau) \right\} - \frac{\omega_I}{M(\tau) g_-(\tau) \omega_0} \int d\Omega \Omega \left\{ b^{(+)}_{\Omega_1} G_+(\Omega, \tau) - b^{(+)}_{\Omega_1} G^*_+(\Omega, \tau) \right\}, \]

where \(G_+(\Omega, \tau) = e^{i\Theta(\tau)} g(\Omega, \tau) + e^{-i\Theta(\tau)} g^*(-\Omega, \tau). \)
We are interested in the final state of the system, which can be obtained by taking the asymptotic limit of a constant coupling. In this paper, the asymptotic limit is to take \( \tau_0 \rightarrow -\infty \). From now on, we set \( e(\tau) = e \).

The homogeneous solution of the oscillator decays to zero:

\[
q_0(\tau) = \left[ q(\tau_0) \frac{\omega_0}{\omega_I} \cos\{\omega_I(\tau - \tau_0) - \theta\} + \frac{p(\tau_0)}{m\omega_I} \sin \omega_I(\tau - \tau_0) \right] e^{-\epsilon(\tau - \tau_0)}. \tag{69}
\]

The inhomogeneous solution is

\[
G_-(\Omega, \tau) = i\epsilon \chi(\Omega) u_\Omega^{(+)}(\tau) \tag{70}
\]

\[
+ i\epsilon \chi(\Omega) u_\Omega^{(+)}(\tau_0) e^{-\epsilon(\tau - \tau_0)} \left[ -\cos[\Omega I(\tau - \tau_0) - \theta] - i \frac{\Omega}{\omega_I} \sin \omega_I(\tau - \tau_0) \right],
\]

where

\[
\chi(\Omega) = \frac{1}{m[\omega_0^2 - \Omega^2 - 2i\epsilon \Omega]} \tag{71}
\]

The first term of Eq. (70) vanishes asymptotically. From now on, we ignore terms which vanishes for the limit \( \tau_0 \rightarrow -\infty \). The asymptotic form of \( a(\tau) \) is

\[
a(\tau) = i\epsilon \sqrt{m\omega_0} \left\{ \frac{\omega_0 + i\epsilon}{\omega_0} \int d\Omega \Omega \left[ u_\Omega^{(+)}(\tau, 0) \chi(\Omega) b_{\Omega+}^{(+)} + u_\Omega^{(+)*}(\tau, 0) \chi^*(\Omega) b_{\Omega+}^{(+)*} \right] \right\}
\]

\[
- \frac{1}{\omega_0} \int d\Omega \Omega \left[ b_{\Omega+}^{(+)}(\Omega + i\epsilon) \chi(\Omega) u_\Omega(\tau) + b_{\Omega+}^{(+)*}(\Omega - i\epsilon) \chi^*(\Omega) u_\Omega^{(+)*}(\tau, 0) \right] \}
\]

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Now we are ready to get the asymptotic relation between $B^{(\sigma)}_\kappa$ and $b^{(\sigma)}_\kappa$. The expansion coefficients (67) are

$$\bar{G}(\Omega, \Omega', \tau) = \frac{i e^2 \chi(\Omega)}{\sqrt{2\Omega}} \delta(\Omega - \Omega') + O(1/\tau_0), \quad (73)$$

and $\Omega \int d\tau' u^{(+)*}_k(\tau', 0) q_\Omega(\tau') \sim O(1/\tau_0)$. In these calculations, we use the formula:

$$\lim_{x \to \infty} e^{i\alpha x} \simeq \frac{\delta(\alpha)}{x}. \quad (74)$$

From Eq. (73) we get

$$B^{(\sigma)}_\kappa(\tau) = b^{(\sigma)}_\kappa + 2i\epsilon \Omega \chi(\Omega) \sqrt{2} b^{(+)\delta_\sigma}. \quad (74)$$

One can easily verify the commutation relation (21) using the Kramer-Kroig type relation

$$\chi(\Omega) - \chi^*(\Omega) = 4i\epsilon \Omega |\chi(\Omega)|^2. \quad (75)$$

This transformation is non-singular in a real $\kappa$ axis so the inverse can be obtained easily:

$$b^{(+)}_\kappa = \frac{B^{(+)}_\kappa(\tau) + 2i\epsilon \Omega \chi(\Omega) \left[ B^{(+)}_\kappa(\tau) - B^{(+)}_\kappa(\tau) \right]}{1 + 4i\epsilon \Omega \chi(\Omega)}. \quad (75)$$
REFERENCES


[14] Hyeong-Chan Kim, Min-Ho Lee, Jeong-Young Ji, and Jae Kwan Kim, Phys. Rev. A 53,
3767 (1996).