Radiative decays of $D$ mesons

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The short-distance contribution due to the $c \to u\gamma$ penguin as well as long-distance contributions are considered in an analysis of $D \to V\gamma$ decays, the latter being dominant. The matrix elements for nine $D \to V\gamma$ transitions are calculated using a hybrid model which combines heavy quark effective theory and the chiral Lagrangian. We present the expected range of the branching ratios for these decays, the most frequent ones $D^0 \to \bar{K}^{*0}\gamma$ and $D^+_s \to \rho^+\gamma$, being a few times $10^{-4}$.

1. INTRODUCTION AND THE $Q \to q\gamma$ TRANSITION

Radiative flavour changing quark transitions $Q \to q\gamma$, occur at the one loop level in the electroweak Standard Model and would show up at four different scales $s \to d\gamma$, $c \to u\gamma$, $b \to s(d)\gamma$ and $t \to c(u)\gamma$. These transitions are of intensive theoretical and experimental interest, especially since it has been pointed out [1] that the QCD enhancement of $b \to s\gamma$ brings it into the realm of observability [2]. Here, we are concerned with the $c \to u\gamma$ transition, which can be expected to induce hadronic decays like $D \to V\gamma$.

The amplitude $A^{Q\to q\gamma}$ for the transition of a heavy quark $Q$ to a light quark $q$ and an on-shell photon is given by [3]

$$A^{Q\to q\gamma} = \frac{eG_F}{4\pi^2\sqrt{2}} \sum_{\lambda} V_{\lambda Q}^* V_{\lambda q} (q) F_{2,\lambda}(k^2) i\sigma_{\mu\nu} k^\nu$$

$$\left( m_Q \frac{1 + \gamma_5}{2} + m_q \frac{1 - \gamma_5}{2} \right) u(Q)e^{i\mu}$$  \hspace{1cm} (1)

where in our case $F_2 = \Sigma_{\lambda} \lambda V_{\lambda q}^* F_{2,\lambda}$, and the summation is over $\lambda = d, s, b$. $F_2$ is calculable in the electroweak model [3]; furthermore, the inclusion of QCD corrections leads to a strong enhancement of its value [4,5]. Despite this increase, the short distance $c \to u\gamma$ amplitude leads to an inclusive branching ratio of $10^{-8}$ only [5]; accordingly, one expects for the exclusive transitions $D \to V\gamma$ a branching ratio of about $10^{-9}$ from this short-distance contribution.

The long-distance (LD) contribution to $c \to u\gamma$ has also been estimated recently [6]. Considering the process $c \to u + f\bar{f}_1$ with the quark pairs $f_1f_i$ hadronizing into vector mesons, one derives the amplitudes for the $c \to uV_i$ processes $(V_i = \rho, \omega, \phi)$ using the QCD corrected nonleptonic weak Lagrangian. Taking then the transverse part of the $c \to uV_i$ amplitude, with gauge invariance and vector meson dominance one arrives at the amplitude [6]

$$LD A(c \to u\gamma) = \frac{G_F}{\sqrt{2}} a_2(m_c^2) V_{ud} V_{cd}^* C'_{\text{VMD}} \left[ \bar{u} \gamma^{\mu} \theta \gamma_{\epsilon}^{\mu} k \epsilon^* \right]$$  \hspace{1cm} (2)

where $a_2$ is a Wilson coefficient and $C'_{\text{VMD}}$ is given by

$$C'_{\text{VMD}} = -\frac{1}{2} \frac{g_2^2(0)}{m_\rho^2} + \frac{1}{6} \frac{g_2^2(0)}{m_\omega^2} + \frac{1}{3} \frac{g_2^2(0)}{m_\phi^2}.$$  \hspace{1cm} (3)

Using the vector-photon couplings measured in leptonic decays, and assuming $g_{\gamma_V}(0) \simeq g_{\gamma_V}(m_{V_i}^2)$ one finds [7] a strong cancellation in (3) due to
There are three terms in the factorized amplitude. The first term is schematically approximated as

\[ VV_0 \approx \text{annihilation part} + \text{V-} - \text{spectator part} + \text{V-} - \text{spectator part} \]

In the present treatment, we aim for a more comprehensive and systematic treatment for these decays, employing an effective hybrid Lagrangian which combines two approximate symmetries of QCD, the infinite heavy quark \((Q = c)\) mass limit and the chiral limit for light quarks.

We assume the radiative decays to originate from the nonleptonic weak transition \(D \rightarrow VV_0\) followed by the conversion \(V_0 \rightarrow \gamma\) via vector meson dominance. In addition, there are transitions due to direct photon emission from the initial \(D\) state. The effective nonleptonic Lagrangian we use is given by

\[
\mathcal{L}_{LD} = \frac{-G_F}{\sqrt{2}} V_{uq}, V_{cq}^* \left[ a_1(\bar{u}q_i)\gamma^\mu(\bar{c}j) \right] \rho^\mu + a_2(\bar{u}c)_\mu(\bar{q}jq_i) \nu^\mu \tag{4}
\]

and in order to evaluate the matrix elements of the product of two currents we use systematically [11] the factorization approximation. The quark bilinears in (4) are treated as interpolating fields for the appropriate mesons, the relevant hadronic degrees of freedom being the charm pseudoscalar \((D)\) and vector mesons \((D^*)\) and the light pseudoscalar \((P)\) and vector mesons \((V)\). In the factorization approach we use, the \(D \rightarrow VV_0\) amplitude is schematically approximated as

\[
(VV_0)(\bar{q}K) \mu = V_{uq}\bar{u}q_j V_{cq}^* \left[ a_1(\bar{u}q_i)\gamma^\mu(\bar{c}j) \right] \rho^\mu + a_2(\bar{u}c)_\mu(\bar{q}jq_i) \nu^\mu \tag{5}
\]

There are three terms in the factorized \(D \rightarrow VV_0\) amplitude, referring to an annihilation part, a \(V_0\)-spectator part and a \(V\)-spectator part, each of them involving several explicit expressions. For the QCD-induced constants \(a_1, a_2\) we take \(a_1 = 1.26, a_2 = 0.55\) as determined from an extensive application of (4) to nonleptonic \(D\) decays [13].

The general gauge invariant amplitude for the decay \(D(p) \rightarrow (p\nu)\gamma(k)\) is

\[
A(D \rightarrow V + \gamma) = \frac{\epsilon_g}{\sqrt{2}} V_{uq}, V_{cq}^* \left[ \epsilon_{\mu\nu\alpha\beta}(\bar{u}q_i)\gamma^\nu(\bar{c}j)e^\mu_{\alpha\beta} A_{PC} + \epsilon_{\mu\nu}(\bar{u}q_i)\gamma^\nu(\bar{c}j)A_{PV} \right] \tag{6}
\]

We have classified all diagrams contributing to \(A_{PC}, A_{PV}\), and the explicit expressions are given in references [6] and [11].

The calculation requires the \(\langle V| (V_\mu - A_\mu) D \rangle\) matrix element, which has one vector \(V(q^2)\) and four axial-vector \(A_i(q^2)\) form factors. Actually, gauge invariance and finiteness reduce the number of independent form factors to three. For these we take a pole dominance behaviour, with known values of pole masses. Information on \(V(0), A_1(0), A_2(0)\) we extract from the semileptonic decays \(D^+ \rightarrow K^{*0}\ell\nu_\ell\) and \(D_s^+ \rightarrow \phi\nu_\ell\).

3. RESULTS

The calculation of the decay amplitudes involves various constants of the hybrid Lagrangian. Their imprecise knowledge is the first source of uncertainty. In addition, the amplitudes contain several components with unknown relative phases, which is yet another source of uncertainty. This allows us to give only an expected range, for each of the calculated transitions. In the Table below, we give the predicted range of the calculated values for the branching ratios. The first two decays are Cabibbo-allowed, the next five are Cabibbo-forbidden and the last two are doubly forbidden. To give an indication, the photon energy in the first two decays is 717 and 834 MeV respectively. As the table shows, a few of these decays are in a range which should become experimentally feasible soon. Their detection will contribute decisively to the understanding of long-distance dynamics in D decays.

On the other hand, we are still left with the
question of how to detect directly the $c \rightarrow u\gamma$ transition. One possibility is related to accidental cancellations between the various other contributions to the amplitudes of Cabibbo-forbidden decays [6], which is probably unlikely. The alternative would be to search for weak radiative transitions among baryonic states containing several $c$ quarks, like $\Xi^{++}_{cc} \rightarrow \Sigma^{++}_{c}\gamma$ and $\Omega^{++}_{ccc} \rightarrow \Xi^{++}_{cc}\gamma$. In these cases, due to the valence structure of the participating baryons, the $c \rightarrow u\gamma$ process would play a dominant role[14].

Table 1
The predicted branching ratios for nine $D \rightarrow V\gamma$ decays

<table>
<thead>
<tr>
<th>$D \rightarrow V\gamma$ Transition</th>
<th>$\text{Br Ratio} \times 10^5$[11]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0 \rightarrow \bar{K}^0\gamma$</td>
<td>6-36</td>
</tr>
<tr>
<td>$D^+_s \rightarrow \rho^+\gamma$</td>
<td>20-80</td>
</tr>
<tr>
<td>$D^0 \rightarrow \rho^0\gamma$</td>
<td>0.1-1</td>
</tr>
<tr>
<td>$D^0 \rightarrow \omega\gamma$</td>
<td>0.1-0.9</td>
</tr>
<tr>
<td>$D^0 \rightarrow \phi\gamma$</td>
<td>0.4-1.9</td>
</tr>
<tr>
<td>$D^+ \rightarrow \rho^+\gamma$</td>
<td>0.4-6.3</td>
</tr>
<tr>
<td>$D^+_s \rightarrow K^{*+}\gamma$</td>
<td>1.2-5.1</td>
</tr>
<tr>
<td>$D^+ \rightarrow K^{*+}\gamma$</td>
<td>0.03-0.44</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^{*0}\gamma$</td>
<td>0.03-0.2</td>
</tr>
</tbody>
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REFERENCES