The Age Dependent Luminosities of the
Red Giant Branch Bump, Asymptotic Giant Branch Bump,
and Horizontal Branch Red Clump

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ABSTRACT

Color-magnitude diagrams of globular clusters often exhibit a prominent horizontal branch (HB) and may also show features such as the red giant branch (RGB) bump and the asymptotic giant branch (AGB) bump. Stellar evolution theory predicts that the luminosities of these features will depend on the metallicity and age of the cluster. We calculate theoretical lines of 2 to 12 Gyr constant age RGB-bumps and AGB-bumps in the $\Delta V_{HB \text{Bump}}^{\text{HB}} - [\text{Fe/H}]$ diagram, which shows the brightness difference between the bump and the HB as a function of metallicity. In order to test the predictions, we identify giant branch bumps in new Hubble Space Telescope color-magnitude diagrams for 8 SMC clusters. First, we conclude that the SMC cluster bumps are RGB-bumps. The data for clusters younger than $\sim 6$ Gyr are in fair agreement with our predictions for the relative age dependent luminosities of the HB and RGB-bump. The $\Delta V_{HB \text{Bump}}^{\text{HB}}$ data for clusters older than $\sim 6$ Gyr demonstrate a less satisfactory agreement with our calculations. We conclude that $\sim 6$ Gyr is lower bound to the age of clusters for which the Galactic globular cluster, age independent $\Delta V_{HB \text{Bump}}^{\text{HB}} - [\text{Fe/H}]$ calibration is valid. Application of the $\Delta V_{HB \text{Bump}}^{\text{HB}} - [\text{Fe/H}]$ diagram to stellar population studies is discussed.

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1. Introduction

Color-magnitude diagram studies of globular clusters are ideally suited to test stellar evolution theory (Renzini & Fusi Pecci 1988). The stars in globular clusters are typically coeval and chemically homogeneous. Thus, the globular clusters represent stellar populations pure in composition.

A landmark feature in most color-magnitude diagrams is the horizontal branch (HB). Horizontal branches are populated by near constant luminosity core helium-burning giants, which can exhibit a range of temperatures. The well known RR Lyrae variable stars are HB stars with temperatures inside the boundaries of the instability strip. Some color-magnitude diagrams show predominantly red HBs, also known as red clumps (Cannon 1970), where most of the HB stars are cooler than the instability strip. Old metal-rich globular clusters usually have red clumps, as do younger clusters, which have more massive HB stars. We designate all HB stars cooler than the instability strip, and evolved from red giant branch (RGB) stars with degenerate helium cores, as red clump stars. Our definition therefore applies to all red horizontal branch stars in stellar populations older than $\sim 0.6$ Gyr, i.e. those that have undergone the RGB phase transition (Sweigart, Greggio, & Renzini 1990).

Stellar evolution theory predicts that the red HB clump luminosity depends on age. When comparing two clusters of similar metallicity, the red HB clump of the younger cluster will generally be brighter (Lattanzio 1986; Seidel, Demarque, & Weinberg 1987). The effect is $\sim 0.3$ mag for an age difference of $\sim 10$ Gyr$^3$. The rate of change in red clump luminosity with age may also depend on metallicity (Sarajedini, Lee & Lee 1995). Unfortunately, little observational data exists with which to study the dependence of red clump luminosity on age, or test for any dependence at all$^4$. We therefore seek a new observational test of the theoretically predicted age dependent red HB clump luminosity in order to lay the foundation for more accurate interpretations of color-magnitude diagrams of globular clusters and field populations in local group galaxies.

In addition to the HB, globular cluster color-magnitude diagrams may show features such as the RGB-bump, an evolutionary pause on the first-ascent red giant branch (Thomas 1967; Iben 1968), or the AGB-bump, a clustering of stars at the base of the asymptotic

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$^3$The red HB clump luminosity will actually decrease for very young clusters, i.e. those just older than the RGB phase transition (e.g Lattanzio 1986).

$^4$The notable exception is the Carina dwarf galaxy color-magnitude diagram, which shows a prominent blue (old) horizontal branch, and a (young) red clump $\sim 0.25$ mag brighter (Smecker-Hane et al. 1994).
giant branch (Castellani, Chieffi, & Pulone 1991). RGB-bumps and AGB-bumps are much more rapid phases of stellar evolution than the HB, which makes their detection challenging. However, if an RGB-bump or AGB-bump is identified, the brightness difference between the bump and HB provides a new test of the relative age dependent luminosities of these features, free from uncertainties due to distance or reddening.

To make our study of the luminosities of the RGB-bump, AGB-bump, and red HB clump, we first collect the relevant theoretical evolutionary results and predict the behavior of these features for clusters of different ages and metallicities. We endeavor to make this exercise simple, and make all of the underlying theoretical assumptions and calibrations as transparent as possible. We then seek a test of our predictions with a suitable set of observations. Although high quality photometric studies of many Galactic globular clusters have been available for years, these clusters are predominantly old, and do not span an adequately wide range of ages (Sarajedini, Chaboyer & Demarque 1997). For these reasons, Galactic globulars are a poor place to look. On the otherhand, the globular clusters of the Small Magellanic Cloud (SMC) range in age from $\sim$2 to 12 Gyr and are ideally suited to our study. Color-magnitude diagram photometric data for clusters in the SMC are best obtained with the high resolution of the Hubble Space Telescope. These new data are just becoming available (Mighell, Sarajedini & French 1998).

Our paper is organized as follows. In §2 we present a theoretical examination of the age dependencies of $M_{V,HB}$, $M_{V,RR}$, and $M_{V,AB}$, the absolute visual magnitudes of the horizontal branch, RGB-bump, and AGB-bump respectively. In §3 we present new Hubble Space Telescope data for SMC clusters and review extant detections of RGB-bumps and AGB-bumps in the literature. In §4 we use our theoretical data to predict constant age lines in the $\Delta V_{HB}^{Bump} - [\text{Fe/H}]$ diagram, which shows the brightness difference between a bump (RGB-bump or AGB-bump) and the HB. We then test these predictions with our SMC cluster data. In §5, we make our conclusions.

2. Theoretical Data

As a wide variety of theoretical data exist in the literature, we endeavor to make some comparisons of different studies, which allows a quick estimation of the level of consistency found. Wherever possible, we employ easily reproducible analytic calibrations and simple, plausible approximations to the theoretical data. We rely on empirical calibrations when available.
2.1. Analytic Calibrations

We require a calibration between turn-off mass and age. The absolute age scale is less important than the relative ages. Following Iben and Laughlin (1989) and using the theoretical data of Mengel et al. (1979) we derive,

\[
\log(Age) = 10.15 + 0.16[\text{Fe/H}] - 3.86 \log(M) - 0.2[\text{Fe/H}] \log(M) + 2 \log^2(M)
\]  

(1)

where age is in units of years (throughout this paper) and \( M \) is turn-off mass in solar units. Equation 1 is applicable for \( M \approx 0.7 \) to 2.0 \( M_\odot \) and \( [\text{Fe/H}] > -1.3 \) dex. For \( [\text{Fe/H}] < -1.3 \) dex, we substitute \([\text{Fe/H}] = -1.3 \) dex, as the age–mass calibration is increasingly insensitive to metallicity. Including metallicity in the age–mass calibration has only a small effect on the results of this paper. Equation 1 predicts relative ages for different mass and metallicity stellar evolution models with a precision of \( \pm 1 \) Gyr.

To an excellent approximation, the bolometric correction \( (M_{\text{bol}} = M_V + BC_V) \) for giant branch and red horizontal branch stars with \( \log(T) = 3.55 \) to 3.75 is given by,

\[
BC_V = -529.226 + 282.524 \log(T) - 37.719 \log^2(T)
\]  

(2)

We adopt the usual relation between \( M_{\text{bol}} \) and \( \log(L) \), and \( M_{\text{bol}, \odot} = 4.69 \) mag. Equation 2 is derived from the Padova isochrones (Bertelli et al. 1994) at metallicities \([\text{Fe/H}] = -0.4, -0.7, -1.3, -1.7 \) dex. We find a negligible systematic dependence on \([\text{Fe/H}] \). Equation 2 will typically predict \( BC_V \) to within 0.01 mag of the Padova data. To transform the luminosity of an RGB-bump to an observable \( M_V \) magnitude, we require \( \log(T) \) from an interpolation along reference RGBs of appropriate age and metallicity. The reference RGBs have the form,

\[
\log(L) = \alpha + \beta \log(T) + \gamma \log^2(T) + f(Age)
\]  

(3)

\[
f(Age) = -0.502 \log(Age) - 4.168
\]  

(4)

where the coefficients \( \alpha, \beta \) and \( \gamma \) depend on metallicity and are given in Table 1. These are also derived from the Padova isochrones, valid in the luminosity range \( \log(L) = 1.3 \) to 3.0, and ages of 2 to 15 Gyr. These give \( \log(T) \) along the giant branch with an accuracy better than \( \delta \log(T) = 0.005 \), which corresponds to \( \pm 0.05 \) mag in \( M_V \) using Equation 2.

2.2. Horizontal Branch

The dependence of the absolute visual magnitude of the HB on metallicity has been the subject of numerous theoretical and observational studies. Without further discussion,
we will begin with an empirical $M_{V,HB}$–[Fe/H] calibration, which is applicable to ancient globular clusters, assuming their ages are the same to within a few Gyr. We adopt,

$$M_{V,HB} = 0.15[Fe/H] + 0.70$$

from Walker (1992), which is consistent with that found by Alcock et al. (1997) from LMC RR Lyrae stars. The zero-point of Equation 5 only enters our analysis as a negligible second order effect through the bolometric corrections.

In Figure 1, we plot $M_{V,HB}$ as a function of log($Age$), comparing theoretical data from several different authors; the different data are described in the caption. We have used Equations 1 & 2 to calculate $M_{V,HB}$ and age for the data from Seidel, Demarque, and Weinberg\(^5\) (1987), Vassiliadis and Wood (1993), and Lattanzio (1986). The data of Sarajedini, Lee and Lee (1995) were presented as $M_{V,HB}$ and age, and we have adopted them as published. The data of Sarajedini, Lee and Lee (1995) suggest that the dependence of $M_{V,HB}$ on age may be a function of metallicity. The data of Vassiliadis and Wood (1993) do not show such a metallicity dependence, although the limited number of data-points precludes any strong statements to this effect. We will simply adopt the following relation for all metallicities, ignoring a possible age-metallicity cross-term,

$$M_{V,HB} \propto 0.5 \log(Age)$$

which is shown in the lower right corner of Figure 1 and labeled. As a check of Equation 6, consider the dual horizontal branches of the Carina dwarf (Smecker-Hane et al. 1994; see also Hesser et al. 1996). Carina is $\sim80\%$ of a 2.5 to 7 Gyr old population and $\sim20\%$ of 10 to 14 Gyr old population; both components have similar metallicities. Assuming the red clump represents a population $\sim4$ Gyr old, and the blue HB represents a population $\sim12$ Gyr old, Equation 6 predicts a brightness difference of 0.24 mag, in good agreement with 0.25 mag observed.

2.3. RGB-Bump

The RGB-bump is theoretically understood as a luminosity dip due to the hydrogen-burning shell crossing a discontinuity in chemical composition left by the deepest penetration

\(^5\)These theoretical data are HB stellar evolution models. We have assumed no mass-loss on the first-ascent RGB, and equate total HB model mass with turn-off mass. This assumption may be poor for the 0.9 and 1.0 mass models, but is likely satisfactory for the higher mass (younger) models. See also discussion in §2.4 of this paper.
of the convective envelope (Thomas 1967; Iben 1968). The discontinuity represents a drop in the mean molecular weight of fuel and an increased opacity for the hydrogen-burning shell as it moves radially outward, which causes a drop in luminosity and small increase in temperature (Refsdal & Weigert 1968; Sweigart, Greggio & Renzini 1990). The RGB-bump is predicted for a wide range of masses, and metal and helium abundances (Sweigart & Gross 1978; Sweigart, Greggio & Renzini 1989), and is insensitive to “smoothing” of the composition discontinuity due to convective undershooting (Bono & Castellani 1992).

Alongi et al. (1993) showed that the dependence of RGB-bump luminosity on mass is independent of overshoot or classical mixing schemes. The number of stars decreases with increasing luminosity along the RGB (Castellani, Chieffi & Norci 1989), which makes observational detection of the RGB-bump challenging for low metallicity or young populations (these RGB-bumps would be bright). The difficulty is compounded in young populations, as the duration of the pause is shorter relative to the pace of evolution up the RGB, and thus the contrast of RGB-bump stars against the underlying RGB luminosity function is less.

As noted by Fusi Pecci et al. (1990), theory predicts absolute RGB-bump luminosities brighter than observed. However, they found good agreement between theory and observation for the metallicity dependence of the RGB-bump luminosity. Cassisi and Salaris (1997) found that updated input physics resolves the discrepancy between the predicted and observed absolute luminosities of RGB-bumps. We will adopt the dependence of RGB-bump luminosity on mass from theory, but empirically calibrate the absolute luminosities to globular cluster data. Fortunately, the number of observational detections of RGB-bumps for Galactic globular clusters has grown considerably since the first detection by King et al. (1985) for 47 Tuc. In an important paper, Fusi Pecci et al. (1990) identified the RGB-bump in 11 Galactic globular cluster luminosity functions. Sarajedini and Forrester (1995) added data for 5 additional clusters and found,

\[
[\text{Fe/H}] = -1.33 + 1.43\Delta V_{\text{Bump}}^{HB}
\]

where \(\sigma_{\text{RMS}} = 0.04\) dex about this fit. \(\Delta V_{\text{Bump}}^{HB}\) is defined as the brightness difference, \(\Delta V(\text{Bump} - \text{HB})\), between the RGB-bump and the HB (see §4 and Fig. 5 of this paper). From Equations 5 & 7, applicable to ancient globular clusters, we find,

\[
M_{V,\text{RB}} = 0.85[\text{Fe/H}] + 1.63
\]

which yields \(M_{V,\text{RB}} = 0.185, 0.525, 1.035, \text{ and } 1.290\) mag for \([\text{Fe/H}] = -1.7, -1.3, -0.7, \text{ and } -0.4\) dex respectively, our normalization points. We are using the subscript “RB” to designate the RGB-bump. To maintain consistency with the zero-point in Equation 5, we will assume that all of the Galactic globular clusters are 12 Gyrs old (Chaboyer, Demarque
& Sarajedini 1996; see their Table 3, Column 5), which yields turn-off masses $M = 0.865, 0.865, 0.920,$ and $0.947 \, M_\odot$ for the normalization points as above, from Equation 1.

In Figure 2 we plot $\log(L)$ of the RGB-bump as a function of model mass, which we will equate with turn-off mass. The theoretical data from Alongi et al. (1993), Sweigart and Gross (1978), Vassiliadis and Wood (1993), and Fusi Pecci et al. (1990) illustrate the various dependencies of RGB-bump luminosity on $Z$, $Y$, and mixing schemes, and are described more completely in the caption. In all of the data, the dependence of $\log(L)$ on mass is quite similar. The solid line in the lower right corner of Figure 2 (labeled) represents the dependence of RGB-bump luminosity on mass adopted in this paper,

$$\log(L) \propto 0.75 M$$ (9)

The zero-point is arbitrary. We now calculate $M$ for ages of 10, 5, and 2 Gyr and $\Delta \log(L)$ for the RGB-bump using $\Delta \log(L)_{Age} = 0.75 \times (M_{Age} - M_{12})$, which follows from Equation 9. We calculate $\log(L)$ for the 12 Gyr RGB-bumps from the values of $M_{V,RB}$ given above (Eqn. 8) using the appropriate reference RGBs. These values, coupled with $\Delta \log(L)$ as above, yields $\log(L)$ for the younger RGB-bumps, calibrated to the Galactic globular cluster data. Interpolating along the appropriate reference RGB gives $\log(T)$, $BC_v$, and $M_{V,RB}$.

Our calculations show the dependence of $M_{V,RB}$ on age varies some with metallicity. For easy comparison with Equation 6, which gives the age dependence of $M_{V,HB}$, we present the following approximation to the different metallicity $M_{V,RB}$ data,

$$M_{V,RB} \propto 1.7 \log(Age)$$ (10)

Equation 10 is a reasonable approximation because we have adopted a single mass-luminosity calibration for all metallicities, i.e. there is no $M \times [\text{Fe/H}]$ term in Equation 9. We conclude that bolometric corrections and the shape of the different age and metallicity RGBs (in the $\log(L)$–$\log(T)$ plane) have only a small effect on the age dependence of $M_{V,RB}$. In any case, $M_{V,RB}$ increases more rapidly with age than $M_{V,HB}$.

Equations 6 & 10, which give the age dependencies of $M_{V,HB}$ and $M_{V,RB}$, illustrate a fundamental point of this paper. Theory predicts the age dependencies of the red HB clump and RGB-bump luminosities to be sufficiently different, such that, in young clusters, the brightness difference $\Delta V_{HB}^{Bump}$ will be observably “shifted” off the Galactic

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6Straniero and Chieffi (1991) give $M_{V,RB} \propto 0.9562 \log(Age)$, applicable to globular clusters with ages from 10 to 20 Gyr. We speculate that the shallower dependence on age derives from consideration of theoretical data constructed for only very ancient clusters. It may indicate a more complex age dependence than the simple proportionality given in Eqn. 10.
globular cluster $\Delta V_{Bump}^{HB}$-[Fe/H] calibration (Eqn. 7). We emphasize that theoretical uncertainties associated with mass-loss on the RGB, the reference RGBs, the turn-off mass/age calibration, and the absolute luminosities of the RGB-bump and HB are second order effects because we will restrict ourselves to an empirically calibrated, differential comparison of $M_{V,HB}$ and $M_{V,RB}$.

2.4. AGB-Bump

The AGB-bump occurs at the beginning of helium shell-burning asymptotic giant branch (AGB) evolution. Caputo et al. (1989) found the AGB-bump approaches the Hayashi line in higher mass models. From an observational perspective, particularly in young clusters, there is potential for confusion between the RGB-bump and AGB-bump. Theory tells us that the luminosity difference between the AGB-bump and HB is fairly insensitive to metallicity (Castellani, Chieffi & Pulone 1991) or helium abundance (Bono et al. 1995). Theoretical treatment of central helium-burning convective core instabilities known as breathing pulses$^7$ have a $\sim0.25$ mag effect on the luminosity of the AGB-bump (Castellani et al. 1985; Caputo et al. 1989; Bono et al. 1995). Breathing pulses have been suppressed in the stellar evolution models examined in this paper. Pulone (1992) and Ferraro (1992) both claim the AGB-bump is a standard candle, with $M_{V,AB} = -0.4 \pm 0.1$ mag for a limited range of metallicities (low) and ages (ancient). There is little discussion of the age dependence of the AGB-bump luminosity in the literature.

In contradistinction to our empirical calibrations of $M_{V,HB}$ and $M_{V,RB}$, we will predict $M_{V,AB}$ as a function of age and metallicity directly from theory. In a classical paper, Castellani, Chieffi, and Pulone (1991) presented a grid of horizontal branch stellar evolution models, intended to represent Galactic globular clusters. The models were evolved through to the re-ignition of the hydrogen shell, i.e. the beginning of double shell-burning and the putative onset of the thermal-pulsing AGB phase. Following standard procedure, for each metallicity, the mass of the helium core was fixed according to theoretical prescriptions of the helium flash at the tip of first ascent RGB ($M_{Core} \approx 0.5M_{\odot}$). Total HB mass was treated as a free parameter, covering a range from 0.80 to 0.525 $M_{\odot}$. To convert HB model mass to turn-off mass, we must account for RGB mass-loss. We adopt 0.10 $M_{\odot}$ lost on the

$^7$Modeling the transition from core helium-burning to helium shell-burning is a matter of considerable debate, with implications for the measurement of the primordial helium content, a parameter of strong cosmological relevance (Caputo et al. 1989).
RGB\textsuperscript{8}. To extend the theoretical data to higher masses, we use the models of Vassiliadis and Wood (1993).

In Figure 3, top panel, we plot \( \log(L) \) of the AGB-bump as a function of turn-off mass for \( Z=0.001 \). The Castellani et al. data are shown as open squares. The Vassiliadis and Wood data are shown as filled circles. In Figure 3, bottom panel, we plot \( \log(L) \) of the AGB-bump as a function of \([Fe/H]\). Symbols are the same as in the top panel. The Castellani et al. data are for a turn-off mass of 0.85 \( M_\odot \), while the Vassiliadis and Wood data are for 1.50 \( M_\odot \). We adopt the following analytic fit to these data,

\[
\log(L) = 1.90 + 0.215(M) - 0.155(M)[Fe/H] - 0.047[Fe/H]^2
\]

(11)

In the top panel, we project lines from Equation 11 for \([Fe/H] = -0.4, -1.3, \) and \(-1.7 \) dex; each line is labeled. In the bottom panel we project lines from Equation 11 for turn-off masses 0.85, 0.95, 1.10, 1.30 and 1.50 \( M_\odot \); each line is labeled. In the absence of more theoretical data, Equation 11 is a plausible extrapolation of the Castellani et al. data to higher masses. Clearly, more theoretical models are needed. To calculate absolute \( M_{V,AB} \) we assume the AGBs are uniformly hotter than the appropriate age and metallicity reference RGBs by \( \delta \log(T) = 0.02 \).

We find \( M_{V,AB} \) increases with age more rapidly for low metallicities, which is a direct result of the \( M \times [Fe/H] \) cross-term included in Equation 11. In all cases, the dependence of \( M_{V,AB} \) on age is steeper than that for \( M_{V,HB} \) (Eqn. 6) and shallower than that for \( M_{V,RB} \) (Eqn. 10). Ignoring the metallicity dependence and adopting the mean slope of linear regressions of the \( M_{V,AB} \) data yields the following approximation,

\[
M_{V,AB} \propto 0.90 \log(Age)
\]

(12)

where the slope ranges from \( \sim 0.8 \) for the high metallicity AGB-bumps to \( \sim 1.0 \) for the low metallicity AGB-bumps. We predict \( M_{V,AB} \) will always be brighter in the younger of two stellar populations with similar metallicity, which allows unambiguous identification of RGB-bumps in certain regions of the \( \Delta V_{HB}^{Bump} - [Fe/H] \) diagram. For example, \( \Delta V_{HB}^{Bump} > -0.8 \) mag is likely to be an RGB-bump, regardless of the age or metallicity of the stellar population. If \( \Delta V_{HB}^{Bump} < -0.8 \) mag, the situation is less clear. The bump maybe a young RGB-bump or an AGB-bump.

\textsuperscript{8}See Figure 8 of Sweigart, Greggio, and Renzini (1990). Our adopted RGB mass-loss is roughly equivalent to a Reimers scaling of \( \eta = 1/3 \).
2.5. Summary of Theoretical Data

Table 2 summarizes our theoretical calculations for the absolute visual magnitudes of the red HB clump, RGB-bump, and AGB-bump. Column 1 lists [Fe/H] in units of dex. Column 2 gives age in Gyr. Column 3 gives turn-off mass calculated from the age according to Equation 1. Column 4 gives $M_{V,HB}$ in units of magnitudes for our 12 Gyr old calibrators according to Equation 5, and for 10, 5, and 2 Gyr assuming an age dependence given by Equation 6, relative to the 12 Gyr old calibrators. Columns 5 & 6 list the values of $M_{V,RB}$ and $M_{V,AB}$ respectively. Figure 4 illustrates the dependencies on age for $M_{V,HB}$, $M_{V,RB}$, and $M_{V,AB}$ given Equations 6, 10 & 12 of this paper respectively, taken directly from Table 2. We plot $M_V$ as a function of log(Age); each of the four panels represents a different metallicity, which is labeled. The HB data are shown as filled squares, the RGB-bump data as open circles, and the AGB-bump data as open triangles.

3. Observational Data

In order to test the predictions of the theoretical calculations, we must compare them to observational data. In this case, we require information on the ages, metallicities and $\Delta V_{HB}$ values for several clusters. The ideal dataset for this purpose is that of Mighell et al. (1998), where Hubble Space Telescope (HST) observations are combined with ground-based photometry to estimate the ages and abundances of 7 star clusters in the SMC (Lindsay 113, Kron 3, NGC 339, NGC 416, NGC 361, Lindsay 1, and NGC 121). We refer the reader to that paper for a description of how these quantities were measured. Note that in the present work, the uncertainties quoted for the $V(HB)$ values are the standard error of the mean (random error), whereas in the paper by Mighell et al. (1998), the $V(HB)$ errors represent the total uncertainty (random as well as systematic). This is done because we are primarily interested in the difference in magnitude between the bump and the HB, not the absolute value of each quantity. To establish values for the magnitude of the bump in each cluster, we proceed as follows. First, we construct a luminosity function (LF) of the giant branch. We then perform a Gaussian fit to the LF in the region of the bump. The peak of this fit is the value of $V(Bump)$. The $\sigma$ of the Gaussian divided by the square-root of the number of points used in the fit (i.e. the area under the Gaussian) is then our estimate for the error in $V(Bump)$.

In order to supplement this dataset with clusters of even younger age, we searched the HST archive and constructed color-magnitude diagrams for approximately two dozen populous LMC and SMC clusters that have Wide Field Planetary Camera 2 (WFPC2) observations. This search yielded one SMC cluster, NGC 411, that exhibits a noticeable
clustering of stars on its giant branch. The photometry has been reduced following the procedure outlined by Sarajedini (1998). The HST color-magnitude diagram of NGC 411, showing only the data from the Planetary Camera, which was centered on the cluster, is shown in Figure 5. We plot $V$ as a function of $(B - V)$; the red HB clump and giant branch bump are marked with two arrows. We note that the giant branch bump looks like it lies on the first ascent RGB, and not along an AGB.

In measuring the age, abundance, and $\Delta V_{HB}^{Bump}$ of NGC 411, we follow procedures consistent with those of Mighell et al. (1998, see above). To determine the red HB clump magnitude of NGC 411, we begin by selecting stars in the range $0.56 < (B - V) < 0.81$ and $18.85 < V < 19.90$; we find $V(HB) = 19.43 \pm 0.01$, where the quoted error is the standard error of the mean. The V magnitude of the bump in NGC 411 is measured in the manner as described above. The metallicity of NGC 411 can be estimated by employing the simultaneous reddening and metallicity (SRM) method of Sarajedini (1994). We note, however, that since NGC 411 appears to be significantly younger than the clusters considered by Sarajedini (1994) and Mighell et al. (1998), we must employ a different relation between metallicity and RGB position/shape. Recently, Noriega-Mendoza & Ruelas-Mayorga (1997) have presented such relations derived from photometric observations of Galactic open clusters, which nicely encompass the age and likely abundance of NGC 411. When coupled with a polynomial describing the shape and location of the NGC 411 RGB, the SRM method then yields the metallicity that we seek, $[\text{Fe}/\text{H}] = -0.68 \pm 0.07$. Finally, an estimate of the age of NGC 411 can be derived by appealing to the isochrones of Bertelli et al. (1994). Utilizing the tracks for $Z = 0.004$ and offsetting the isochrones to match the magnitude of the red HB clump and the color of the main sequence, we find an age between 1.26 and 1.58 Gyr for NGC 411. This is in excellent agreement with the results of Da Costa & Mould (1986) who conclude that the age is $1.5 \pm 0.5$ Gyr. For the present paper, we will adopt $1.4 \pm 0.2$ Gyr.

Table 3 lists the relevant observational quantities for these 8 SMC clusters. Column 1 is cluster name, Columns 2 & 3 are $V(HB)$ and $V(Bump)$ respectively, in units of magnitudes. Column 4 is age in units of Gyr. Table 3 also summarizes the Galactic globular cluster RGB-bump data assembled from Fusi Pecci et al. (1990) and Sarajedini and Forrester (1995). Where available, we have listed ages from Chaboyer, Demarque, and Sarajedini (1996), appropriate for the zero-point adopted in Equation 5 of this paper.

Observational detections of AGB-bumps in local group field population and globular cluster color-magnitude diagrams have been discussed recently by Gallart (1998). For Galactic globular clusters, we find four marginal detections of AGB-bumps in the literature. From the luminosity functions presented by Ferraro (1992; see also their Table 2), we
find $\Delta V_{Bump}^{HB} = -1.0, -1.0, \text{ and } -0.95 \text{ mag for the AGB-bumps in M5, NGC 1261, and NGC 2808 respectively. From the color-magnitude diagram of Hesser et al. (1987) and the discussion given by Castellani et al. (1991) of this data, we adopt $\Delta V_{Bump}^{HB} = -0.95 \text{ for 47 Tuc. We adopt a metallicity of } [\text{Fe/H}] = -1.40 \text{ dex for M5 following Ferraro (1992). The metallicities for NGC 1261, NGC 2808 and 47 Tuc are given in Table 3 (47 Tuc = NGC 104). We caution that $\Delta V_{Bump}^{HB}$ for 47 Tuc is determined by eye, following Castellani et al. (1991, their Fig. 10), and should be regarded with proper skepticism. For M5, NGC 1261, and NGC 2808 we estimate an uncertainty of $\pm 0.07 \text{ mag for each value $\Delta V_{Bump}^{HB}$ by inspection of the Ferraro (1992) luminosity functions. We note that Caputo et al. (1989) derive $\Delta V_{Bump}^{HB} = -0.95 \pm 0.09 \text{ mag for the M5 AGB-bump, in agreement with our estimation.}$

4. The $\Delta V_{Bump}^{HB}$–[Fe/H] Diagram

In Figure 6, we present the theoretical $\Delta V_{Bump}^{HB}$–[Fe/H] diagram, which shows the brightness difference between the bump (AGB-bump or RGB-bump) and the HB. The observational data for RGB-bumps in Galactic globular clusters are shown as solid black squares. Error bars are omitted for clarity, but see Table 3 of this paper. The solid line through these points, and labeled “12 Gyr” is the best fit regression of these points, Equation 7 of this paper. Assuming the horizontal branch has no dependence on age, the locations of RGB-bumps in stellar populations of ages 10, 5, and 2 Gyr are shown as solid lines, and labeled (R10, R5, and R2 respectively). Here, we are adopting the 12 Gyr $M_{V,HB}$ values from Table 2 for all ages. The short-dashed lines show the locations of RGB-bumps assuming $M_{V,HB}$ depends on age as given by Equation 6, and listed in Table 2. The effect of an age dependent $M_{V,HB}$ is to move the young RGB-bumps closer to the ancient cluster calibration in the $\Delta V_{Bump}^{HB}$–[Fe/H] diagram. At ages of $\sim 5$ Gyr or younger, we should clearly see the age-shift effect in observational data. The dotted lines, labeled A10/12, A5, and A2, show the locations of AGB-bumps in this diagram, assuming the age-dependent values of $M_{V,HB}$, as listed in Table 2. The 12 and 10 Gyr lines are virtually indistinguishable. The 5 and 2 Gyr old AGB-bump lines have increasingly more negative $\Delta V_{Bump}^{HB}$ values. The observational detections of AGB-bumps in Galactic globular clusters, as discussed in §3, are plotted as filled circles.

Several aspects of Figure 6 warrant further discussion. We have plotted the RGB-bump constant age lines under two assumptions of the dependence of $M_{V,HB}$ on age, the case of no dependence and that given by theory (Eqn. 6), to illustrate the discriminating power of RGB-bumps in young clusters. Assuming the age dependence of $M_{V,RB}$ is accurately predicted by theory, this is a new test of the age dependence of $M_{V,HB}$. In reality, we have
no empirical data with which to constrain the age dependencies of either \( M_{V,HB} \) or \( M_{V,RB} \), and we will only test the relative age dependencies. However, empirical confirmation of the relative age dependencies of \( M_{V,RB} \) and \( M_{V,HB} \) is an important new test of our theories. As for the AGB-bumps, our theoretical A10/12 line is in excellent agreement with the detections in Galactic globular clusters. While we do not wish to make strong conclusions from this small and inhomogeneous observational dataset, the agreement implies the bright zero-point in our \( M_{V,HB} - [\text{Fe/H}] \) calibration (Eqn. 5) is correct. An increase in the zero-point of \( \sim 0.3 \) mag, as advocated by, for example, Carney et al. (1992), would move the A10/12 line 0.3 mag to the left in Figure 6, and the observed AGB-bumps would be too faint. In turn, if breathing pulses had not been suppressed in the stellar evolution models examined in this paper, the theoretical A10/12 line would move \( \sim 0.25 \) mag to the right, in better accord with a fainter absolute calibration of \( M_{V,HB} \). More theoretical studies of the AGB-bump and more robust observational detections of AGB-bumps in Galactic globular clusters would be desirable.

Also shown in Figure 6 are the data for the 8 new SMC cluster giant branch bumps, shown as open circles with error bars. We find four of the SMC clusters (NGC 416, NGC 361, Lindsay 1, and NGC 121; mean age = 9.0 Gyr) lie right on the Galactic globular cluster RGB-bump calibration. Three of the clusters (Lindsay 113, Kron 3, NGC 339; mean age = 5.9 Gyr) are shifted “off” of the Galactic globular cluster RGB-bump calibration by \( \sim 2-3\sigma \) in \( \Delta V_{HB}^{\text{Bump}} \) and \( \sim 1-2\sigma \) in \([\text{Fe/H}]\). The youngest SMC cluster (NGC 411; age = 1.4 Gyr) deviates from the Galactic globular cluster RGB-bump calibration with an extremely high significance. Moreover, it lies \( \sim 8\sigma \) away from the A10/12 AGB-bump line and \( \sim 16\sigma \) from the A2 AGB-bump line, where the latter is the most appropriate comparison given the age of NGC 411. We conclude that all 8 SMC cluster bumps are RGB-bumps. It is also clear that the SMC cluster data, particularly NGC 411, are in better agreement with interpolations between the R2, R5, and R10 lines calculated with an age dependent \( M_{V,HB} \) (the dashed lines) as opposed to an age independent \( M_{V,HB} \) (the solid lines).

In Figure 7, we plot the deviation from the Galactic globular cluster RGB-bump line in the \( \Delta V_{HB}^{\text{Bump}} - [\text{Fe/H}] \) diagram (\( \delta \Delta V_{HB}^{\text{Bump}} \)) as a function of age, in Gyr. Plotted as open circles with error bars are the SMC cluster data from Table 3. In the lower portion of the figure, underneath each SMC cluster datapoint, we have labeled the metallicity for each cluster. We have plotted the Galactic globular cluster RGB-bump data as filled squares. Error bars are omitted for clarity. Clusters NGC 5927 and NGC 6637 are not plotted (see Table 3). The case of \( \delta \Delta V_{HB}^{\text{Bump}} = 0 \) is plotted as a solid line. Shown as dotted lines are the theoretical predictions calculated in this paper for \([\text{Fe/H}] = -1.7, -1.3, -0.7, \) and \(-0.4 \) dex, using the data in Table 2. The \([\text{Fe/H}] = -1.7 \) line deviates from the other three metallicity lines, which lie close together. Shown as a dashed line is the approximation
to our theoretical calculations derived from Equations 6 & 10 of this paper, which yield 
\[ \delta \Delta V_{HB} \propto 1.2 \log(Age). \] 
For the SMC cluster younger than \( \sim 6 \) Gyr, we find reasonable agreement with the theoretical calculations in this paper, lending support to our conclusion that these giant branch bumps, particularly the NGC 411 bump, are RGB-bumps. These data demonstrate, for the first time, deviations from the \( \Delta V_{HB}^{\text{Bump}} - [\text{Fe/H}] \) Galactic globular cluster calibration, which are in fair agreement with the theoretically predicted age dependence of the luminosities of the RGB-bump and red HB clump. Interestingly, the data appear to show a “break” near 6 Gyr, where the clusters older than this follow the \( \Delta V_{HB}^{\text{Bump}} - [\text{Fe/H}] \) Galactic globular cluster calibration remarkably well.

5. Conclusions

We have examined a variety of theoretical data regarding the age and metallicity dependent luminosities of the red HB clump, the RGB-bump, and AGB-bump. We make no claim to have provided an exhaustive review. Rather, we have endeavoured to make some comparisons of different theoretical studies, which allows a quick estimation of the level of consistency found. We then adopt the simple, plausible approximations to the theoretical data, such as our linear \( M_{V,HB} - \log(Age) \) calibration (Eqn. 6) or the linear \( \log(L) - M \) calibration for the RGB-bump luminosity (Eqn. 9). Transformation of the theoretical data to the observable plane (i.e. visual magnitudes) is accomplished with analytic formulae, which can be easily reproduced. Where possible, we have empirically calibrated our calculations; employing, for example, the \( M_{V,HB} - [\text{Fe/H}] \) calibration (Eqn. 5) and the \( \Delta V_{HB}^{\text{Bump}} - [\text{Fe/H}] \) calibration (Eqn. 7). As for the AGB-bump, we have appealed to the models of Castellani et al. (1991) and Vassiliadis and Wood (1993) to derive an analytic approximation for the luminosity as a function of mass and metallicity (Eqn. 11). Our final product is a theoretical \( \Delta V_{HB}^{\text{Bump}} - [\text{Fe/H}] \) diagram, showing the brightness difference between the bump (RGB-bump or AGB-bump) and HB as a function of age and metallicity. Our diagram shows good agreement with extant observational data on RGB-bumps and AGB-bumps in ancient Galactic globular clusters.

To test the (younger) constant age RGB-bump and AGB-bump lines in the \( \Delta V_{HB}^{\text{Bump}} - [\text{Fe/H}] \) diagram, we have identified giant branch bumps in the Hubble Space Telescope color-magnitude diagrams (Mighell et al. 1998) of 7 SMC globular clusters ranging in age from \( \sim 5 \) to 12 Gyr. To supplement these data, we have reduced archival Hubble Space Telescope data for the SMC cluster NGC 411, which also shows a giant branch bump. As the ages and metallicities of these 8 SMC clusters are constrained by other techniques, we conclude from their locations in the \( \Delta V_{HB}^{\text{Bump}} - [\text{Fe/H}] \) diagram that these bumps are
RGB-bumps. We make three principal conclusions from our comparison of the observational data and theoretically predicted values of $\Delta V_{HB}^{Bump}$, (1) Deviations from the Galactic globular cluster $\Delta V_{HB}^{Bump} - [Fe/H]$ calibration are consistent with a general trend of increasing RGB-bump brightness with decreasing age, in agreement with the theoretical prediction. (2) The data are in poor agreement with the hypothesis of no dependence on age for the absolute visual magnitude of the red HB clump. Assuming the age dependent luminosity of the RGB-bump is accurately predicted by theory, the red HB clump becomes brighter with decreasing age. (3) The data for clusters older than $\sim 6$ Gyr follow the empirical, age independent, Galactic globular cluster $\Delta V_{HB}^{Bump} - [Fe/H]$ calibration (Eqn. 7) well, which indicates a lower bound to the age of clusters for which this calibration is applicable. The calibration would appear to be an accurate metallicity diagnostic ($\sigma_{Fe/H} = 0.04$ dex; Sarajedini & Forrester 1995) for clusters older than $\sim 6$ Gyr.

The SMC cluster data show the worst agreement with our predictions near ages of $\sim 6$ to 10 Gyr, which we have designated as the 6 Gyr break. The 6 Gyr break likely reflects an over-simplification in our construction of the theoretical $\Delta V_{HB}^{Bump} - [Fe/H]$ diagram. One explanation would be a more complex dependence of either $M_{V,HB}$ or $M_{V,RB}$ on age than employed here. In this case, the relative age dependencies of the red HB clump and RGB-bump luminosities likely approaches our calculation for young stellar populations. The luminosities of the red HB clump and RGB-bump for more ancient populations are not necessarily independent of age, but likely have a similar dependence on age. Alternatively, we note the effects of helium abundance (e.g. Catelan & De Freitas Pacheco 1996) have been neglected in our analysis. A detailed accounting for the dependence of $M_{V,HB}$ and $M_{V,RB}$ on helium abundance is beyond the scope of our paper. We have assumed possible differences in the helium abundances of the clusters observed are small. The observational dataset is still limited. The identification of RGB-bumps in other globular cluster color-magnitude diagrams, in particular those ranging in age from $\sim 1$ to 10 Gyr would be highly desirable.

The $\Delta V_{HB}^{Bump} - [Fe/H]$ diagram may be used as a consistency check for age and metallicity determinations measured via other techniques for globular clusters. In particular, $\Delta V_{HB}^{Bump}$ is shown to be an accurate predictor of metallicity for clusters as young as $\sim 6$ Gyr, which likely encompasses the ages of all Galactic globular clusters. Additionally, the $\Delta V_{HB}^{Bump} - [Fe/H]$ diagram may be employed as a diagnostic tool to aid in the interpretation of color-magnitude diagrams of field populations in nearby galaxies, where the different ages and metallicities of the component stellar populations are not necessarily known with high accuracy. In this case, giant branch bumps may be identified as either RGB-bumps or AGB-bumps by their locations in the $\Delta V_{HB}^{Bump} - [Fe/H]$ diagram. Moreover, the bumps may yield new clues to the component stellar populations of local group galaxies, which has important implications for our theories of galaxy formation. An excellent example of
this would be the 9 million star color-magnitude diagram of the LMC bar, assembled from the MACHO Project’s microlensing survey photometry (Alves et al. 1998, 1998b; Alcock et al. 1998), where the very large number of stars will make giant branch bump detections statistically very significant.

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Fig. 1.— Theoretical data showing the dependence of the absolute visual magnitude of the red horizontal branch, $M_{V,HB}$, as a function of $\log(Age)$, where in age is in units of years. Open circles connected with solid lines are data from Vassiliadis and Wood (1993), for $Y = 0.25$, classical mixing and $Z = 0.008, 0.004,$ and $0.001$, decreasing in luminosity in that order. Open triangles connected by a dashed line are data from Seidel, Demarque, and Weinberg (1987) for $Y = 0.25$ and $Z = 0.01$. Open squares connected by a dotted line are data from Lattanzio (1991 for $Y = 0.25$ and $Z = 0.001$. The filled squares connected by short dashed lines are from Sarajedini, Lee, and Lee (1995) for $[\text{Fe/H}] = -1.7$ and $-0.7$ dex, where the former is brighter. The solid line in the lower right corner of the figure (labeled) represents the dependence of $M_{V,HB}$ on age adopted in this paper. The zero-point is arbitrary, and will be empirically calibrated.

Fig. 2.— Theoretical data showing the dependence of RGB-bump luminosity, $\log(L)$, and model mass, both in solar units. Filled triangles connected by solid lines are data from Alongi et al. (1993) for $Z = 0.008$, $Y = 0.25$, and classical and overshoot mixing schemes (brighter and fainter respectively). Open squares connected by short-dashed lines are data from Sweigart and Gross (1978) for $Z = 0.01$, classical mixing, and $Y = 0.30$ and 0.20 (brighter and fainter respectively). Filled circles connected with solid lines are data from Vassiliadis and Wood (1993), for $Y = 0.25$, classical mixing and $Z = 0.008, 0.004,$ and $0.001$, increasing in luminosity in that order. The long-dashed line is the mass dependence given by Fusi Pecci et al. (1990), with an arbitrary $\log(L)$ zero-point. The solid line in the lower right corner of the figure (labeled) represents the dependence of RGB-bump luminosity on mass adopted in this paper. The zero-point is arbitrary, and will be empirically calibrated.

Fig. 3.— Theoretical data for the luminosity of the AGB-bump as a function of mass and metallicity. Top panel shows $\log(L)$ as function of $M$ (solar units) for $Z = 0.001$ data; open squares from Castellani et al. (1991) and filled circles from Vassiliadis and Wood (1993). Bottom panel shows $\log(L)$ as a function of $[\text{Fe/H}]$, for $M = 1.50$ (filled circles) and $M = 0.85$ (open squares), same symbols as above. Projected lines from our analytic fit to the data are shown as solid lines. In the top panel, projections at constant metallicities of $[\text{Fe/H}] = -1.7, -1.3,$ and $-0.4$ dex are plotted and labeled. In the bottom panel, projections at constant mass are shown and labeled ($M = 0.85, 0.95, 1.10, 1.30, 1.50$).

Fig. 4.— Summary of theoretical data for the absolute visual magnitudes of the horizontal branch ($M_{V,HB}$, filled squares), the RGB-bump ($M_{V,RB}$, open circles), and the AGB-bump ($M_{V,AB}$, open triangles) plotted as a function of $\log(Age)$, where age is units of years. Each panel represents a different metallicity, $[\text{Fe/H}]$, and is labeled. See also Table 2.
Fig. 5.— Color-magnitude diagram of the SMC cluster NGC 411, derived from archival Hubble Space Telescope WFPC2 data (PC-chip only). Arrow at \( V = 19.43 \) marks the red HB clump mean magnitude. Arrow at \( V = 18.74 \) is the giant branch bump. See text for details of measuring these values.

Fig. 6.— \( \Delta V_{\text{HB Bump}} \)–[Fe/H] diagram, which shows metallicity versus brightness difference between an RGB-bump or AGB-bump and the HB. The filled squares are RGB-bump observational data for Galactic globular clusters (see Table 3), and the solid line labeled “12 Gyr” is the regression through these points. The solid lines show the expected age-shift of \( \Delta V_{\text{HB Bump}} \) for the RGB-bump for ages of 10, 5, and 2 Gyr (labeled), assuming the brightness of the HB has no dependence on age, and an age of 12 Gyr for the Galactic globular clusters. The short-dashed lines show the \( \Delta V_{\text{HB Bump}} \) age-shifts assuming \( M_{V,HB} \) depends on age for the RGB-bump for 10, 5, and 2 Gyr (see text). The dotted lines show the location of AGB-bumps in this diagram for ages of 12, 10, 5 and 2 Gyr, assuming the \( M_{V,HB} \) depends on age. Filled circles are observational data for 4 Galactic globular cluster AGB-bumps (see text). Observational data for 8 SMC clusters shown as open circles with error bars.

Fig. 7.— \( \delta \Delta V_{\text{HB Bump}} \), the distance from the “12 Gyr” line in Figure 5 (Eqn. 7) in magnitudes, as a function of age, in units of Gyr. SMC cluster observational data shown as open circles with error bars. Below each SMC cluster point, we have labeled the metallicity. RGB-bump observational data for Galactic globular clusters shown as filled squares, error bars omitted for clarity (see Table 3). \( \delta \Delta V_{\text{HB Bump}} = 0 \) shown as a solid line. Dotted lines show our theoretical predictions for [Fe/H] = −1.7, −1.3, −0.7, and −0.4 dex, where the latter three lines are very similar. The dashed line shows the approximations to our theoretical predictions, given by Equations 6 & 10 (see text). The normalization of the theoretical lines to \( \delta \Delta V_{\text{HB Bump}} = 0 \) at 12 Gyr is also illustrated here.
Table 1. Reference RGB Coefficients

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Table 2. Theoretical Data

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</tr>
<tr>
<td>NGC 1261</td>
<td>16.70 ± 0.10</td>
<td>16.70 ± 0.05</td>
<td>−1.31 ± 0.09</td>
<td>13.1 ± 1.1</td>
</tr>
<tr>
<td>NGC 1851</td>
<td>16.15 ± 0.02</td>
<td>16.15 ± 0.03</td>
<td>−1.36 ± 0.09</td>
<td>11.6 ± 0.8</td>
</tr>
<tr>
<td>NGC 2808</td>
<td>16.25 ± 0.10</td>
<td>16.20 ± 0.05</td>
<td>−1.37 ± 0.09</td>
<td>14.1 ± 1.3</td>
</tr>
<tr>
<td>NGC 5904</td>
<td>15.11 ± 0.05</td>
<td>15.05 ± 0.05</td>
<td>−1.40 ± 0.06</td>
<td>14.0 ± 1.0</td>
</tr>
<tr>
<td>NGC 6752</td>
<td>13.75 ± 0.15</td>
<td>13.65 ± 0.05</td>
<td>−1.54 ± 0.09</td>
<td>17.0 ± 1.8</td>
</tr>
<tr>
<td>NGC 7006</td>
<td>18.72 ± 0.10</td>
<td>18.55 ± 0.05</td>
<td>−1.59 ± 0.07</td>
<td>13.5 ± 1.1</td>
</tr>
<tr>
<td>NGC 5272</td>
<td>15.65 ± 0.05</td>
<td>15.40 ± 0.05</td>
<td>−1.66 ± 0.06</td>
<td>13.6 ± 0.8</td>
</tr>
<tr>
<td>NGC 5897</td>
<td>16.35 ± 0.15</td>
<td>16.08 ± 0.05</td>
<td>−1.68 ± 0.11</td>
<td>14.6 ± 1.8</td>
</tr>
<tr>
<td>NGC 1904</td>
<td>16.25 ± 0.10</td>
<td>15.95 ± 0.05</td>
<td>−1.69 ± 0.09</td>
<td>14.1 ± 1.3</td>
</tr>
<tr>
<td>NGC 6397</td>
<td>13.00 ± 0.10</td>
<td>12.60 ± 0.10</td>
<td>−1.91 ± 0.14</td>
<td>17.8 ± 1.7</td>
</tr>
<tr>
<td>$MP^B$</td>
<td>15.86 ± 0.05</td>
<td>15.35 ± 0.05</td>
<td>−2.15 ± 0.08</td>
<td>17.2 ± 2.0</td>
</tr>
</tbody>
</table>

$^A$References: Mighell et al. (1998), Fusi Pecci et al. (1990), Sarajedini & Forrester (1995), Chaboyer, Demarque, & Sarajedini (1996), this paper.

$^B$MP is metal-poor, a co-added CMD of M15, M92, & NGC 5466.