A head-tail effect due to lattice nonlinearities in storage rings

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Abstract

The head-tail effect due to the transverse wake force has been studied in relation to the chromaticity by many people. We discuss another type of head-tail effect produced by lattice nonlinearities, especially by amplitude dependent tune shifts. We show that damping of the transverse coherent motion due to the nonlinear smear (Landau damping) is strongly affected by this head-tail effect. An experiment which supports this effect has been already performed at KEK Photon Factory, and surprisingly coincides with the results presented here.

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We present a transverse wake effect which couples to the lattice nonlinearities. The wake force is assumed to be linear in the dipole moment of the beam and is conventionally expressed by $W_1(z)$. The chromaticity, which is an energy dependent tune shift, induces the ordinary head-tail effect. We discuss another type of head-tail effect that is induced by the amplitude dependent tune shift. Phenomenologically, we will observe this effect as a kind of interference between the Landau damping and head-tail damping. Our discussion is devoted to the study in electron storage rings, but similar effect may occur in proton and other storage rings.

This work was motivated by experiments[1, 2, 3, 4] in which the damping of the kicked beam has been observed. The most typical experiment was performed at the KEK Photon Factory (PF) using a fast kicker and a turn by turn monitor[1]. In the experiment, the amplitude of a bunch, that had been kicked transversely, was measured turn by turn for various strengths of the octupole magnets. The bunch coherent motion damps into the equilibrium orbit in a characteristic time. We know three of damping mechanisms: i.e., the Landau (nonlinear smear), the head-tail, and the radiation damping. The kicked bunch showed quite different behaviors as one changes the polarity of the octupole magnets in a parameter region where the Landau damping dominates. The results[2] are summarized as

1. In the lower beam current ($0.5mA$, $N_e = 1.95 \times 10^9/bunch$), only the Landau damping behavior was observed for both polarity of the octupole magnets.
2. In higher beam current,
 - (a) the Landau damping was observed for the negative polarity.
 - (b) the head-tail damping was observed for the positive polarity.
3. This phenomenon was more pronounced when the beam current was higher.

The damping rate seems to be far from the simple sum of the head-tail and Landau damping rates. The radiation damping does not seem to be important, since its characteristic time is much longer. The present theory and simulation explain this damping phenomenon. We treat only the horizontal (x) motion for simplicity. The concrete study was performed for the PF ring whose parameters are shown in Table 1. The current dependent tune shift is the measured value ($d\nu_{\beta,I}/dI = 0.169A^{-1}$), and the wake force was

Table 1: Parameters of the PF ring used for the calculation.

Circumference (C)	187 m
Beam energy (E)	$2.5 GeV$
Current (I)	$5 mA$
Number of electrons (N_e)	1.9×10^{10}
Transverse tunes (ν_x, ν_y)	8.4, 3.305
Synchrotron tune (ν_s)	0.023
Natural bunch length (σ_z)	15mm
Energy spread (σ_δ)	0.00073
Emittances ($\varepsilon_x, \varepsilon_y$)	130, 1.5nm

estimated from the tune shift[5] as $W_1(z) = 1.95 \times 10^{17} z \text{ V/Cm}$. Here we assume that the transverse wake force contributes the tune shift dominantly, and increases linearly in the longitudinal distance ($z(m)$).

The amplitude dependent tune shift is defined by

$$\omega_\beta(J) = \omega_{\beta 0}(1 + aJ), \quad (1)$$

where J is a half of the Courant Snyder invariant: i.e., $J = (\gamma x^2 + 2\alpha x x' + \beta x'^2)/2$, and ω_β is the betatron frequency of the beam particles. The amplitude dependent tune shift causes a tune spread, because beam particles have a distribution, which is characterized by the beam size, in the horizontal amplitude. The head-tail effect including the tune spread has been discussed elsewhere[6]. We would like to emphasize that not only the tune spread but also the tune shift (the sign of a in Eq.(1)) plays an important role in this head-tail effect.

We first consider two particles under a constant transverse wake. We take into account only the dipole mode of the beam, but not the longitudinal motion in this two particle model. The head-tail interactions and synchrotron motion are essential, though they are not visible in the model. We discuss the interactions between the two-particles with large and small amplitude. The horizontal positions of the two particles are represented by x_1 and x_2 . The wake force which particles feel is a function of their barycenter. The equation of motion is expressed as

$$x''_{1,2} + \left(\frac{\omega_{\beta 0}(1 + aJ_{1,2})}{c} \right)^2 x_{1,2} = \mathcal{W} \frac{x_1 + x_2}{2}, \quad (2)$$

where $\mathcal{W} = N_e e^2 W_1(2\sigma_z)/4EC$ (see Table 1). We assume that it is possible to separate the

solution into a slow changing factor and an exponential oscillation factor: i.e., $x_{1,2}(s) = X_{1,2}(s) \exp(-i\omega_{\beta,0} \int (1 + aJ_{1,2}) ds/c)$. We obtain a set of differential equations

$$X'_{1,2} = i\frac{\mathcal{W}}{4}\beta \left[X_{1,2} + \exp \left\{ \pm i a \omega_{\beta,0} \int (J_1 - J_2) ds \right\} X_{2,1} \right]. \quad (3)$$

The solution for $a = 0$ ($\omega_{\beta}(J) = \omega_{\beta,0}$) is well-known[7]. Two eigenvectors, whose norms are approximately invariant, are given as follows,

$$\begin{aligned} X_+ &= \frac{X_1 + X_2}{2} & X_- &= X_1 - X_2 \\ X_+(C) &= \exp(i\Upsilon\nu_s)X_+(0) & X_-(C) &= X_-(0), \end{aligned} \quad (4)$$

where $\Upsilon = \mathcal{W}\beta C/2\nu_s$ and $\beta = c/\omega_{\beta}$ is the horizontal beta function where the wake source is installed. $\Upsilon\nu_s/2\pi$ corresponds to the tune shift due to the transverse wake force. The amplitude of each particle is expressed as

$$|X_{1,2}(C)|^2 = \frac{|X_1(0)|^2 + |X_2(0)|^2}{2} \pm \frac{|X_1(0)|^2 - |X_2(0)|^2}{2} \cos(\Upsilon\nu_s) \mp \text{Im}(X_1^*(0)X_2(0)) \sin(\Upsilon\nu_s). \quad (5)$$

The amplitudes are modulated by the wake force with a period of $2\pi/\Upsilon\nu_s$ turns. We consider using the realistic parameters of Table 1. If the source of wake is put at a position with $\beta = 5m$, then $\Upsilon = 0.19$ and the amplitude modulates with a period of 1400 turns.

We next include the amplitude dependent tune shift. The differential equations Eq.(3) have the integral $|x_1|^2 + |x_2|^2 = \text{const}$. Introducing the new variable $\zeta \equiv |x_1|^2 - |x_2|^2$, a differential equation is obtained as follows[8],

$$\zeta''' = \frac{\zeta'\zeta''}{\zeta} - \left(\frac{a}{2\beta^2} \right)^2 \zeta^2 \zeta'. \quad (6)$$

The differential equation can be solved by the elliptic functions. We obtained relations at $s = 0$ as follows

$$\zeta_0'' = -\frac{\mathcal{W}}{2}\beta\zeta_0 \left\{ \frac{a}{2\beta^2} \text{Re}(X_1^* X_2)_0 + \frac{\mathcal{W}}{2}\beta \right\}. \quad (7)$$

$$\zeta_0' = -\frac{\mathcal{W}}{2}\beta \text{Im}(X_1^* X_2)_0 \quad (8)$$

where the 0 indicates initial values. When the two particles have the same initial betatron phase ($\text{Im}(X_1^* X_2)_0 = 0$), we find $\zeta_0' = 0$. When $\zeta_0 > 0$ ($|X_{1,0}|^2 > |X_{2,0}|^2$), ζ_0'' is expressed as

$$\zeta_0'' \propto -\left(\frac{a\nu_{\beta}}{2\beta} \text{Re}(X_1^* X_2)_0 + \frac{\mathcal{W}\beta C}{4\pi} \right) \sim -a\nu_{\beta}J - \delta\nu_I. \quad (9)$$

We found that the behavior of ζ near $s = 0$ is determined by a competition between the amplitude dependent tune shift and the head-tail tune shift. If a is larger than a value determined by Eq.(9), the difference between $|X_1|^2$ and $|X_2|^2$ decreases near $s = 0$, elsewhere it increases.

Here we do not go into the analytical approach further more, since we got qualitative features. Numerical analysis is presented hereafter. We first solved the two particle model of Eq.(2) numerically using the parameters in Table 1. The amplitude dependent tune shift is examined for $a\nu_\beta = \pm 1000$. These two $a\nu_\beta$ correspond to negative and positive ζ_0'' of Eq.(9): i.e., $a\nu_\beta J = \pm 2.5 \times 10^{-3}$ and $\delta\nu_I = 0.85 \times 10^{-3}$. We consider the case that the beam, whose size is $\sigma_x \sim 1mm$, is kicked $5mm$ horizontally. Initial amplitudes are $x_1 = 6mm$ and $x_2 = 4mm$, and $x'_{1,2} = 0$. Fig.1 shows evolutions of each $|X|$ obtained by solving the two particle model. We obtained different behaviors of $|X|$ for each sign of a as is expected in Eq.(9). The amplitudes of two particles intersect each other for positive a , while are separated for negative a . The modulation frequency for positive a ($\sim 550\text{turn}$) was faster than that of $a = 0$ (1400turn).

Figure 1: Evolution of $|X_1|$ and $|X_2|$. (a) and (b) are evolution of $|X|$'s for a positive and negative amplitude dependent tune shift.

This modulation of $|X|$ is essential for the Landau damping due to the amplitude dependent tune shift (spread). In the symplectic motion conserving $|X|$, the betatron phases, whose phase advances are determined by the amplitudes of particles, are dispersed by the tune spread corresponding the amplitude difference. If $|X|$ exchange in the process, the phase which is dispersed is gathered again, with the result that the coherent amplitude does not decrease. Conversely, splitting $|X|$ amplifies the betatron phase dis-

persion and the damping of the coherent amplitude. To treat the phenomenon caused by this mechanism accurately, the two particle model seems to be a bit inadequate. With the process of the Landau damping, the wake force is reduced by the damping of the dipole moment, and the modulation will be also reduced. In the two particle model, the dipole moment could not be estimated exactly because of poor statistics. Multi-particle tracking is essential to understand the actual phenomenon due to this head-tail effect.

We performed a realistic simulation for the PF ring. All magnets and cavities are expressed by 6 dimensional symplectic maps[9]. An element inducing the wake force was installed at one position in the ring. Needless to say, the synchrotron motion and the head-tail interactions are taken into account by the maps. A simplified model using a linear map, an octupole magnet and the wake force element showed the same results qualitatively, therefore the exact lattice information was not needed. It is better to present the result using the exact map in order to compare with the experiment more quantitatively. Macro-particles were generated randomly using a Gaussian distribution in the 6 dimensional phase space with the beam size in Table 1. Macro-particles are shifted $5mm$ to x direction at the launching point. The octupole magnets were excited by the same field strength for both polarity. The amplitude dependent tune shift is $a\nu_\beta = 2344$ and -2848 [5], and the difference of their absolute values is offset due to the sextupole magnets. We tracked 1000 macro-particles in normal simulation, and tracked also 2000 particles to check the statistics. There was no difference between the two cases.

Fig.2 shows evolutions of $|X| = \sqrt{2\beta J}$ for five particles chosen arbitrary. We found intersections of $|X|$ around 200-th and 500-th turn for $a > 0$, as seen in the two particle model. For $a < 0$, $|X|$'s of some particles decrease and do not increase again. The difference from the two particle model is due to the statistics. Though there are a few differences between the two particle model and the exact simulation, the important point of view is kept: that is, positive a causes the mixing of the amplitudes, while negative a causes a split.

Fig.3 shows the phase space distributions of the macro-particles after 500 turns. For $a < 0$, the betatron phases of macro-particles are ordered like a vortex, and the nonlinear smear evolves smoothly as is expected. On the other hand, for $a > 0$, many particles are localized around a betatron phase, and the nonlinear smear is much weaker than for $a < 0$.

The evolution of x obtained by the simulation ((a) and (b)) and the experiment ((c)

Figure 2: Evolution of $|X| = \sqrt{2\beta J}$ for five macro-particles chosen arbitrary. (a) and (b) are evolution of $|X|$'s for a positive and negative amplitude dependent tune shift.

Figure 3: Phase space distribution of macro-particles after 500 revolutions. (a) and (b) are distributions for a positive and negative amplitude dependent tune shift.

and (d)) are shown in Fig.4. We first focus at the result of the simulation. Those for $a > 0$ and $a < 0$ are evidently different. We found very long damping time for $a > 0$, and something like an echo around ~ 450 -th turn. This structure will occur due to the exchange of $|X|$ shown in Fig.1 and 2. On the other hand, very rapid damping was found for $a < 0$. We next look at the result of experiment. The behavior of the betatron oscillation for each sign of a completely coincided with the simulation. The echo was observed around ~ 550 -th turn at the experiment. It is certain that the phenomenon discussed above actually comes into existence in the experiment.

Figure 4: Betatron oscillation affected by the wake force. (a) and (b) show betatron oscillation obtained by the simulation for positive and negative amplitude dependent tune shift. (c) and (d) show betatron oscillation by the experiment at the PF ring with the same condition of (a) and (b).

We investigated a new type of head-tail effect due to the transverse wake force. The head-tail effect is caused by the amplitude dependent tune shift, and is closely related to the Landau damping. The wake force works by mixing the amplitudes of particles. The positive amplitude dependent tune shift amplifies the mixing, while negative shift

suppresses the mixing or leads to a split of the amplitudes. The amplitude mixing works to suppress the Landau damping. The betatron phases of the particles, which are normally dispersed by the amplitude dependent tune shift, are not dispersed, though they are modulated by the mixing. On the other hand, the splitting of the amplitude amplifies the dispersion of the betatron phase. The Landau damping is suppressed for the positive amplitude tune shift, while is amplified for the negative tune shift. The simulation with the multi-particle tracking reproduced this mechanism, and surprisingly coincided with the experiment at the PF ring. Typically the phenomenon occurs in a region where the amplitude dependent tune shift and head-tail tune shift are comparable. This phenomenon will be instructive in the sense that the corrective effect interferes with the Landau damping.

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