Small-Angle Scattering of X-Rays from Extragalactic Sources by Dust in Intervening Galaxies

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ABSTRACT

Gamma-ray bursts are now known to be a cosmological population of objects, which are often accompanied by X-ray and optical afterglows. The total energy emitted in the afterglow can be similar to the energy radiated in the gamma-ray burst itself. If a galaxy containing a large column density of dust is near the line of sight to a gamma-ray burst, small-angle scattering of the X-rays due to diffraction by the dust grains will give rise to an X-ray echo of the afterglow. A measurement of the angular size of the echo at a certain time after the afterglow is observed yields a combination of the angular diameter distances to the scattering galaxy and the gamma-ray burst that can be used to constrain cosmological models in the same way as a time delay in a gravitational lens. The scattering galaxy will generally cause gravitational lensing as well, and this should modify the shape of the X-ray echo from a circular ring.

The main difficulty in detecting this phenomenon is the very low flux expected for the echo. The flux can be increased when the gamma-ray burst is highly magnified by gravitational lensing, or when the deflecting galaxy is at low redshift. X-ray echos of continuous (but variable) sources, such as quasars, may also be detectable with high-resolution instruments and would allow similar measurements.

Subject headings: dust - gamma rays: bursts - gravitational lensing - X-rays: general

1. Introduction

The presence of dust grains along the line of sight to an X-ray point source will result in a halo of scattered X-rays around the source, which will be time delayed relative to the X-rays that were not scattered. The effect is caused by small-angle scattering of the X-rays by the dust grains, due to the diffraction of the wavefront when some of the X-rays are absorbed by the dust grain. The process has been discussed previously by several authors (Overbeck 1965; Slysh 1969; Hayakawa 1970; Trümper & Schönfelder 1973; Alcock & Hatchett 1978). The applications that were considered were generally restricted to galactic sources. The first observations of the
phenomenon were made by the Einstein observatory (Mauche and Gorenstein 1986 and references therein).

In the case of an extragalactic X-ray source with a large column density of dust on the line of sight, the dust is likely to be concentrated on an intervening galaxy. Therefore, all the scattering takes place on a single screen, implying that the time delay is uniquely determined by the angular separation from the point source. Thus, a variable X-ray quasar located behind a dusty galaxy should show a variable intensity pattern in the surrounding X-ray halo, with the radial dependence reflecting the past lightcurve of the quasar. If the time delay at a given angular separation from the source can be measured, a combination of the angular diameter distances to the scattering galaxy and the X-ray source is obtained, using a method analogous to the measurement of angular diameter distances from time delays in gravitational lensing (Refsdal 1966). This method of measuring distances was discussed by Trümper & Schönfelder for galactic sources; in this case, the dust is likely to be distributed along the line of sight, and the past lightcurve of the X-ray source should be smeared out in the X-ray halo depending on the dust distribution.

Recently, gamma-ray bursts have been proved to be of cosmological origin, and the associated X-ray and optical afterglows have been discovered (van Paradijs et al. 1997; Djorgovski et al. 1997; Metzger et al. 1997; Costa et al. 1997). The presence of the afterglow was predicted from the generic model of interaction of a relativistic fireball with a surrounding medium (Vietri 1997; Mészáros & Rees 1997; Wijers, Rees, & Mészáros 1997). Gamma-ray bursts may prove to be a useful source for doing this type of observation.

The paper is organized as follows. In §2 we derive the general characteristics of the scattered X-rays: the angular size of the halo, the time delay, and the intensity. In general, gravitational lensing effects are likely to be present, since a dusty galaxy on the line of sight will typically cause a gravitational deflection of the same order as the scattering angle of the X-rays; this is analyzed in §3. Our results are discussed in §4.

2. Characteristics of the X-ray Echo

We consider an X-ray source from which, in the absence of scattering, a total fluence (or energy per unit area) $\epsilon_0$ is received instantaneously; i.e., the flux from the source can be written as $F = \epsilon_0 \delta(t - t_0)$, where $t_0$ is the time when the non-scattered X-rays arrive. This may be a good approximation for a gamma-ray burst afterglow. The source is at redshift $z_s$ and angular diameter distance $D_s$, and the dusty galaxy is at redshift $z_d$ and angular diameter distance $D_d$. Any scattered X-rays received at an angle $\theta$ from the source will arrive with a time delay $t_d$ given by:

$$c t_d = \frac{(1 + z_d) D_d D_s}{D_{ds}} \frac{\theta^2}{2} ,$$

(1)
where $D_{ds}$ is the angular diameter distance from the deflector to the source. This equation is generally valid for any cosmological model, and is derived in the same way as the equation for the time delay in gravitational lensing (Refsdal 1966).

Thus, the “echo” of an instantaneous pulse of X-rays consists of a ring of angular size $\theta$ at time $t_d$ after the pulse has been received.

### 2.1. Intensity of the X-ray Echo

To calculate the intensity of the X-ray echo, we need to know the scattering cross section as a function of the scattering angle $\alpha$. We consider spherical dust grains of radius $a$. The typical scattering angle of X-rays of wavelength $\lambda$ is $\alpha \sim \lambda/(2\pi a)$; therefore, X-rays will generally be scattered at very small angles by grains with sizes $a \sim 1\mu m$. The total scattering cross section of a dust grain is given by $\sigma_T = Q_s \pi a^2$, where $Q_s$ is the scattering efficiency. For the case of magnesium silicate, a typical material in dust grains, the scattering efficiency can be approximated as (see van de Hulst 1957, Alcock & Hatchett 1978)

$$Q_s \simeq 0.7 \left( \frac{\lambda}{2 \ \mu m} \right)^2 \left( \frac{a}{1 \mu m} \right)^2,$$

if $Q_s < 1$;

$$Q_s \simeq 1.5,$$ otherwise. \hspace{1cm} (2)

The transition between the two regimes occurs when the phase shift across the dust grain, $| \rho | = (4\pi a)/\lambda | m - 1 |$ (where $m$ is the refractive index of the dust), reaches unity. For magnesium silicate, $| \rho | \simeq (\lambda/2 \ \mu m) (a/1 \mu m)$. The detailed form of $Q_s$ is shown in Figure 1 of Alcock & Hatchett (the numbers in the horizontal top and bottom axes in this figure are erroneous; see Figs. 32 and 33 in van de Hulst 1957 for similar curves).

The differential cross section can be written as

$$\frac{d\sigma}{d\alpha} = \sigma_T \frac{2\pi a}{\lambda} \sigma_n\left( \frac{2\pi a}{\lambda} \alpha \right).$$ \hspace{1cm} (3)

The dimensionless function $\sigma_n$ is normalized to $\int_0^\infty dx \sigma_n(x) = 1$. This expression is valid in the limit of small scattering angles (this is why the upper limit in the integral normalizing $\sigma_n$ is replaced by infinity). The function $\sigma_n$ can be calculated for spherical grains under different approximations, depending on the value of $| \rho |$. The result is shown in Figure 2 of Alcock & Hatchett; generally, $\sigma_n(x)$ rises linearly with $x$ for $x \ll 1$ (as required by continuity of the scattering cross section per unit solid angle for zero deflection), peaks near $x \simeq 1.5$, and then declines rapidly and may have oscillations of decreasing amplitude at large $x$ (analogous to the familiar diffraction pattern of a circular aperture). Practically all the X-rays are scattered at angles with $x < 4$. Of course, dust grains should not be exactly spherical in practice, and there should be a wide distribution of grain sizes.
Considering now a variable X-ray source with an unabsorbed flux as a function of time $F_0(t)$, and assuming that the fraction of X-rays scattered in the intervening galaxy is small (so that multiple scatterings can be neglected), the surface brightness $S$ of the X-ray echo at angular separation $\theta$ and position angle $\phi$ from the source is

$$S(\theta, \phi; t) \sin \theta \, d\theta \, d\phi = F_0(t - t_d) \, \tau(\theta, \phi) \, \sigma_T^{-1} \, \frac{d\sigma}{d\alpha} \, d\alpha \, (d\phi/2\pi).$$

(4)

This equation expresses the fact that the flux received along a direction within $(d\theta, d\phi)$ is equal to the total flux of the source, times the probability of scattering (equal to $\tau(\theta, \phi)$, which is assumed to be small), times the probability that the photon is scattered in the direction within $(d\alpha, d\phi)$. Expressing the scattering angle in terms of the angular separation, $\alpha = \theta (D_s/D_d)$, we have

$$S(\theta, \phi; t) = \frac{F_0(t - t_d)}{\sin \theta} \, \tau(\theta, \phi) \, \frac{a D_s}{\lambda D_d} \, \sigma_n \left( \frac{2\pi a D_s}{\lambda D_d} \theta \right).$$

(5)

When the variable source emits most of its energy on a time much shorter than $t_d$ (i.e., the flux is approximated as $F_0(t) = \epsilon_0 \delta(t - t_0)$ as before), the echo is a narrow ring with total flux $F_e$ given by

$$F_e(t) = \frac{\epsilon_0}{t_d} \, \frac{a D_s}{2\lambda D_d} \, \theta \, \sigma_n \left( \frac{2\pi a D_s}{\lambda D_d} \theta \right) \int_0^{2\pi} d\phi \, \tau(\theta, \phi),$$

(6)

where the angle $\theta$ is to be calculated from the time delay $t_d = t - t_0$ from equation (1).

The optical depth for X-ray scattering is due to dust grains of different sizes, and can be expressed as $\tau = \int N_d(a) \, \sigma_T(a) \, da$, where $N_d(a) \, da$ is the column density of dust grains of radius $a$. The dust column density can be written as $N_d(a) = N_b Z_d(a) (4\pi a^3 \rho_d/3m_p)^{-1}$, where $N_b$ is the column density of baryons in the interstellar medium of the intervening galaxy, $Z_d(a) \, da$ is the fraction of mass of the interstellar matter in the form of grains of radius $a$ (i.e., the “metallicity” in dust grains of radius $a$), $\rho_d$ is the dust density, and $m_p$ is the proton mass. We assume that $Z_d(a)$ is independent of position. Substituting into equation (6), we obtain

$$F_e(t) = \frac{\epsilon_0}{t_d} \, \frac{3m_p D_s}{8\rho_d \lambda D_d} \, \theta \, \int_0^{2\pi} d\phi \, N_b(\theta, \phi) \, \int da \, Z_d(a) \, Q_s(\lambda, a) \, \sigma_n \left( \frac{2\pi a D_s}{\lambda D_d} \theta \right).$$

(7)

In order to proceed further, we will need to discuss the model for the function $Z_d(a)$. Studies of interstellar reddening indicate that $Z_d(a) \propto a^{-0.5}$ for small grains, $a \lesssim 0.1 \mu$m, (Mathis, Rumpf, & Nordsieck 1977). The function must turn over at larger size so that the total mass in grains converges. While most of the mass seems to be in grains of size near 0.1 $\mu$m (Martin 1978; Savage & Mathis 1979; Mitsuda et al. 1990), it seems natural that the mass is distributed over a wide range of $a$. Thus, as an example, we shall assume $Z_d(a) \propto a^{-1.5}$ for $a > 0.1 \mu$m. The scattering of X-rays in the regime we shall discuss is dominated by grains with $a \gtrsim 1 \mu$m, so our estimate of the flux is sensitive to the uncertain form of $Z_d(a)$, which may vary depending on the interstellar environment of the galaxy. For the adopted model, if $Z_t$ is the total metallicity in dust grains, we have $aZ_d(a) = Z_t/4 (a/0.1 \mu$m)$^{-0.5}$. A chemically evolved galaxy can be reasonably assumed to have $Z_t \sim Z_\odot$. 


Let us now examine carefully the terms appearing in the integral over the dust grain radius in equation 7. The function $\sigma_n$ is sharply peaked at the radius $a_p$ given by $2\pi a_p D_s \theta / (\lambda D_{ds}) \simeq 1.5$ (see Fig. 2 of Alcock & Hatchett), and is normalized to $\int \sigma_n(x) dx = 1$. As long as $a > 1 \mu m (3 \AA / \lambda) \equiv a_1$, we can use $Q_s \simeq 1.5$ (see the discussion above; this value for $Q_s$ is an average over the range $1 \lesssim a/a_1 \lesssim 10$, which will be the range of interest). Smaller grains are not effective for X-ray scattering because $Q_s$ is very small. For the assumed shape $Z_d(a) \propto a^{-1.5}$, and if $a_p > a_1$, a sufficient approximation is obtained by substituting $Z_d(a)$ in equation 7 by its value at the peak of the function $\sigma_n$. Thus, when $a_p > a_1$, the integral can be approximated as
\[
\int da Z_d(a) Q_s(\lambda, a) \sigma_n \left( \frac{2\pi a D_s}{\lambda D_{ds}} \theta \right) \simeq a_p Z_d(a_p) = Z_t/4 - \frac{a_p}{0.1 \mu m}^{-0.5}.
\]

The condition $a_p > a_1$ implies
\[
a_p \simeq 1.5 \frac{\lambda D_{ds}}{2\pi D_s \theta} > 3.8 \mu m \left( \frac{D_s}{D_{ds}} \frac{1'''}{\theta} \right)^{1/2},
\]
or
\[
\lambda > 0.78 \AA \left( \frac{D_s}{D_{ds}} \frac{\theta \lambda}{1'''} \right)^{1/2}.
\]
For shorter wavelengths (when the angular separation $\theta$, and therefore the time delay, is fixed), the integral rapidly becomes much smaller; essentially, the intensity of the scattered rays is greatly reduced because the grains that are small enough to cause scattering at an angle $\theta$ are mostly transparent to the radiation.

Thus, when equation (10) is satisfied, the flux of the X-ray echo is
\[
F_e \simeq \frac{\epsilon_0}{t_d} \frac{6\pi m_p D_s \theta}{8\rho_d \lambda D_{ds}} \bar{N}_b(\theta) a_p Z_d(a_p)
\]
\[
\simeq 0.03 \frac{\epsilon_0}{t_d} \frac{D_s}{D_{ds}} \frac{1 \AA}{\lambda} \frac{\theta}{1'''} \frac{\bar{N}_b}{10^{22} \text{cm}^{-2}} \frac{Z_t}{0.02} \frac{a_p}{0.1 \mu m}^{-0.5},
\]
where $\bar{N}_b(\theta)$ is the mean gas column density over the ring of angular size $\theta$, and we have used the value $\rho_d = 3 \text{g cm}^{-3}$ (appropriate for magnesium silicate). Notice that $\lambda$ is the wavelength in the frame of the deflector in all our equations (i.e., the observed wavelength is $\lambda(1 + z_d)$).

As an example, we consider a gamma-ray burst at redshift $z \sim 1$ similar to GRB970228. The total X-ray fluence of this burst in the energy band from 2 to 10 keV was $4 \times 10^{-6} \text{erg cm}^{-2}$, about 40% of the fluence in the gamma-ray burst itself in the band 40 – 700 keV (Costa et al. 1997; van Paradijs et al. 1997; Wijers, Rees, & Mészáros 1997). A large fraction of the X-ray fluence was emitted in the beginning of the afterglow, about 50 seconds after the burst, and the flux decayed as $t^{-1.33}$ (Costa et al. 1997). The X-rays that can be scattered most effectively are in the range 2 to 6 keV (at lower energies, the scattering is large and the X-rays arrive with a very long time delay, and at higher energies the condition $a_p > a_1$ is not obeyed). We take the fluence in this narrower
energy band to be $2 \times 10^{-6}$ erg cm$^{-2}$. For a time delay of one year, $\epsilon_0/t_d = 10^{-13.2}$ erg s$^{-1}$ cm$^{-2}$. If the deflector is at $(1 + z_d)D_d \sim 10^3$ Mpc, and taking $D_s/D_{ds} \simeq 2$, then equation (1) gives $\theta \simeq 3''$, and equations (9) and (10) give $\lambda > 2$ Å, $a_p > 1.5\mu$m. Using the fiducial values of $\bar{N}_b$ and $Z_t$ in equation (12), and $\lambda = 2$ Å, we find a flux $F_e \sim 10^{-15}$ erg cm$^{-2}$ s$^{-1}$. At $z_s = 1$, the corresponding luminosity is $L_e \equiv 4\pi(1 + z_s)^4D^2_sF_e \simeq 0.02E_0/t_d \sim 5 \times 10^{42}$ erg s$^{-1}$. This luminosity is much larger than that of normal galaxies, so the X-ray echo can be much brighter than any intrinsic emission from the intervening galaxy.

We see that the dust grains that can produce these X-ray echos are generally of a large size compared to most of the grains, which are responsible for interstellar reddening. Small dust grains should of course give rise to X-ray echos of larger angular size, but these are more difficult to observe because of the longer time delay they imply, and because most random intervening galaxies are not larger than a few arc seconds.

If the X-ray echo can be resolved, and the angular radius of the ring is measured at a time $t_d$ after the known occurrence of the gamma-ray burst, then equation (1) can be used to calculate the value $D_dD_s/D_{ds}$. Assuming that the redshifts $z_d$ and $z_s$ are known, this gives a useful constraint on the Hubble constant, as well as the value of the radius of curvature and cosmological constant of the universe. These can be constrained separately once the observation is done in several sources at different redshifts. If the X-ray echo is not resolved, one might still be able to infer the angular size of the ring by studying the intervening galaxy in optical and infrared light, and inferring the distribution of dust. The lightcurve of the unresolved X-ray echo might then reveal the region of the galaxy that the echo is moving through. For example, if the intervening galaxy has a smooth dust density distribution peaked at the center, but the position of the gamma-ray burst is displaced from the center of the galaxy, then the time when the X-ray echo peaks in intensity should correspond to the time when the echo has passed through the center of the galaxy.

The main difficulty in detecting the X-ray echo will be the very low flux expected, $\sim 10^{-15}$ erg cm$^{-2}$ s$^{-1}$. For example, the few X-ray clusters of galaxies detected so far at high redshift have fluxes above $10^{-13}$ erg cm$^{-2}$ s$^{-1}$ (Luppino & Gioia 1995) (however, the X-ray echo should be detectable at a lower flux than an X-ray cluster, because of its small angular extent). The faintest X-ray flux detectable with the Advanced X-ray Astrophysics Facility should be of order $10^{-15}$ erg cm$^{-2}$ s$^{-1}$. Other future missions (such as XMM and Constellation X) have much larger apertures and could reach fainter fluxes, but with lower spatial resolution, so the photon noise is then dominated by the X-ray background for this very low flux.

### 3. Effects of Gravitational Lensing

The X-ray echo we have discussed should be visible when an intervening galaxy containing large amounts of dust is close to the line of sight to a gamma-ray burst. The gravitational field of the intervening galaxy will very often also deflect the light-rays in a very significant way. In fact,
the typical deflection angle of gravitational lenses caused by ordinary galaxies is of the same order as the scattering angles of the X-ray echos we have discussed in the previous section.

In the presence of gravitational lensing, equation (1) giving the time delay needs to be modified to

\[ ct_d = \frac{(1 + z_d) D_d D_s}{D_{ds}} \left[ \frac{\theta^2}{2} - \psi(\theta, \phi) \right], \]

where \( \phi \) is the azimuthal angle. The projected potential \( \psi \) is determined by \( \nabla^2 \psi = 2\kappa \), where the Laplacian operator differentiates with respect to the angular coordinates, \( \kappa = \Sigma/\Sigma_{\text{crit}} \) is the convergence, \( \Sigma \) is the surface density of mass, and \( \Sigma_{\text{crit}} = c^2/(4\pi G) D_s/(D_d D_{ds}) \) is the critical surface density (e.g., Blandford & Narayan 1992).

If the mass of the galaxy itself were negligible, and the gravitational lensing were caused by a smooth mass distribution around the galaxy (due to a possible cluster of galaxies of which the intervening dusty galaxy could be part of, or any large-scale structure density fluctuations present along the line of sight), then we can approximate the time delay surface with a second-order expansion. The images of background sources are then stretched along the two orthogonal axes of the shear by factors \((1 - \kappa - \gamma)^{-1}\) and \((1 - \kappa + \gamma)^{-1}\), where \( \gamma \) is the shear (determined by the second derivatives of the projected potential). The X-ray echo would also be stretched by the same factors, becoming an ellipse. For high magnification, the X-ray echo increases to a given angular size in a much shorter time than in the absence of lensing, therefore increasing its flux through the factor \( \epsilon_0/t_d \) in equation (12).

When the mass of the galaxy is important, the gamma-ray burst may be multiply imaged. In this case, the X-ray echo should follow a curious evolution that could in principle provide a detailed map of the potential of the lensing mass. Considering a typical case of a three-image lens, with the third image being near the core of a galaxy and generally demagnified by a large factor, one should first see the image of the gamma-ray burst in the minimum of the time delay surface. This image would be followed by a highly elongated, arc-like X-ray echo expanding around the image. The arc would then increase in size and progressively bend around the center of the galaxy, until its two tips would touch on the opposite side of the first image. The second image of the gamma-ray burst should then arrive from the saddle-point of the time delay surface. The X-ray echo should then break up into two rings, one expanding out (with a shape that would continue mapping the mass distribution around the galaxy), and the other collapsing toward the center to eventually disappear in the third image of the gamma-ray burst (at the maximum of the time delay surface), expected to be very faint for the usual centrally peaked density profiles of galaxies.

Gravitationally lensed X-ray echos should generally be brighter than non-lensed ones, because of the shorter time it takes for the echo to cover a given solid angle. In addition, the required scattering angle is reduced, and this can only result in an increased surface brightness for the echo (this increase is probably small, because most grains are small enough to produce scattering angles much larger than a few arc seconds). The observed time delays can again be used to obtain the
product \( D_d D_s / D_{ds} \), as in other gravitational lenses. The advantage here is that, if the evolution of the X-ray echo can be closely monitored, the shape of the lensing potential can be mapped, eliminating the usual modeling uncertainties in other gravitational lenses. Notice, however, that the well-known “\( \kappa \)-degeneracy” is not eliminated: if a uniform sheet of matter with surface density \( \Sigma = \kappa_0 \Sigma_{\text{crit}} \) is added to the lens, resulting in the addition of \( \kappa_0 \theta^2 / 2 \) to the projected potential, and the potential \( \psi \) is multiplied by \( 1 - \kappa_0 \), the new time delay surface differs from the initial one only by the constant \( 1 - \kappa_0 \), which can be absorbed into the factor \( D_d D_s / D_{ds} \). Fortunately, this is the only systematic uncertainty that remains if the mapping of the X-ray echo can yield an accurate measurement of the shape of the time delay surface.

4. Discussion

Gamma-ray bursts having a dusty galaxy near their line of sight should be followed by an X-ray echo of the X-ray afterglow resulting from small-angle X-ray scattering by the dust. We have discussed the expected properties of these echoes, including their angular extent as a function of time, and their flux. The main difficulty in detecting these X-ray echoes will be their very low flux. Given the intensity of the observed X-ray afterglow following a gamma-ray burst, and the identification of any dust-rich galaxy near the line of sight from optical and infrared observations, it should be possible to predict approximately the flux of and time when the echo should be observable, and to estimate if it could be detected.

There are some cases where the flux of the X-ray echo of a gamma-ray burst afterglow could be substantially higher than our estimate in §2. One possibility is the presence of gravitational lensing, which we have discussed in §3. Another possible case where a particularly bright X-ray echo might be observed is when the scattering galaxy is at very low redshift. In this case, the dust could have a larger angular extent, and the time delay should be much shorter for a given angle. For example, if \((1 + z_d)D_d = 100 \text{ Mpc}\), then the angular size is \( \theta \simeq 6'' \) when the time delay is two months, and for the fluence \( \epsilon_0 = 2 \times 10^{-6} \text{ erg cm}^{-2} \), the flux of the echo is now close to \( 10^{-14} \text{ erg cm}^{-2} \text{ s}^{-1} \). It needs to be pointed out that, given the presence of intervening dust, the probability density of the redshift of the intervening galaxy is constant at low redshift, because the cross section of a galaxy to produce an X-ray echo is proportional to its angular size (as opposed to the cross section for gravitational lensing, which is constant at low redshift for lenses with an isothermal profile).

X-ray echos may also be searched around X-ray quasars with a dusty galaxy near their line of sight. In this case, the fluence \( \epsilon_0 \) should be the integrated variable flux over some feature in the lightcurve of the quasar, which can then be seen reverberated in the scattered echo with a measured time delay. Of course, the detection of the X-ray echo of a constant source would also be of great interest, even without the measurement of the time delay; in this case, high angular resolution is needed to separate the X-ray echo from the point spread function of the quasar, and any X-ray emission from a possible cluster around the quasar or intervening galaxy.
I thank the anonymous referee for helpful comments.
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This preprint was prepared with the AAS LaTeX macros v4.0.