Properties of U(1) lattice gauge theory with monopole term *

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In 4D compact U(1) lattice gauge theory with a monopole term added to the Wilson action we first reveal some properties of a third phase region at negative $\beta$. Then at some larger values of the monopole coupling $\lambda$ by a finite-size analysis we find values of the critical exponent $\nu$ close to, however, different from the Gaussian value.

1. INTRODUCTION

In recent higher-statistics studies of 4D compact U(1) lattice gauge theory with the Wilson action [1] and with this action supplemented by a double charge term [2] for increasing lattice size the critical exponent $\nu$ has turned out to decrease towards $\frac{1}{4}$, the value characteristic of a first-order transition. Also stabilization of the latent heat has been observed. Thus, there are now rather strong indications that in those cases the transition is of first order. In contrast to this, for the action where a monopole term with coupling $\lambda$ is added, at $\lambda = 0.9$ the critical exponent has been found to be characteristic of second order [3].

In [3] the value $\beta_{cr}$ at the transition from the confinement phase to the Coulomb phase has turned out to decrease with $\lambda$ and to get negative below $\lambda = 1.2$. The symmetry $\beta \rightarrow -\beta$, $U_0 \rightarrow -U_0$ of the Wilson action at $\lambda = 0$ gives rise to a transition at $\beta = -1$ in addition to the one at $\beta = 1$ [4]. For $\lambda \neq 0$ the indicated symmetry is violated by the monopole term. At $\lambda = \infty$ only the transition at negative $\beta$ persists and occurs at about $\beta = -0.7$ [4,5].

Here we have checked the occurrence of such transition at negative $\beta$ also at intermediate values of $\lambda$ determining the maximum of the specific heat $C_{max}$. It turns out that there is a transition line extending from $(\lambda, \beta) = (0, -1)$ to $(\infty, -0.7)$. Since from the properties we have observed so far we have no indication of a further subdivision of the region below this line, we consider it a third phase. Figure 1 gives an overview of the phase regions as they are according to our present knowledge. It includes the line separating confinement and Coulomb phases obtained in [3] and the data at negative $\beta$ found here.

$P_{\text{net}}$, the probability to find an infinite network of monopole currents (where “infinite” on finite lattices is to be defined in accordance with the boundary conditions), has turned out to provide an unambiguous characterization of the confinement phase and the Coulomb phase [6]. For the finite-size analyses. The respective results are presented in Sect. 3.

2. PHASE REGIONS

In [3] the value $\beta_{cr}$ at the transition from the confinement phase to the Coulomb phase has turned out to decrease with $\lambda$ and to get negative below $\lambda = 1.2$. The symmetry $\beta \rightarrow -\beta$, $U_0 \rightarrow -U_0$ of the Wilson action at $\lambda = 0$ gives rise to a transition at $\beta = -1$ in addition to the one at $\beta = 1$ [4]. For $\lambda \neq 0$ the indicated symmetry is violated by the monopole term. At $\lambda = \infty$ only the transition at negative $\beta$ persists and occurs at about $\beta = -0.7$ [4,5].
periodic boundary conditions used here “infinite” means topologically nontrivial in all directions. Taking the values 1 and 0 in the confinement and Coulomb phases, respectively, $P_{\text{net}}$ is very efficient to discriminate between those phases. In contrast to this in the third phase at fixed $(\lambda, \beta)$ we have observed values 0 as well as 1 for $P_{\text{net}}$. That $P_{\text{net}}$ provides no longer a criterion in the third phase can be understood by noting that the monopole quantities are not invariant under the transformation $\beta \rightarrow -\beta, U_\square \rightarrow -U_\square$.

As a characteristic feature of the third phase we have found that different states exist between which transitions in the simulations are strongly suppressed. We have observed this phenomenon at various negative $\beta$ in the $\lambda$ range from 0 to 2.5. Typical examples of time histories of the average plaquette $\epsilon$ are given in Figure 2.

The values of $\epsilon$ increase as $\beta$ gets more negative. They are somewhat below 2 which indicates that at sufficiently negative $\beta$ the average value of $\cos \Theta$ gets close to $-1$. In view of the the symmetry $\beta \rightarrow -\beta, U_\square \rightarrow -U_\square$ of the Wilson action the correspondence of positive $\cos \Theta$ to positive $\beta$ and of negative $\cos \Theta$ to negative $\beta$ is conceivable.

The origin of the different states is not yet clear. Such states are similarly observed in spin glasses and frustrated systems, and also with spontaneous breaking of $Z(N)$ in finite-temperature SU($N$) gauge theory.

3. CRITICAL PROPERTIES

At $\lambda = 1.1$ and $\lambda = 0.8$ for each lattice size considered Monte Carlo simulations have been performed at a number of $\beta$ values in the critical region. Multihistogram techniques have been applied to evaluate the data and the errors have been estimated by Jackknife methods. In the finite-size analysis in addition to the specific heat and the Challa-Landau-Binder (CLB) cumulant complex zeros of the partition function, in particular the Fisher zero $z_0$ closest to the $\beta$ axis, have been used.
Figure 3. Maximum of specific heat $C_{\text{max}}$ as function of lattice size $L$ for $\lambda = 0.8$, 0.9 and 1.1 at transition point $\beta_{\text{cr}}$ between confinement and Coulomb phases.

For $d = 4$ the maximum of the specific heat is expected to behave as $C_{\text{max}} \sim L^4$ if the phase transition is of first order and as $C_{\text{max}} \sim L^2$ if it is of second order, where $\alpha$ is the critical exponent of the specific heat and $\nu$ the critical exponent of the correlation length.

In Figure 3 we present the results for $C_{\text{max}}$ obtained on various lattices. They include data from simulations of the present investigation with $\lambda = 1.1$ and $\lambda = 0.8$ and ones from simulations of Ref. [3] with $\lambda = 0.9$.

The fits to the data in Figure 3 give values for $\nu$ far from 4, i.e. far from what would be expected for first order. Using these values and the hyperscaling relation $\alpha = 2 - d \nu$ the values for $\nu$ listed in Table 1 are obtained. They are seen to be close to $\frac{1}{2}$. Thus in any case to conclude on second order appears quite safe.

Similar results are obtained for the minimum of the CLB cumulant

$$U_{\text{CLB}} = \frac{1}{3} (1 - \frac{\langle E^4 \rangle}{\langle E^2 \rangle^2})$$

and for the imaginary part of the closest Fisher zero $z_0$. For these quantities finite-size scaling predicts the behaviors

$$\text{Im}(z_0) \sim L^{-\frac{1}{2}} \ ,$$

$$U_{\text{CLB,min}} \sim L^{\frac{d}{4}}.$$ (3.3)

The results of the respective fits are also listed in Table 1.

The values of $\nu$ obtained are seen to be close to the Gaussian value $\frac{1}{2}$, however, different from it. The observed increase of $\nu$ with $\lambda$ could indicate a nonuniversal behavior. Another possibility is that it is related to finite-size effects. Then the increase should disappear on much larger lattices. In that case the universal value of $\nu$ taken on the infinite lattice could even be the Gaussian one.

### Table 1

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\text{Im}(z_0)$</th>
<th>$C_V$</th>
<th>$U_{\text{CLB}}$</th>
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<tr>
<td>0.8</td>
<td>0.404(5)</td>
<td>0.433(2)</td>
<td>0.421(3)</td>
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<tr>
<td>0.9</td>
<td>0.446(5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>0.421(8)</td>
<td>0.467(2)</td>
<td>0.455(2)</td>
</tr>
</tbody>
</table>

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### REFERENCES

1. C. Roiesnel, hep-lat/9709081; B. Klaus and C. Roiesnel, hep-lat/9801036.
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