Searching for QCD–Instantons at HERA

A. Ringwald\textsuperscript{a} and F. Schrempp\textsuperscript{a}

\textsuperscript{a}DESY, Notkestr. 85, D-22603 Hamburg, Germany

We review the present status of our ongoing systematic study of the discovery potential of QCD-instanton induced events in deep-inelastic scattering at HERA.

1. INTRODUCTION

Instantons \cite{1}, fluctuations of non-abelian gauge fields representing topology changing tunneling transitions in Yang-Mills gauge theories, induce hard processes which are absent in conventional perturbation theory \cite{2}: In accord with the ABJ-anomaly, they violate certain fermionic quantum numbers, notably, chirality ($Q_5$) in (massless) QCD and baryon plus lepton number ($B+L$) in electro-weak interactions.

While implications of QCD-instantons, notably for long-distance phenomena, have been intensively studied for a long time, mainly in the context of the phenomenological instanton liquid model \cite{3} and of lattice simulations \cite{4}, the direct experimental verification of their existence is lacking up to now. Clearly, an experimental discovery of such a novel, non-perturbative manifestation of non-abelian gauge theories would be of basic significance.

The deep-inelastic regime is distinguished by the fact that here hard QCD-instanton induced processes may both be calculated \cite{5–7} within instanton-perturbation theory and possibly detected experimentally \cite{8–11}.

In this paper, we review the present status of our ongoing systematic study \cite{6–11} of the discovery potential of QCD-instanton induced events in deep-inelastic scattering (DIS) at HERA.

2. CROSS-SECTION ESTIMATES

The leading instanton ($I$)-induced process in the DIS regime of $e^\pm P$ scattering is displayed in Fig. 1. The dashed box emphasizes the so-called instanton-subprocess with its own Bjorken vari-

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{The leading instanton-induced process in the DIS regime of $e^\pm P$ scattering, violating chirality by $\Delta Q_5 = 2n_f$.}
\end{figure}

\[
Q^2 = -q'^2 \geq 0; \quad x' = \frac{Q'^2}{2p \cdot q'} \leq 1. \tag{1}
\]

The inclusive $I$-induced cross-section in unpolarized deep-inelastic $e^\pm P$ scattering can be expressed (in the Bjorken limit) as \cite{7}

\[
\frac{d\sigma^{(I)}_{eP}}{dx' dQ'^2} \sim \sum_{p',\bar{p}'} \frac{dL^{(I)}_{p'\bar{p}'}}{dx' dQ'^2} \sigma^{(I)}_{p'\bar{p}'}(x', Q'^2), \tag{2}
\]

where $p' = q', \bar{q}'$ denotes the virtual quarks entering the $I$-subprocess, with corresponding total cross-section $\sigma^{(I)}_{p'\bar{p}'}$ from the photon side and $p = q, \bar{q}, g$ denotes the target partons. The differential luminosity $dL^{(I)}_{p'\bar{p}'}$, accounting for the number of $p'\bar{p}'$ collisions per $eP$ collision, has a convolution-like structure \cite{8}, involving integrations over the target-parton density, $f_p$, the $\gamma^*$-flux, $P_{\gamma^*}$, and the known \cite{10} flux $P^{(1)}_{p'}$ of the parton $p'$ in the $I$-background.
In Eq. (2), the I-subprocess total cross-section $\sigma^{(I)}_{p'p}$ contains the essential instanton dynamics. We have evaluated the latter [7] by means of the optical theorem and the so-called $I(T)$-valley approximation [12] for the relevant $qg \rightarrow q'g$ forward elastic scattering amplitude in the $I(T)$ background. This method resums the exponentiating final state gluons in form of the known background. This method resums the exponential cut-off \[6\] forward elastic scattering amplitude in the $I(T)$ background.

Corresponding to the symmetries of the theory, the instanton calculus introduces at the classical level certain (undetermined) “collective coordinates” like the $I(T)$-size parameters $\rho(\vec{p})$ and the $I(T)$ distance $\sqrt{R^2/\rho}$ (in units of the size). Observables like $\sigma^{(I)}_{p'p}$ must be independent thereof and thus involve integrations over all collective coordinates. Hence, we have generically,

$$\sigma^{(I)}_{p'p} = \int_0^\infty d\rho D(\rho) \int_0^\infty d\vec{p} D(\vec{p}) \int d^4R \ldots \tag{3}$$

$$\times e^{-(\rho+\vec{p})Q'} e^{(p+p')} R e^{-\frac{\alpha_s}{\pi} S^{(I(T)}(\xi)}.$$

The first important quantity of interest, entering Eq. (3), is the $I$-density, $D(\rho)$ (tunnelling amplitude). It has been worked out a long time ago [2,13] in the framework of $I(T)$-perturbation theory: (renormalization scale $\mu_r$)

$$D(\rho) = d \left( \frac{2\pi}{\alpha_s(\mu_r)} \right)^6 \exp\left(-\frac{2\pi}{\alpha_s(\mu_r)}(\rho \mu_r)^b\right). \tag{4}$$

$$b = \beta_0 + \frac{\alpha_s(\mu_r)}{4\pi}(\beta_1 - 12\beta_0), \tag{5}$$

in terms of the QCD $\beta$-function coefficients, $\beta_0 = 11 - 2n_f, \beta_1 = 102 - \frac{36n_f}{4}$. In this form it satisfies renormalization-group invariance at the two-loop level [13]. Note that the large, positive power $b$ of $\rho$ in the $I(T)$ density (4) would make the integrations over the $I(T)$-sizes in Eq. (3) infrared divergent without the crucial exponential cut-off [6] $e^{-(\rho+\vec{p})Q'}$ arising from the virtual quark entering the $I$-subprocess from the photon side.

The second important quantity of interest, entering Eq. (3), is the $I(T)$-interaction, $S^{(I(T)} - 1$. In the valley approximation, the $I(T)$-valley action, $S^{(I(T)} = \frac{\alpha_s}{\pi} S[A^{(I(T)}]$ is restricted by conformal invariance to depend only on the “conformal separation”, $\xi = R^2/\rho |p|^2 + |\vec{p}|^2/\rho$, and its functional form is explicitly known [12]. It is important to note that, for all separations $\xi$, the interaction between $I$ and $T$ is attractive; in particular, the $I(T)$-valley action monotonically decreases from 1 at infinite conformal separation to 0 at $\xi = 2$, corresponding to $R^2 = 0, \rho = \vec{p}$.

The collective coordinate integration in the cross-section (3) can be performed via saddle-point techniques. One finds $R^2 = (\rho^* \sqrt{\xi^* - 2} = 0, \rho^* \equiv \vec{p}^*$, where the saddle-point solutions $\rho^*$ and $\xi^*$ behave qualitatively as

$$\rho^* \sim \frac{4\pi}{\alpha_s Q^*}; \quad \sqrt{\xi^* - 2} = \frac{R^2}{\rho^*} \sim 2 \sqrt{\frac{x'}{1 - x'}}. \tag{6}$$

Thus, the virtuality $Q'$ controls the effective $I(T)$-size: as one might have expected intuitively, highly virtual quarks probe only small instantons. The Bjorken-variable $x'$, on the other hand, controls the conformal separation between $I$ and $T$: for decreasing $x'$, the conformal separation decreases.

Our quantitative results [7] on the dominating cross-section for a target gluon, $\sigma^{(I)}_{qg}$, are shown in detail in Fig. 2, both as functions of $Q'^2$ (top) and of $x'$ (bottom). The residual dependence on the renormalization scale turns out [7] to be strongly reduced by using the two-loop renormalization-group invariant form of the $I$-density $D(\rho)$ from Eqs. (4) and (5). Intuitively one may expect [6,5] $\mu_r \sim 1/\rho \sim Q'/\beta_0 = \mathcal{O}(0.1)Q'$. Indeed, this guess turns out to match quite well our actual choice of the “best” scale, $\mu_r = 0.15 Q'$, determined by $\partial\sigma^{(I)}_{qg}/\partial\mu_r \approx 0$. The dotted curves in Fig. 2, indicating lines of constant $\rho^*$ (top) and of constant $R^*/\rho^*$ (bottom), nicely illustrate the qualitative relations (6) and their consequences: the $Q'$ dependence essentially maps the $I$-density, whereas the $x'$ dependence mainly maps the $I(T)$-interaction.

Fortunately, important information about the range of validity of $I(T)$-perturbation for the $I$-density and the $I(T)$-interaction, in terms of the instanton collective coordinates ($\rho \leq \rho_{max}, R/\rho \geq (R/\rho)_{min}$), can be obtained from recent (non-
perturbative) lattice simulations of QCD and translated via the saddle-point relations (6) into a “fiducial” kinematical region ($Q' \geq Q'_{\text{min}}, x' \geq x'_{\text{min}}$). In fact, from a comparison of the perturbative expression of the $I$-density (4) with recent lattice “data” [14] one infers [7] semi-classical $I$-perturbation theory to be valid for $\rho < \rho_{\text{max}} \approx 0.3$ fm. Similarly, it is found [7] that the attractive, semi-classical valley result for the $II$-interaction applies down to a minimum conformal separation $\xi_{\text{min}} \approx 3$, corresponding to $(R^*/\rho)_{\text{min}} \approx 1$. The corresponding “fiducial” kinematical region for our cross-section predictions in DIS is then obtained as

$$\rho^* \lesssim 0.3 \text{ fm};$$
$$R^*/\rho \gtrsim 1 \Rightarrow \begin{cases} Q' \geq Q'_{\text{min}} \approx 8 \text{ GeV}; \\ x' \geq x'_{\text{min}} \approx 0.35. \end{cases}$$

Fig. 3 displays the finalized $I$-induced cross-section at HERA, as function of the cuts $x'_{\text{min}}$ and $Q'_{\text{min}}$, as obtained with the new release “QCDINS 1.6.0” [11] of our $I$-event generator. For the following “standard cuts”,

$$C_{\text{std}} = x' \geq 0.35, Q' \geq 8 \text{ GeV}, x_BJ \geq 10^{-3}, 0.1 \leq y_BJ \leq 0.9,$$

involving the minimal cuts (7) extracted from lattice simulations, we specifically obtain

$$\sigma_{\text{HERA}}(C_{\text{std}}) = \mathcal{O}(100) \text{ pb}.$$  \hfill (9)

The main inherent uncertainties are discussed in Ref. [7]. With the total luminosity accumulated by experiments at HERA, $L = \mathcal{O}(80) \text{ pb}^{-1}$, there should be already $\mathcal{O}(10^4)$ $I$-induced events from the kinematical region (8) on tape. Note also that the cross-section quoted in Eq. (9) corresponds to a fraction of $I$-induced to normal DIS events of $f^{(I)}(C_{\text{std}}) = \mathcal{O}(1) \%$.

3. SEARCHES AT HERA

Thus, it seems to be a question of signature rather than a question of rate to discover $I$-induced scattering processes at HERA. Hence, we turn now to the final states of $I$-induced events in DIS.

In Fig. 4 we display the lego plot of a typical $I$-induced event at HERA, as generated by our Monte-Carlo generator QCDINS [9–11]. Its characteristics can be easily understood on the basis of the underlying $I$-subprocess:

The current quark in Fig. 1 gives rise, after hadronization, to a current-quark jet. The partons from the $I$-subprocess, on the other hand, are emitted spherically symmetric in the $p'/p$ c.m.
system. The gluon multiplicities are generated according to a Poisson distribution with mean multiplicity \( \langle n_g \rangle^{(I)} \sim \frac{1}{\alpha_s} \sim 3 \). The total mean parton multiplicity is large, of the order of ten. After hadronization we therefore expect from the \( I \)-subprocess a final state structure reminiscent of a decaying fireball: Production of the order of 20 hadrons, always containing strange mesons, concentrated in a “band” at fixed pseudorapidity \( \eta \) in the \( (\eta, \text{azimuth angle } \phi) \)-plane. Due to the boost from the \( p'p \) c.m. system to the HERA-lab system, the center of the band is shifted in \( \eta \) away from zero, and its width is of order \( \Delta \eta \simeq 1.8 \), as typical for a spherically symmetric event. The total invariant mass of the \( I \)-system, \( \sqrt{s'} = Q' \sqrt{1/x'} - 1 \), is expected to be in the 10 GeV range, for \( x' \simeq 0.35 \), \( Q' \simeq 8 \text{ GeV} \). All these expectations are clearly reproduced by our Monte-Carlo simulation.

These features have been exploited by experimentalists at HERA to place first upper limits on the fraction of \( I \)-induced events to normal DIS (nDIS) events, in a similar kinematical region as our standard cuts (8): From the search of a \( K^0 \) excess in the “band” region the H1 Collaboration could establish a limit of \( f_{\text{lim}}^{(I)} = 6 \% \), while the search of an excess in charged multiplicity yields \( f_{\text{lim}}^{(I)} = 2.7 \% \) [15]. The limit from the charged multiplicity distribution has been further improved in Ref. [16] to about 1 %. Thus, despite of the high rate of \( I \)-induced events at HERA, no single observable is known (yet) with sufficient nDIS rejection. A dedicated multi-observable analysis is required. However, it seems that a decisive search for \( I \)-induced events at HERA is feasible.

REFERENCES