The U(1) phase transition on toroidal and spherical lattices
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We have studied the properties of the phase transition in the U(1) compact pure gauge model paying special attention to the influence of the topology of the boundary conditions. From the behavior of the energy cumulants and the observation of an effective $\nu \sim 1/d$ on toroidal and spherical lattices, we conclude that the transition is first order.

We have focussed on the problem of the influence of the topology of the boundary conditions on the properties of the phase transition in the compact pure gauge U(1) model. Its most popular lattice formulation is obtained through the Wilson action:

$$S_W = \beta \sum_P [1 - \text{Re} \text{tr } U_P]$$

for which most of the numerical work has been done. Lattices as large as $L = 16$ have been simulated [1], finding two-state distributions from $L = 6$ on, increasing free energy gaps, and a $\nu$ exponent compatible with $1/d$ has been observed as well [1]. Altogether the transition is commonly believed to be first order, and since $\xi_c$ remains finite, no consequences for possible continuum limits should be expected.

Enlarging the parameter space by adding a term to the action in the adjoint representation

$$S_{EW} = S_W + \gamma \sum_P [1 - \text{Re} \text{tr } U_P^2]$$

does not seem to change qualitatively the first order scenario [5].

Mainly two issues remain still to be clarified in order to definitively discard the possibility of, despite all numerical evidence, having a continuous transition in the thermodynamical limit: 1) On the one hand, a complete stabilization of the latent heat has not been observed in numerical simulations, and hence the possibility of a thermodynamical limit where the observed two peaks superimpose is still open; 2) on the other hand, the role of non trivial topological structures appearing on finite toroidal lattices is a source of controversy since wrapping monopoles were conjectured to be responsible for the energy jump. Using in the simulations lattices homotopic to the sphere allows monopole loops to be contracted to a single point. Should this hypothesis be correct, no energy jump will be observed in the simulations on spherical lattices. Indeed this is the behavior observed in [2–4]. However, the spherical lattices constructed in those simulations are not homogeneous, and presumably, larger lattices should be needed to get rid of uncontrolled finite size effects.

We have performed a comparative study on the toroidal and spherical topologies to shed some light on both problems [5]. The spherical lattice is constructed by considering the surface of a 5D cube which is topologically equivalent to a 4D sphere [2,4]. On this 4D surface there is a number of sites with less than eight surrounding links, the homogeneity being only restored in the thermodynamical limit. To alleviate these inhomogeneities authors in [4] increase the contribution to the action of the inhomogeneous sites by an amount proportional to its lack of neighbors. We do not expect this smoothing to affect the existence of two states nor the Finite Size Scaling properties in large enough lattices.

We use the extended Wilson action (2) and define the plaquette energy in the usual way, $E_p = \frac{1}{N_p} (\sum_p \cos \theta_p)$, where $N_p$ denotes the number of plaquettes. On the torus $N_p = 6L^4$; on the
spherical lattice $N_p$ is not simply proportional to the number of sites, but it can be computed as a function of the base length, $N$, of the 5D cube, $N_p = 60(N - 1)^4 + 20(N - 1)^2$.

We typically perform trial runs to locate the peak of the specific heat, $\beta^*(L)$, where we perform a simulation to get the energy distribution, $P_E(\beta^*)_L$.

On the torus we worked at $\gamma = -0.1, -0.2$, in lattice sizes up to $L = 20$ and up to $L = 24$ at $\gamma = -0.3, -0.4$; on the sphere we first study the Wilson action ($\gamma = 0$) to check for the absence of two state signals claimed by authors in [2,4]. To compare with our results on the torus we also simulate at $\gamma = -0.2$.

From our simulations on the torus a two state signal together with an increasing free energy gap is revealed by the histograms for all lattice sizes, at all $\gamma$ values we simulate (see Figure 1). The lattice size at which the free energy gap starts appearing is larger the more negative $\gamma$ is, and the latent heat decreases as $\gamma$ gets more negative.

On the spherical lattice, at $\gamma = 0$ we find two state signals from $N = 12$ on. An increasing energy gap is observed when simulating $N = 14$, (see Figure 2) together with a volume increasing rate in the specific heat maximum. At $\gamma = -0.2$ the two state signal sets in at $N = 16$. Comparing with the toroidal lattices for which equivalent signals are observed ($L = 6$ at $\gamma = 0$; $L = 12$ at $\gamma = -0.2$) a first observation is the retard on the onset of double peak distributions on the sphere by a factor around 100 in volume.

From the energy distributions we compute the position of the nearby partition function zero closest to the real axis, whose scaling law allows to compute the $\nu$ exponent: $Im(\omega_0) \sim L^{-1/\nu}$. In order to monitorize the phase transition with increasing lattice size an effective exponent, $\nu_{eff}$, is computed following:

$$\nu_{eff} = -\frac{\ln L_2/L_1}{\ln(Im\omega_0(L_2)/Im\omega_0(L_1))}$$

(3)

On the torus (see Figure 3) the quasi-stabilization of the latent heat coincides with the falling of $\nu_{eff}$ from a value around 1/3 towards the first order value 1/d. This is the typical behavior expected for the effective exponents in weak first order phase transitions [6].

From the energy jump on finite lattices, $C_{lat}(L)$, we compute the latent heat on the thermodynamical limit, $C_{lat}(\infty)$, on the torus. On a lattice with periodic boundary conditions the FSS behavior of a $\xi$ dependent quantity, such as the latent heat, is expected to be controlled by the $\nu$ exponent with a law:

$$C_{lat}(L) = C_{lat}(\infty) + AL^{-1/\nu}$$

(4)

We get a $C_{lat}(\infty) \neq 0$ for all $\gamma$ we investigate.

On the sphere the behavior is not simple due
to the inhomogeneities. At $\gamma = 0$ a shifting of the peak in the Coulomb phase is observed. The explanation for this fact is the smaller contribution to the action of the sites with less than maximum connectivity. In the low $T$ region (Coulomb phase) their influence is stronger since the system is more ordered. As larger lattices are considered those sites contributions are less and less important, and the distance between the two peaks tends to the latent heat obtained by extrapolating the results obtained in the torus (see Figure 4). The conjectures about a possible superimposition of the two peaks in the thermodynamical limit becomes rather unplausible when the results on spherical lattices are taken into account, since both, the sphere and the torus share a common behavior in this limit.

To summarize, our results show a clear first order behavior in all the observables we have studied. The observation of an energy jump in the spherical lattice seems to rule out the conjectures about the influence of non trivial monopoles on such jump. It is worth remarking that simulations suppresing wrapping monopoles by other techniques were performed some time ago [7], their results showing the existence of an energy jump as well.

REFERENCES