In search of a scaling scalar glueball

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Anisotropic lattices are an efficient means of studying the glueballs of QCD, however problems arise with simulations of the lightest, scalar state. The mass is strongly dependent on the lattice spacing, even when a mean-field improved gluon action is used. The nature and cause of these errors are discussed and the scaling properties of the scalar from different lattice actions are presented.

1. THE SCALAR DIP

Efficient studies of the QCD glueball spectrum are made possible on coarse spatial lattices, provided the temporal discretisation is finer. This permits sufficient measurements of the glueball correlator before statistical fluctuations dominate [1]. These studies found good scaling for the $2^{++}$ and $1^{+-}$ glueballs. However for the lightest, scalar state, the mass (in units of $r_0$) was seen to fall rapidly to a minimum when the spatial lattice spacing is $\approx 0.25$ fm, where the scaling violations are $\approx 25\%$ and then rise as the lattice spacing is increased further; the “scalar dip”. This effect was also seen in $SU(2)$ simulations [2], although the dip is less pronounced.

The first question is whether Symanzik improvement of the action has worked. Fig. 1 shows the scaling of the scalar from both Wilson and improved simulations. In this figure, the mass of the $E$-irrep tensor glueball is used to set the scale as these mass ratios are independent of any anisotropy renormalisations. The two curves are results of fitting the datasets to a parabola. Fig. 1 demonstrates the improvement reduces the depth of the dip, but only by about a third.

We are examining a number of other explanations for this behaviour; the first is that using the plaquette definition for the mean-link parameters leads to an under-estimate of the effects of the tadpole graphs. Secondly, the influence of a critical end-point in a higher-dimensional space of couplings may be at fault.

In our preliminary study of the mean-link definition no significant improvement is observed. This is in contrast to findings for $SU(2)$ where the weaker scalar dip was reduced further by employing the Landau-link definition [2].

Simulations of QCD in the fundamental-adjoint space of plaquette couplings demonstrate the existence of a critical point close to the fundamental axis. As this point is approached, the mass of the scalar (in units of the string tension) has been shown to fall rapidly [3]. At the critical point, at least one lattice correlation length must diverge and this is seen to be the scalar channel. If this effect is responsible, addition of negatively coupled adjoint terms to the action might define a scaling trajectory where the influence of this critical point is diminished [4].

This solution ought to be independent of any Symanzik-improvement programme. To test this, we have added adjoint-like terms to the anisotropic Wilson action. This report discusses progress with these studies. Simulations of Symanzik-improved actions with these new terms are in progress.

2. DESIGNING NEW ACTIONS

Based on the observation of the nearby critical point, we consider one member of a class of new actions which include “adjoint-like” terms. The term we add is designed to be easy to simulate with existing update methods.
Beginning with the plaquette operator,

\[ P_{\mu\nu}(x) = \frac{1}{N} \text{ReTr} \, U_{\mu}(x) U_{\nu}(x + \hat{\mu}) U^{\dagger}_{\mu}(x + \hat{\nu}) U^{\dagger}_{\nu}(x) \]

the Wilson discretisations of the magnetic and electric field strengths are

\[ \Omega_s = \sum_{x,i>j} \{1 - P_{ij}(x)\} = \frac{\xi}{\beta} \int d^4x \, \text{Tr} \, B^2 + O(a^2) \]

and

\[ \Omega_t = \sum_{x,i} \{1 - P_{i0}(x)\} = \frac{1}{\xi \beta} \int d^4x \, \text{Tr} \, E^2 + O(a^2) \]

respectively, where \( i, j \) are spatial indices and \( \xi \) is the tree-level anisotropy, \( a_s/a_t \). We introduce a term which correlates pairs of spatial plaquettes separated by one site temporally

\[ \Omega_s^{(2t)} = \frac{1}{2} \sum_{x,i>j} \{1 - P_{ij}(x)P_{ij}(x + \hat{t})\} . \]

A similar term with spatially correlated plaquette pairs can be constructed but is not considered here.

The separation of the two plaquettes allows the standard Cabibbo-Marinari and over-relaxation gauge-field update methods to be applied. Including two-plaquette terms adds a computational overhead of just 10% to our anisotropic Wilson action workstation codes.

It can be shown that the operator combination

\[ \tilde{\Omega}_s = (1 + \omega) \Omega_s - \omega \Omega_s^{(2t)} \]

has an identical expansion in powers of \( a_{s,t} \) to \( \Omega_s \) up to \( O(a_{s,t}^4) \). Thus a signal for being closer to the QCD fixed point would be physical ratios becoming weakly dependent on the free parameter, \( \omega \).

A candidate action (without Symanzik improvement) is then

\[ S_\omega = \beta \left( \xi \, \Omega_t + \frac{1}{\xi} \, \Omega_s \right) . \]

Mean-link improvement is applied in the standard manner.

**2.1. Simulation results**

We have performed a range of simulations at a number of values of \( \omega \) and lattice spacings.
Fig. 2 shows the dependence of ratios of glueball masses on $\omega$ at fixed $\beta$ (= 2.7) for $S_\omega$. Close to $\omega = 0$ (anisotropic Wilson), these scaling ratios show a strong dependence on $\omega$, both in the scalar and pseudoscalar channels. In the range $\omega = 1$ to 3, the dependence is much weaker for all three channels, and the ratios are closer to their continuum values. The lattice spacing does not change significantly over this range of $\omega$; the lattice mass of the $E$ irrep varies by just 7%.

Fig. 3 compares the scaling properties of $S_\omega$ for two values of $\omega$ to the anisotropic Wilson action. The cut-off dependence in the scalar channel is seen to be reduced significantly. More data are required to resolve the continuum limit fully.

Fig. 4 shows the tensor irrep rotational symmetry violations and the scaling of the pseudoscalar glueball. Remarkably, the large scaling violations of the Wilson action in these channels are seen almost to vanish for $S_\omega$ with $\omega = 1$ or 2. This was not anticipated from critical point arguments.

3. CONCLUSIONS

By comparing data from anisotropic Wilson simulations to their Symanzik-improved counterparts, we have shown that improvement reduces the scaling violations of the scalar glueball, however large effects remain.

We found that adding to the action a new term which includes the product of two plaquette traces leads to significant reduction in the curious finite-lattice-spacing effects seen for the scalar glueball mass. Surprisingly, the pseudoscalar and $T_2$ tensor irrep masses (in units of the $E$ irrep mass) also show far less cut-off dependence. We emphasise that the action, $S_\omega$ used here is not Symanzik improved, and has leading discretisation errors at $O(a^2_{s,t})$.

We are performing simulations where two-plaquette terms are added to Symanzik-improved actions of the type considered in Ref. [1].

REFERENCES