The Center-Symmetric Phase of QCD.

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Abstract

An investigation of the center symmetric phase of SU(2) QCD is presented. The role of the center-symmetry, the dynamics of Polyakov loops and the structure of Abelian monopoles are studied within the axial gauge representation of QCD. Realization of the center symmetry is shown to result from non-perturbative gauge fixing and concomitant confinement like properties emerging even at the perturbative level are displayed. In an analysis of the Polyakov loop dynamics, non-perturbative gauge fixing is also shown to inevitably lead to singular gauge field configurations whose dynamics are briefly discussed.

1 Introduction

The center symmetry [1, 2, 3] distinguishes the phases of Yang Mills theories. This symmetry is realized in the confining and spontaneously broken in the deconfined phase with the Polyakov loops serving as order parameter. Unlike in studies of lattice QCD, the center symmetry has not been the subject of systematic analytical investigations of QCD. For instance, perturbative approaches in general imply a change of the underlying gauge symmetry from SU(N) to U(1)^N-1 when the coupling vanishes and thereby break the Z_N center symmetry. For the center symmetry to be preserved in the path integral formulation, the Faddeev–Popov determinant arising in the process of gauge fixing cannot be treated perturbatively. Likewise, for the center symmetry to be preserved in the canonical formalism, the Gauss law has to be resolved non-perturbatively. Only then the center symmetry is guaranteed to appear as the correct residual gauge symmetry. The process of non-perturbative gauge fixing unveils another fundamental and possibly far reaching property of the
formal structure of QCD. Unlike in QED, a global — for all field configurations valid — elimination of redundant variables is possible in QCD only at the expense of introducing coordinate singularities and thus of including singular gauge field configurations in such gauge fixed formulations [4]. Formation of Gribov horizons [5] or appearance of magnetic monopoles [6, 7] represent two prominent examples of the occurrence of singular field configurations as a result of non-perturbative gauge fixing. It is thus tempting to connect the realization of the center symmetry with the emergence of singular field configurations and to identify these non-perturbative basic structures as the origin of the characteristic properties of the confining phase of QCD. I will present a study of the role of the center symmetry and the structure and dynamics of monopole like singular field configurations in gauge fixed QCD. This discussion summarizes the results of a series of investigations of QCD in axial gauge [8, 9, 10, 11].

For the formulation of the center symmetry and definition of the Polyakov loops, QCD has to be considered in a geometry where the system is of finite extent \((L)\) in one direction \((x_3)\), and in general of infinite extent in the other directions. For finite temperature QCD, one has to choose the time direction to be compact and the associated fields to be periodic (gauge fields) or antiperiodic (quark fields). In the following discussion, the space-like 3-direction is assumed to be compact. In this way, the center symmetry will appear as an ordinary symmetry represented canonically by an operator which commutes with the Hamiltonian. Such a standard interpretation of the center symmetry is not possible for finite temperature QCD and conceptual difficulties arise [12] concerning for instance the existence of domain walls. By covariance, QCD at finite (spatial) extension is equivalent to finite temperature QCD. By rotational invariance in the Euclidean, the value of the partition function of a system with finite extension \(L\) in 3 direction and \(\beta\) in 0 direction is invariant under the exchange of these two extensions,

\[
Z (\beta, L) = Z (L, \beta)
\]

 provided standard boundary conditions in both time and 3 coordinate are imposed on the fields. As a consequence of (1), energy density and pressure are related by

\[
\epsilon (\beta, L) = -p (L, \beta)
\]

For a system of non-interacting particles this relation connects energy density or pressure of the Stefan Boltzmann law with the corresponding quantities measured in the Casimir effect.

In QCD, covariance also implies by Eq. (2) that at zero temperature a confinement-deconfinement transition occurs when compressing the QCD vacuum (i.e. decreasing \(L\)). From lattice gauge calculations [13] it can be inferred that this transition occurs at a critical extension \(L_c \approx 0.8\) fm in the absence of quarks and at \(L_c \approx 1.3\) fm when quarks are included. For extensions smaller than \(L_c\), the energy density and pressure reach values which are typically 80 % of the corresponding “Casimir” energy and pressure. When compressing the system beyond the typical length scales
of strong interaction physics, correlation functions at transverse momenta or energies \(|p| \ll 1/L\) are dominated by the zero “Matsubara wave-numbers” in 3-direction and, as confirmed by lattice QCD calculations \([14]\), are given by the dimensionally reduced QCD\(_{2+1}\).

2 Center Symmetry

The order parameter which characterizes the phases of QCD is the vacuum expectation value of the trace of the Polyakov loop operator at finite temperature \([16]\) and correspondingly of the operator

\[
W(x_{\perp}) = P \exp \left\{ ig \int_0^L dx^3 A_3(x) \right\}
\]

at finite extension \((x_{\perp} = (x_0, x_1, x_2))\). I will refer in the following also to \(W\) as the Polyakov loop operator. Under gauge transformations \(U(x), W(x_{\perp})\) transforms as

\[
W(x_{\perp}) \rightarrow U(x_{\perp}, L) W(x_{\perp}) U^\dagger(x_{\perp}, 0).
\]

The coordinates \(x = (x_\perp, 0)\) and \(x = (x_\perp, L)\) describe identical points, and we require the periodicity properties imposed on the field strengths not to change under gauge transformation. This is achieved if \(U\) satisfies

\[
U(x_{\perp}, L) = c_U \cdot U(x_{\perp}, 0)
\]

with \(c_U\) being an element of the center of the group. Thus gauge transformations can be classified according to the value of \(c_U\) \((\pm 1 \text{ in SU}(2))\). Therefore under gauge transformations

\[
\text{tr}(W(x_{\perp})) \rightarrow \text{tr}(c_U W(x_{\perp})) \overset{\text{SU}(2)}{=} \pm \text{tr}(W(x_{\perp})).
\]

A simple example of an SU(2) gauge transformation \(u_-\) with \(c = -1\) is

\[
u_-= e^{-i\pi \hat{\psi} x_3/L}, \quad c_{u_-} = -1.
\]

with the arbitrary unit vector \(\hat{\psi}\). Its effect on an arbitrary gauge field is

\[
A_{\mu}^{u_-} = e^{i\pi \hat{\psi} x_3/L} A_{\mu} e^{-i\pi \hat{\psi} x_3/L} - \frac{\pi}{gL} \hat{\tau} \hat{\psi} \delta_{\mu 3}.
\]

This representative \(u_-\) can be used to generate any other gauge transformation changing the sign of \(\text{tr}(W)\) by multiplication with a strictly periodic \((c = 1)\) but otherwise arbitrary gauge transformation. The decomposition of SU(2) gauge transformations into two classes according to \(c = \pm 1\) implies a decomposition of each gauge orbit \(O\), into sub-orbits \(O_{\pm}\) which are characterized by the sign of the Polyakov loop

\[
A(x) \in O_{\pm}, \quad \text{if} \quad \pm \text{tr}(W(x_{\perp})) \geq 0.
\]
Thus strictly speaking, the trace of the Polyakov loop is not a gauge invariant quantity. Only $|\text{tr}(W(x_{\perp})|)$ is invariant under all of the gauge transformations. Furthermore, the spontaneous breakdown of the center symmetry as it supposedly happens at small extension or high temperature is a breakdown of the underlying gauge symmetry. It implies that the wave functional describing such a state is different for gauge field configurations which belong to $\mathcal{O}_+$ and $\mathcal{O}_-$ respectively, and which therefore are connected by gauge transformations such as $u_-$ in Eq.(7). These considerations are also of relevance for understanding the structure of gauge fixed theories. Whenever gauge fixing is carried out exactly and with the help of strictly periodic gauge fixing transformations ($\Omega, c_\Omega = 1$) the resulting formalism must contain the center symmetry

$$\text{tr}(W(x_{\perp})) \rightarrow -\text{tr}(W(x_{\perp})). \hspace{1cm} (10)$$

as residual gauge symmetry. In other words, gauge fixing via strictly periodic gauge transformations does not lead to a complete gauge fixing. Each gauge orbit is represented by two gauge field configurations. This could be circumvented by allowing for more general gauge fixing transformations which are periodic only up to a center element, i.e. by including gauge fixing transformations with $c_\Omega = -1$.

In the following we will carry out explicitly a gauge fixing procedure and represent QCD in the axial gauge. As will be seen, the gauge fixing leading to axial gauge is incomplete in the above sense and will therefore exhibit the center symmetry as a residual gauge symmetry. This gauge is of particular relevance for properties related to the center symmetry, since the associated order parameter, the Polyakov loops appear as elementary rather than composite degrees of freedom. For carrying out the gauge fixing procedure the following transformation will be useful

$$v_- = \Omega_D^\dagger (x_{\perp}) e^{-i\pi x_3/L} e^{i\pi \tau_3/2} \Omega_D (x_{\perp}) \hspace{1cm} (11)$$

where $\Omega_D$ diagonalizes the Polyakov loop

$$\Omega_D (x_{\perp}) W(x_{\perp}) \Omega_D^\dagger (x_{\perp}) = e^{ig L a_3 (x_{\perp}) + \pi \tau_3}. \hspace{1cm} (12)$$

By including $\Omega_D$ in the definition of $v_-$, the color 3 direction and the color direction of the Polyakov loop coincide.

### 3 QCD in Axial Gauge

At this point we pass to a gauge fixed formulation by applying the gauge fixing transformation

$$\Omega(x) = \Omega_D (x_{\perp}) (W^\dagger (x_{\perp}))^{x_3/L} P \exp \left\{ ig \int_0^{x_3} dz A_3 (x_{\perp}, z) \right\}. \hspace{1cm} (13)$$

in which the axial gauge is reached in 3 steps [8]. In the presence of the third factor only, the gauge transformation would eliminate $A_3$ completely. In order to preserve
the periodic boundary conditions of the gauge fields the second term reintroduces zero mode fields which in turn are diagonalized by \( \Omega_D \). Thus the gauge condition reads

\[
\Omega(x) \left( A_3(x_\perp) + \frac{1}{i g} \partial_3 \right) \Omega^\dagger(x) = (a_3(x_\perp) + \frac{\pi}{gL}) \tau_3. \tag{14}
\]

By the gauge transformation, the 3 component of the gauge field is transformed to zero apart from the eigenvalues of the Polyakov loops. The elementary rather than composite nature of the Polyakov loop variables \( a_3(x_\perp) \) in axial gauge is manifest.

The gauge fixing transformation \( \Omega \) is periodic and consequently field configurations which before gauge fixing are related by a gauge transformation with \( c = -1 \) are not identified. Therefore center symmetry transformations appear as residual symmetry transformations of the gauge fixed theory. By construction, these symmetry transformations are the gauge fixed transformations of Eq.(11)

\[
C = \Omega(x) v^- \Omega^\dagger(x) = e^{-i\pi\tau_3 x_3/L} e^{i\pi\tau_1/2}. \tag{15}
\]

The effect of \( C \) on an arbitrary gauge field is most conveniently written in a spherical color basis

\[
\Phi_\mu(x) = \frac{1}{\sqrt{2}} (A_\mu^1(x) + i A_\mu^2(x)) e^{-i\pi x_3/L}, \tag{16}
\]

as

\[
C : \ a_3 \rightarrow -a_3 , \ A_3^\mu \rightarrow -A_3^\mu , \ \Phi_\mu \rightarrow \Phi_\mu^\dagger , \ (\mu \neq 3). \tag{17}
\]

The center symmetry transformation \( C \) acts as (Abelian) charge conjugation with the ”photons” described by the neutral fields \( A_3^\mu(x), a_3(x_\perp) \). For identification of the center symmetry with charge conjugation symmetry, the shift in the definition of the Polyakov loop variables in Eq.(12), the rotation around the 1-axis in Eq. (11) as well as the shift in phase in the definition of the charged fields (Eq.(16)) have been introduced. As will be seen shortly, this definitions will also simplify the description of the dynamics. The phase change in Eq.(16) makes the charged fields antiperiodic

\[
\Phi_\mu(x_\perp, x_3 = L) = -\Phi_\mu(x_\perp, x_3 = 0). \tag{18}
\]

If the center symmetry is realized \( gLa_3(x_\perp) \) has to be distributed symmetrically around the origin. As will be seen below, other variables exist in axial gauge which can be used as order parameters of the realization of the charge symmetry \( C \).

Apart from the discrete center symmetry transformation described by the charge conjugation \( C \), all other symmetries related to the gauge invariance have been used to eliminate \( A_3 \). In such a case of a global, non-perturbative gauge fixing we have to expect, as argued above, singular field configurations to emerge. In transforming to the axial gauge, diagonalization of the Polyakov loops (\( \Omega_D \) in Eq.(13)) is the crucial step of the gauge fixing procedure, in which such singular gauge field configurations appear. This diagonalization can be viewed as choice of coordinates in color space in which the color 3-direction is identified with the direction of the Polyakov loop. As is
evident from Eq.(12), this choice of coordinates becomes ambiguous if \( gL a_3 (x^N, S) = \pm \pi \), i.e. if the Polyakov loop is in the center of the group

\[
W(x^N, S) = \pm \mathbb{1}.
\]  

(19)

This requirement determines a point on the group manifold \( S^3 \) and thus, for generic cases, fixes (locally) uniquely the position \( x^N, S \). At these points, the gauge transformed field

\[
A'_\mu (x) = \Omega_D (x) A_\mu (x) \Omega_D^\dagger (x) + s_\mu (x), \quad \mu \neq 3
\]

(20)

with

\[
s_\mu (x) = \Omega_D (x) \frac{1}{ig} \partial_\mu \Omega_D^\dagger (x),
\]

(21)

in general, is singular with \( \Omega_D \).

All the elements are now available for writing down the central result of our investigations, the expression for the axial gauge QCD partition function

\[
Z = \sum_n Z_n = \sum_n \int D[a_3^n] \int \prod_{\mu \neq 3} D[A_\mu] e^{-S[A + s, a_3^n]}.
\]

(22)

The integration variables, the unconstrained degrees of freedom, are the 3 components of the gauge field \( (A_\mu (x), \mu \neq 3) \) and the eigenvalues of the Polyakov loops. The integration over these eigenvalues has been decomposed according to the number \( n = (n_N, n_S) \) of north \( (n_N) \) and south \( (n_S) \) pole singularities; i.e., the path integral in \( Z_n \) is performed over field configurations in which the Polyakov loop passes \( n_{N,S} \) times through north and south pole respectively. For this decomposition to be meaningful, regularization of the generating functional is required. The singular field \( s(x) \) is determined by the Polyakov loop variables

\[
s = s [a_3^n].
\]

(23)

4 Dynamics in Axial Gauge QCD.

We will display the dynamical content of the above expression for the generating functional by discussing a hierarchy of approximations to \( Z \) with increasing complexity.

1. The QCD generating functional in the naive axial (or temporal) gauge is obtained if only the sector without singularities is kept and the dependence on the eigenvalues of the Polyakov loops is disregarded. As a consequence of these approximations, the generating functional becomes actually ill-defined as has been noticed by Schwinger 35 years ago [15]. In definition of propagators certain “\( \epsilon \)” prescriptions have to be applied. Due to the approximations, the center-symmetry is not present anymore.
2. Still, keeping the zero singularity sector only one might proceed by accounting for the dependence of \( Z \) on \( a_3 \). The simplest form of these dynamics results, if these variables are treated as Gaussian variables, i.e. if the non-flat measure

\[
d[a_3] = \prod_{y\perp} \cos^2 \left( gL a_3(y\perp) / 2 \right) \Theta \left( (\pi/gL)^2 - a_3^2(y\perp) \right) da_3(y\perp)
\]

is replaced by the flat measure \( da_3 \). In this way, one effectively treats the Polyakov loop eigenvalues as the zero modes in QED. It is therefore not surprising that the center-symmetry is lost again and Debye screening like in QED [17] is obtained.

3. First characteristic properties of QCD are encountered if, still in the absence of singular field configurations, the non-flat measure of the Polyakov loop variables is properly taken into account. These properties will be the subject of the following section. In particular, the perturbative phase reached in this way will be seen to be center-symmetric.

4. The role of singular field configurations in the \( n \neq 0 \) sectors (cf.Eq.(22)) is very poorly understood. In particular it has not been possible so far to identify those sectors which dominate the partition function nor has the dynamics of the quantum fluctuations around singular fields been studied systematically. Nevertheless, basic and well understood properties of QCD permit a certain indirect characterization of the dynamics in these sectors as will be discussed in the concluding section.

4.1 Polyakov Loop Dynamics

In this subsection we sketch the dynamics in the sector where no singularities are present. Unlike in more standard approaches, the non-Gaussian nature of the Polyakov loop variables \( a_3(x\perp) \) is explicitly taken into account and the finite limit of integration associated with these variables is respected [10]. We first consider the Polyakov loop dynamics in the absence of coupling to the other degrees of freedom. The corresponding generating functional is, in the Euclidean, given by

\[
Z_0 = \int d[a_3] \exp \left\{ -1/2 \int d^4x (\partial_\mu a_3(x\perp))^2 \right\} = \int_{-\pi/2}^{\pi/2} \prod_{x\perp} d\tilde{a}_3(x\perp) \cos^2 \tilde{a}_3(x\perp) \exp \left\{ -2\ell g^2 L \sum_{y\perp,\delta\perp} \left( \tilde{a}_3(y\perp + \delta\perp) - \tilde{a}_3(y\perp) \right)^2 \right\}.
\]

Transverse space time has been discretized with \( \ell \) and \( \delta\perp \) denoting lattice spacing and lattice unit vectors respectively and the Polyakov loop variables have been rescaled

\[
\tilde{a}_3(x\perp) = gL a_3(x\perp) / 2.
\]
In the continuum limit,

\[ \frac{\ell}{g^2 L} \sim \frac{1}{L \ln \frac{\ell}{L}} \to 0 , \tag{26} \]

and therefore the nearest neighbor interaction generated by the Abelian field energy of the Polyakov loop variables is negligible. As a consequence, in the absence of coupling to other degrees of freedom, Polyakov loops do not propagate,

\[ \langle \Omega | T (a_3 (x_\perp) a_3 (0)) | \Omega \rangle \sim \left( \frac{\ell}{g^2 L} \right)^{x_\perp/\ell} \to \delta^3 (x_\perp) . \tag{27} \]

Although the above procedure is similar to the strong coupling limit in lattice gauge theory, here a strong coupling approximation has not been invoked. In the lattice dynamics of single links, the factor \( \frac{1}{g^2} \) appears in the action and, as a consequence, continuum limit and strong coupling limit describe two different regimes of the lattice theory. In the Polyakov loop dynamics on the other hand which is controlled by the factor \( \frac{\ell}{g^2 L} \), strong coupling and continuum limit coincide. Propagation of excitations induced by \( a_3 (x_\perp) \) can consequently only arise by coupling to the other microscopic degrees of freedom. Formally this suggests the Polyakov loop variables \( a_3 \) to be integrated out by disregarding the contribution of the free \( a_3 \) action, but keeping the coupling to the other degrees of freedom. In this way, the following effective action is obtained

\[ S_{\text{eff}} [A_\mu] = S_{\text{YM}} [A_\mu, A_3 = 0] + S_{\text{gf}} \left[ \int_0^L dz A_3^\mu \right] + M^2 \int d^4 x \, \Phi^\dagger_\mu (x) \Phi_\mu (x). \tag{28} \]

The Polyakov loop variables have left their signature in the geometrical mass term of the charged gluons (cf. Eq.(16))

\[ M^2 = \left( \pi^2 / 3 - 2 \right) / L^2 \tag{29} \]

and in the antiperiodic boundary conditions (Eq.18). The neutral gluons remain massless and periodic. The antiperiodic boundary conditions reflect the mean value of the Polyakov loop variables, the geometrical mass their fluctuations; notice that in both of these corrections, the coupling constant has dropped out. I emphasize that periodic boundary conditions for the gluon fields are imposed in the representation (22) of the generating functional. The antiperiodic boundary conditions in (18) describe the appearance of Aharonov-Bohm fluxes in the elimination of the Polyakov loop variables. Periodic charged gluon fields may be used if the differential operator \( \partial_3 \) is replaced by

\[ \partial_3 \to \partial_3 + \frac{i \pi}{2L} [\tau_3] . \tag{30} \]

As for a quantum mechanical particle on a circle, such a magnetic flux is technically most easily accounted for by an appropriate change in boundary conditions – without changing the original periodicity requirements. With regard to the rather
unexpected physical consequences, the space-time independence of this flux is important, since it induces global changes in the theory. These global changes are missed if Polyakov loops are treated as Gaussian variables.

The role of the order parameter is taken over by the neutral color current in 3-direction $u(x_\perp)$ which is generated by the 3-gluon interaction

$$u(x_\perp) = i \int_0^L dx_3 \Phi_\mu^\dagger(x) \partial_{3}\Phi^\mu(x).$$

This composite field is odd under charge conjugation (cf.(17))

$$C: \quad u(x_\perp) \rightarrow -u(x_\perp).$$

It determines the vacuum expectation value of the Polyakov loops

$$\langle \Omega|W(x_\perp)|\Omega \rangle \propto \langle \Omega|u(x_\perp)|\Omega \rangle$$

and the corresponding correlation function

$$\langle \Omega|T[\{W(x_\perp)W(0)\}u(x_\perp)]\Omega \rangle \propto \langle \Omega|T[\{u(x_\perp)u(0)\}u(x_\perp)]\Omega \rangle$$

which in turn yields the static quark-antiquark interaction energy [16]. Up to an irrelevant factor we have after rotation to the Euclidean ($r = |x_\perp|$)

$$\exp\left\{-LV(r)\right\} = \langle \Omega|T[u(x_\perp)u(0)]\Omega \rangle,$$

i.e., the static quark-antiquark potential is given directly by (the $a = b = 3, \mu = \nu = 3$ component of) the vacuum polarization tensor $\Pi^{\mu\nu}$ and not by the zero mass propagator with corresponding self-energy insertions as obtained in the standard Gaussian treatment. This remarkable consequence of the ultralocality property (27) of the Polyakov loop variables provides a direct connection between confinement and certain spectral properties of gluonic states. If, as required in the center symmetric phase, the vacuum expectation value of the Polyakov loop operator vanishes and if the spectrum of states excited by $u$ exhibits a gap $\Delta E$, Eq.(35) implies a linear rise in $V$ for large separations

$$V(r) \rightarrow \sigma r = \Delta Er/L.$$

Thus in axial gauge, confinement is connected to a shift in the spectrum of gluonic excitations to excitation energies

$$E \geq \sigma L$$

which diverges with the extension L becoming infinite. Comparison with the interaction energy of adjoint static charges suggests the negative charge conjugation parity (cf.Eq.(32)) of the intermediate "2-gluon" states contributing to $V$ in Eq.(35) to be the distinctive property which leads to infinite excitation energies.
The system described by the effective action (28) exhibits remarkable properties already at the perturbative level. Most importantly the center symmetry is realized in the perturbative vacuum, i.e. in the ground state obtained by dropping all the terms containing the coupling constant \( g \). Geometrical mass (Eq.(29)) and Aharonov-Bohm flux (Eq.(30)) are not affected by such a perturbative treatment. The perturbative ground-state is even under charge conjugation and the expectation value of the Polyakov loop vanishes

\[
\langle \Omega_{pt} | W(x) | \Omega_{pt} \rangle = 0,
\]

indicating an infinite free energy of a static quark. Indeed perturbative analysis of the correlation function (35) yields a linearly rising static interaction energy. However the perturbative string tension decreases with increasing extension \( \propto L^{-2} \).

The change from this value of the string tension to the physical one together with the emergence of the proper QCD scale is beyond a perturbative treatment also after elimination of the Polyakov loop variables. The perturbative vacuum shares with the QCD vacuum certain properties also after including dynamical quarks. In particular, application of perturbation theory shows the interaction energy of static quarks to cease to rise indefinitely and to be given at asymptotic separations by the non-perturbative value of twice the mass of the dynamical quarks. For small distances, Coulomb-like behavior must emerge if the separation is small on the scale of \( \Lambda_{QCD} \) and small in comparison with the extension \( L \). This is possible only, if the vacuum polarization tensor possess an essential singularity at infinite momentum

\[
\int d^3x e^{ipx} \langle \Omega | T [u(x) u(0)] | \Omega \rangle \rightarrow e^{-\sqrt{g^2Lp/\pi}}.
\]

Obviously, finite order perturbation theory cannot yield such a singularity; it however can be shown that, with increasing order in \( g \), increasingly high powers of \( pL \) appear; two loop evaluation of the short distance behavior indicates exponentiation.

The perturbative phase with its signatures of confinement cannot be relevant for QCD at extensions smaller than \( L_c \). Not only do we expect the center symmetry to be broken at small extensions but also dimensional reduction to QCD_{2+1} to happen. Due to the antiperiodic boundary conditions, charged gluons decouple from the low-lying excitations if dimensional reduction takes place in the center symmetric phase. The small extension or high temperature limit of the center symmetric phase is therefore QED_{2+1}. In order to reach the correct high temperature phase, the deconfinement phase-transition arising when compressing the QCD vacuum, must be accompanied by a change to periodic boundary conditions and simultaneously the geometrical mass must disappear. Connected with this change in the charged gluon boundary condition is a change in Casimir energy density and pressure which for non-interacting gluons (and neglecting the effects of the geometrical mass) is given by

\[
\Delta \epsilon = -\pi^2/12L^4, \quad \Delta p = 3\Delta \epsilon.
\]
This estimate is of the order of magnitude of the change in the energy density across the confinement-deconfinement transition when compressing the system,
\[ \Delta \epsilon = -0.45/L^4, \]
deduced from the finite temperature lattice calculation of Ref. [18].

4.2 Axial Gauge Monopoles

In this concluding section I will characterize qualitatively the structure of the singular field configurations arising in the gauge fixing procedure and address some of the related dynamical issues (cf.[11]). For the following discussion it is convenient to identify, after a rotation to the Euclidean, time with \( x_3 \). In this way the singular fields (cf. Eq.(21)) are static magnetic fields. A simple example of a singular field is that of a Dirac like monopole configuration given by
\[ s(x) = \frac{m}{2g} \left[ -\frac{1}{2} + \cos \theta \hat{\phi} \tau_3 + ((\hat{\phi} + im\hat{\theta})e^{\pm i\phi} \tau_+ + \text{h.c.}) \right], \quad m = \pm 1. \] (42)

Here, vectors denote (after rotation) the spatial components (0,1,2). \( \hat{\phi}, \hat{\theta} \) are azimuthal and polar unit vectors. The neutral component (\( \propto \tau_3 \)) of the singular field \( s(x) \) in Eq. (42) is exactly the vector potential of a Dirac monopole [19] of charge \( 2\pi m/g \), with associated magnetic field
\[ b^3 = \text{rot} s^3 = \frac{m}{2g} \frac{x}{x^3}, \] (43)

and is accompanied by a singular charged field component (\( \propto \tau_\pm \)). The singularity structure of the Dirac monopole configuration is not the most general one. In addition to the longitudinal vector field \( b^3 \), singular transverse magnetic fields are also present whose strength is determined dynamically and not quantized by topological requirements.

In 4-space, the transformed gauge fields are singular on straight lines parallel to the time (3)-axis, and thus represent static singular magnetic fields. The static nature of the singularities is a trivial consequence of the static Polyakov loop which has been selected for introducing coordinates in color space. North and south pole singularities are distinguished by the value of the Polyakov loop (cf.Eq.(19)). In addition to poles, the field \( s(x) \) also exhibits (static) string like singularities representing surfaces in 4-space. The singular neutral magnetic field \( b^3 \), is the central quantity in Abelian projected theories. The complete non-Abelian magnetic field strength built from the inhomogeneous term of Eq. (21) and generated by a gauge transformation of an everywhere regular gauge field cannot be singular and vanishes,
\[ F_{ij}[s] = \partial_i s_j - \partial_j s_i + ig [s_i, s_j] = 0, \] (44)
since $s$ is a pure gauge. “Abelian” magnetic monopoles have vanishing magnetic field energy. Finally I mention the connection between axial gauge monopoles and instantons. As is easily verified, the Polyakov loop of a single instanton of size $\rho$ ($\rho \ll L$) is given by

$$W(x) = e^{i\pi\tau x/\sqrt{x^2 + \rho^2}}$$

which shows that a single instanton contains a north and south pole singularity at its center and at infinity respectively. More generally it can be shown that the topological charge $\nu$ of a field configuration is given by the difference of the net northern and southern charge

$$\nu = \frac{1}{2} \left( \sum_{W(x_i) = 1} m_i - \sum_{W(x_i) = -1} m_i \right).$$

On the basis of the connection between monopole formation and order parameter and using the link between monopoles and instantons, the dynamics of axial gauge monopoles can, to some extent, be characterized. Condensation of monopoles is implied via Eq.(46) by results of the instanton liquid model [20] and of lattice QCD [21] which suggest a finite instanton density in the QCD vacuum. However since it also appears that instantons are not able to account for confinement [22] monopole condensation itself does not appear to be sufficient to induce the dual Meissner effect. This is reminiscent of the difference in the response of a plasma and a superconductor to a static external magnetic field. Obviously, instantons with their rigid correlation between north and south pole singularities give rise to a very particular mode of condensation. Decoupling of the singularities seems to be necessary for generating the confined phase with a symmetric distribution of north and south poles as required by the center symmetry. Beyond the deconfinement transition condensation of axial gauge monopoles must be expected to persist. With the center symmetry broken, the Polyakov loop is not distributed symmetrically around the equator of $S^3$. It rather approaches more and more either the north or the south pole with increasing temperature. An expectation value $W(x_\perp) \neq \pm 1$ in the infinite temperature limit is neither compatible with the Stefan-Boltzmann law [9] nor, as argued above, with the expected dimensional reduction to 2+1 dimensional QCD. Thus, as the Polyakov loop approaches one of the poles, the probability to pass through this pole and therefore the monopole density must be expected to increase. On the other hand, for this increased density to be compatible with perturbation theory and, in particular, not to lead to confinement, one might expect poles and antipoles to compensate each other to a large extent. This would be the case if poles and antipoles are strongly correlated with each other. We thus expect the high-temperature phase to consist of a gas of magnetic dipoles and the deconfinement-confinement transition to be similar to the phase transition in the 2-dimensional XY model which occurs by vortex (monopole) unbinding.
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